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STOCHASTIC PARABOLIC SYSTEMS WITH VARIABLE EXPONENTS OF NONLINEARITY

Some nonlinear parabolic systems of partial differential equations with the white noise is considered. The initial-boundary value problem for a system is investigated and the existence and uniqueness of the weak solution for the problem are proved.

Key words: *partial differential equations, stochastic parabolic system, variable exponent of nonlinearity, white noise, weak solution.*

Introduction. Let $n, N \in \mathbb{N}$ and $T > 0$ be some fixed numbers, $\Omega \subset \mathbb{R}^n$ be a bounded domain with the smooth boundary $\partial\Omega$, $(\mathbb{S}, \mathcal{F}, \mathbb{P})$ be a complete probability space, where $\mathbb{S}$ is a non-empty set, which is interpreted as a space of states or elementary events, $\mathcal{F}$ is a σ-algebra of the subsets of $\mathbb{S}$, and $\mathbb{P}: \mathcal{F} \to \mathbb{R}$ is a probability measure. Let $Q_{0,T} = \Omega \times (0,T)$, $\Pi_{0,T} = \Omega \times (0,T) \times \mathbb{S}$, $\Theta_{0,T} = (0,T) \times \mathbb{S}$. We intend to find a vector-valued function $u = (u_1, \ldots, u_N) : \Pi_{0,T} \to \mathbb{R}^N$ that is dependent on the deterministic variables $x \in \Omega$ and $t \in (0,T)$ and the random variable $\omega \in \mathbb{S}$ such that

$$u_{k,t} + \alpha\Delta^2 u_k - \sum_{i=1}^{n} (a_{ik}(x,t)|u_{x_i}|^{p(x,t)-2} u_{k,x_i})_{x_i} + (\hat{N}u)_k(x,t,\omega) + \varphi_k((\hat{E}u)_k(x,t,\omega)) = F_k(x,t,\omega) + b_{k,t}(x,t,\omega),$$
$$(x,t,\omega) \in \Pi_{0,T}, \quad k = 1,2,\ldots,N, \tag{1}$$

$$u(x,t,\omega) = \Delta u(x,t,\omega) = 0, \quad x \in \partial\Omega, \quad (t,\omega) \in \Theta_{0,T}, \tag{2}$$

$$u(x,0,\omega) = u_0(x,\omega), \quad x \in \Omega, \quad \omega \in \mathbb{S}, \tag{3}$$

where $\alpha > 0$ is a number, $\Delta = \partial^2/\partial x_1^2 + \ldots + \partial^2/\partial x_n^2$ is the Laplacian, $\Delta^{s+1} = \Delta(\Delta^s)$, $s \in \mathbb{N}$,

$$(\hat{N}u)_k(x,t,\omega) := g_k(x,t)|u(x,t,\omega)|^{q(x,t)-2} u_k(x,t,\omega),$$

$$(\hat{E}u)_k(x,t,\omega) := \int_{\Omega} \mathfrak{Z}_k(x,t,y) u_k(y,t,\omega)\, dy, \quad (x,t,\omega) \in \Pi_{0,T},$$

a_{ik}, g_k, φ_k, $\mathfrak{Z}_k$, F_k, u_0, b_k, p, q are some functions, $i = 1,2,\ldots,n$, $k = 1,2,\ldots,N$.

We will prove the unique solvability of problem (1)–(3). This work continues the research of [1], where some semilinear stochastic parabolic equations with the variable exponent of nonlinearity are considered. The main conclusion in [1] was based on the methodology shown in [4, 12], that used

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multivalued functions and their selectors. Similar results for parabolic equations with the constant exponents of the nonlinearity were obtained in [10, 13]. Stochastic problems with the variable exponent of the nonlinearity in a formulation different from ours have been studied in [3, 14]. Nonlinear non-stochastic partial differential equations with the variable exponent of the nonlinearity have been studied, e.g., in [2, 5−9].

In contrast to [1], in this article we make significant use of the uniqueness of the solution to problem (1)−(3) without the random parameter $\omega \in \mathbb{S}$ and the white noise component $b_{k,t}$. Problem (1)−(3) is considered, apparently, for the first time.

1. Statement of the problem. We incorporate notations from [9]. In addition, we denote $\mathrm{Lip}(\mathbb{R})$ as the set of all Lipschitz functions on $\mathbb{R}$; for $m \in \mathbb{N}$ and $\mathcal{O} \subset \mathbb{R}^m$, we introduce the sets: $L^0(\mathcal{O}) := \{v : \mathcal{O} \to \mathbb{R} \mid v \text{ is a measurable function}\}$, $\mathcal{B}_+(\mathcal{O}) := \{\mathfrak{q} \in L^\infty(\mathcal{O}) \mid \operatorname*{essinf}_{y\in\mathcal{O}} \mathfrak{q}(y) > 0\}$. For all $\mathfrak{q} \in \mathcal{B}_+(\mathcal{O})$, we define the following:

$$\mathfrak{q}_0 := \operatorname*{essinf}_{y\in\mathcal{O}} \mathfrak{q}(y), \quad \mathfrak{q}^0 := \operatorname*{esssup}_{y\in\mathcal{O}} \mathfrak{q}(y),$$

$$\mathfrak{q}'(y) := \frac{\mathfrak{q}(y)}{\mathfrak{q}(y)-1}, \quad y \in \mathcal{O}.$$

Also, let $L^r(\mathcal{O})$ be a standard Lebesgue space, where $r \geq 1$ is a number, $L^{\mathfrak{q}(y)}(\mathcal{O})$ be a generalized Lebesgue space, where $\mathfrak{q} \in \mathcal{B}_+(\mathcal{O})$ is a function,

$$\mathcal{P}^{\log}(\mathcal{O}) := \{\mathfrak{q} \in L^\infty(\mathcal{O}) \mid \\ \mathfrak{q} \text{ is a globally log-Hölder continuous, } \mathfrak{q}_0 \geq 1\}.$$

Let $W = W(t,\omega) : \Theta_{0,T} \to \mathbb{R}$ denotes the standard Wiener process [1, p. 109] and by $L_r(\mathbb{S}) \equiv L_r(\mathbb{S}, \mathcal{F}, \mathbb{P})$ a random Lebesgue space, consisting of all random variables $\xi : \mathbb{S} \to \mathbb{R}$ that have a finite r-th absolute momentum. Similarly, let $L_p(\mathbb{S}; B)$ be the random Lebesgue − Bochner spaces of all B-valued random variables $\xi : \mathbb{S} \to B$, where B is some Banach space.

Let $\mathfrak{q} \in \mathcal{B}_+(\mathcal{O})$ and $\mathfrak{q}_0 > 1$. Consider a generalized Lebesgue random space

$$L_{q(y)}(\mathcal{O}\times\mathbb{S}) := \Big\{ z : \mathcal{O}\times\mathbb{S} \to \mathbb{R} \mid \\ 1)\ z \text{ is a measurable function}, \\ 2)\ \rho_{\mathfrak{q}}(z;\mathcal{O}\times\mathbb{S}) := \int_{\mathcal{O}}\int_{\mathbb{S}} |z(y,\omega)|^{q(y)}\,\mathbb{P}(d\omega)\,dy < +\infty \Big\}$$

with the Luxembourg norm

$$\big\| z; L_{\mathfrak{q}(y)}(\mathcal{O}\times\mathbb{S}) \big\| := \inf\{\lambda > 0 \mid \rho_{\mathfrak{q}}(z/\lambda;\mathcal{O}\times\mathbb{S}) \leq 1\}.$$

We will substitute $\mathcal{O}$ with $(0,T)$, Ω, $Q_{0,T}$, etc. below. Assume that the following conditions for the problem coefficients are satisfied:

(P): $p, q \in \mathcal{P}^{\log}(Q_{0,T})$;

(Z): $\alpha > 0$, $r_0 = \min\{2, p_0, q_0\}$, $r^0 = \max\{2, p^0, q^0\}$, $s \in \mathbb{N}$,

$$s \geq \frac{1}{2} \max \left\{ 2, 1 + \frac{n(p^0 - 2)}{2p^0}, \frac{n(q^0 - 2)}{2q^0} \right\};$$

(A): $a_{ik} \in L^0(Q_{0,T}),\ \ 0 < a_0 \leq a_{ik}(x,t) \leq a^0 < +\infty,$

for almost every $(x,t) \in Q_{0,T}$, where $i = 1,\dots,n,\ \ k = 1,\dots,N$;

(G): $g_k \in L^0(Q_{0,T}),\ \ 0 < g_0 \leq g_k(x,t) \leq g^0 < +\infty,$

for almost every $(x,t) \in Q_{0,T}$, where $k = 1,\dots,N$;

(Φ): $\varphi_k \in \mathrm{Lip}(\mathbb{R}),\ \ |\varphi_k(\xi)| \leq \varphi^0 |\xi|,$

for all $\xi \in \mathbb{R}$, where $0 \leq \varphi^0 < +\infty,\ \ k = 1,\dots,N$;

(E): $\mathfrak{Z}_k \in L^0(Q_{0,T} \times \Omega),\ \ |\mathfrak{Z}_k(x,t,y)| \leq \mathfrak{Z}^0 < +\infty,$

for almost every $(x,t,y) \in Q_{0,T} \times \Omega$, where $k = 1,\dots,N$;

(F): $F := (F_1,\dots,F_N) \in L_2(\mathbb{S}; L^2(0,T;H))$;

(U): $u_0 \in L_2(\mathbb{S}; H)$;

(W): W is the standard Wiener process [1, p. 109], $b := (b_1,\dots,b_N)$,

$b_k(x,t,\omega) = b_{0,k}(x) W(t,\omega),\ \ (x,t,\omega) \in \Pi_{0,T},\ \ k = 1,\dots,N$,

and $b_0 := (b_{0,1},\dots,b_{0,N}) \in [C_0^\infty(\Omega)]^N$.

To determine a solution to the problem (1)–(3), let us introduce the following notation:

$$\mathcal{W}_s := \{ v \in H^{2s}(\Omega) \mid v|_{\partial\Omega} = \Delta v|_{\partial\Omega} = \ldots = \Delta^{s-1} v|_{\partial\Omega} = 0 \},$$

$$Z := \mathcal{W}_1, \quad X(t) := W_0^{1,p(x,t)}(\Omega), \quad Y(t) := L^{q(x,t)}(\Omega),$$

$$H := [L^2(\Omega)]^N, \quad V(t) := Z^N \cap [X(t)]^N \cap [Y(t)]^N \cap H,$$

$$X^*(t) := [X(t)]^*, \quad Y^*(t) := [Y(t)]^*, \quad H^* \simeq H, \quad V^*(t) := [V(t)]^*,$$

$$U(Q_{0,T}) := \Big\{ u : (0,T) \to V(t) \ \Big|\ D^\alpha u \in [L^2(Q_{0,T})]^N,\ |\alpha| = 2,$$
$$u_{x_1},\dots,u_{x_n} \in [L^{p(x,t)}(Q_{0,T})]^N,$$
$$u \in [L^{q(x,t)}(Q_{0,T})]^N \cap [L^2(Q_{0,T})]^N \Big\},$$

$$W(Q_{0,T}) := \{ w \in U(Q_{0,T}) \ \mid\ w_t \in [U(Q_{0,T})]^* \}.$$

It is known that $W(Q_{0,T}) \subset C([0,T];H)$ and that for functions from space $W(Q_{0,T})$, the formula for integration by parts with respect to the time variable holds (see, for example, Proposition 3.26 [11, p. 95]). Consider the following operators independent of $\omega \in \mathbb{S}$:

$$(\mathrm{N}z)_k(x,t) := g_k(x,t) |z(x,t)|^{q(x,t)-2} z_k(x,t),$$

$$(\mathrm{E}z)_k(x,t) := \int_\Omega З_k(x,t,y) z_k(y,t)\,dy\,,$$

$$z = (z_1,\dots,z_N), \quad (x,t) \in Q_{0,T}\,,$$

$$\mathrm{N}z := \big((\mathrm{N}z)_1,\dots,(\mathrm{N}z)_N\big), \quad \mathrm{E}z := \big((\mathrm{E}z)_1,\dots,(\mathrm{E}z)_N\big)\,.$$

Let $\mathrm{S} : U(Q_{0,T}) \to [U(Q_{0,T})]^*$ and $S(t) : V(t) \to V^*(t)$ be as follows:

$$\langle S(t)z, w\rangle_{V(t)} := \int_\Omega \sum_{k=1}^N \Big[\alpha \Delta z_k \Delta w_k + \sum_{i=1}^n a_{ik}(x,t)\, |z_{x_i}|^{p(x,t)-2}\, z_{k,x_i} w_{k,x_i} + (\mathrm{N}z)_k w_k + \varphi_k((\mathrm{E}z)_k) w_k \Big] dx\,,$$

$$z, w \in V(t), \quad t \in (0,T)\,;$$

$$\langle \mathrm{S}u, v\rangle_{U(Q_{0,T})} := \int_0^T \langle S(t)u(t), v(t)\rangle_{V(t)}\,dt, \quad u,v \in U(Q_{0,T})\,.$$

To introduce the definition of the solution to problem (1)–(3), we formally substitute the unknown function u with $\tilde{u}$ according to the rule

$$u(x,t,\omega) = \tilde{u}(x,t,\omega) + b(x,t,\omega)\,, \tag{4}$$

where function b is taken from condition **(W)**. Since

$$b\,|_{x\in\partial\Omega} = \Delta b\,|_{x\in\partial\Omega} = 0, \quad b\,|_{t=0} = 0\,,$$

we obtain the following problem for the function $\tilde{u} = (\tilde{u}_1,\dots,\tilde{u}_N) : \Pi_{0,T} \to \mathbb{R}^N$:

$$\tilde{u}_{k,t} + \alpha\Delta^2(\tilde{u}_k + b_k) - \sum_{i=1}^n \Big[a_{ik}(x,t) |\tilde{u}_{x_i} + b_{x_i}(x,t,\omega)|^{p(x,t)-2} (\tilde{u}_k + b_k(x,t,\omega))_{x_i} \Big]_{x_i} + (\hat{\mathrm{N}}(\tilde{u}+b))_k(x,t,\omega) + \varphi_k\big((\hat{\mathrm{E}}(\tilde{u}+b))_k(x,t,\omega)\big) = F_k(x,t,\omega)\,,$$

$$(x,t,\omega) \in \Pi_{0,T}\,, \tag{5}$$

$$\tilde{u}(x,t,\omega) = \Delta\tilde{u}(x,t,\omega) = 0, \quad x \in \partial\Omega, \quad (t,\omega) \in \Theta_{0,T}\,, \tag{6}$$

$$\tilde{u}(x,0,\omega) = u_0(x,\omega), \quad x \in \Omega, \quad \omega \in \mathbb{S}\,. \tag{7}$$

This problem doesn't contain the white noise component b_t anymore. Now we can determine the solution.

Definition 1. *Function* $\tilde{u} : \Pi_{0,T} \to \mathbb{R}^N$ *is called a weak solution to problem* (5)–(7) *if*

1) $\tilde{u} \in W(Q_{0,T})$ *almost surely;*

2) *function* $\tilde{u}$ *almost surely satisfies the equality*

$$\int_{Q_{0,T}} \sum_{k=1}^N \Big[-\tilde{u}_k v_{k,t} + \alpha\Delta(\tilde{u}_k + b_k)\Delta v_k +$$

$$+\sum_{i=1}^{n} a_{ik}(x,t)\left|\tilde{u}_{x_i}+b_{x_i}\right|^{p(x,t)-2}(\tilde{u}_{k,x_i}+b_{k,x_i})v_{k,x_i}+$$

$$+(\hat{\mathrm{N}}(\tilde{u}+b))_k v_k+\varphi_k((\hat{\mathrm{E}}(\tilde{u}+b))_k)v_k\Big]dx\,dt=$$

$$=\int_{Q_{0,T}}\sum_{k=1}^{N}F_k v_k dx\,dt$$

for all test functions $v\in U(Q_{0,T})$, *i.e., in the sense of the spaces* $[U(Q_{0,T})]^*$ *and* $[D^*(Q_{0,T})]^N$ *the equality holds almost surely*

$\tilde{u}_t+\mathrm{S}(\tilde{u}+b)=F$;

3) $\tilde{u}$ *almost surely satisfies condition* (7) *in the sense of the space* $C([0,T];H)$.

***Definition* 2**. Function $u:\Pi_{0,T}\to\mathbb{R}^N$ is called a weak solution to problem (1)–(3) if u has form (4) and the function $\tilde{u}$ is a weak solution to problem (5)–(7) in the sense of Definition 1.

The main result of the article is the following statement.

Theorem 1. *Let conditions* **(P)–(W)** *hold, the constant* s *is taken from condition* **(Z)**, *and* $\partial\Omega\subset C^{2s}$. *Then problem* (1)–(3) *has a unique weak solution* u. *Furthermore,*

$$u\in L_2(\mathbb{S};C([0,T];H)\cap L^2(0,T;Z)),$$

$$u\in L_{q(x,t)}(\Pi_{0,T}),\quad u_{x_1},\dots,u_{x_N}\in L_{p(x,t)}(\Pi_{0,T}). \tag{8}$$

Similar to (1)–(3), the problems without random parameter were considered in [9]. White noise-perturbed integro-differential systems (1) are considered, apparently, for the first time.

P r o o f. First, we introduce the random argument function

$$\Bbbk:\mathbb{S}\to H\times L^2(0,T;H)\times U(Q_{0,T})$$

according to the rule

$$\Bbbk(\omega):=\{u_0(\cdot,\omega),F(\cdot,\cdot,\omega),b(\cdot,\cdot,\omega)\},\quad \omega\in\mathbb{S}. \tag{9}$$

Let

$$\mathfrak{R}:H\times L^2(0,T;H)\times U(Q_{0,T})\to C([0,T];H)\cap U(Q_{0,T})$$

be a function such that

$$\mathfrak{R}\{u_0,F,b\}=\tilde{u}, \tag{10}$$

where $\tilde{u}$ is the solution of problem (5)–(7) with a fixed random parameter $\omega\in\mathbb{S}$. Using the Faedo – Galerkin method, we prove the existence of the solution to this problem. Since the nonlinear components corresponding to the exponents p and q are monotonous, and the integral component belongs to system (1) with the Lipschitz nonlinearity, we can use the standard methods to prove that the solution of problem (5)–(7) with a fixed random parameter $\omega\in\mathbb{S}$ continuously depends on the input data u_0, F, b. Therefore, the function $\mathfrak{R}$ is correctly defined and continuous.

Using (9) and (10) we define the function $\mathfrak{P}:\mathbb{S}\to C([0,T];H)\cap U(Q_{0,T})$ according to the following rule:

$$\mathfrak{P}(\omega) := \mathfrak{R} \circ \Bbbk(\omega) = \mathfrak{R}(\Bbbk(\omega)), \quad \omega \in \mathbb{S}\,. \tag{11}$$

Therefore, for each $\omega \in \mathbb{S}$ the value of $\mathfrak{P}(\omega)$ is equal to $\tilde{u}(\cdot,\cdot,\omega)$, where $\tilde{u}$ is the solution to problem (5)–(7) without the random parameter ω. Since the function $\Bbbk$ from (9) is measurable, and $\mathfrak{R}$ is continuous, the function $\mathfrak{P}$ from (11) is measurable. Thus, $\tilde{u}$ will be a $C([0,T];H) \bigcap U(Q_{0,T})$-valued random variable. In addition, $\tilde{u}$ satisfies the following estimates:

$$\int_\Omega |\tilde{u}(x,\tau,\omega)|^2\,dx \le C_1\mathbf{F}(\tau,\omega)\,, \tag{12}$$

$$\int_{Q_{0,\tau}} \Big[|\Delta\tilde{u}(x,t,\omega)|^2 + \sum_{i=1}^n |\tilde{u}_{x_i}(x,t,\omega)|^{p(x,t)} + + |\tilde{u}(x,t,\omega)|^{q(x,t)} \Big] dx\,dt \le C_1\mathbf{F}(\tau,\omega)\,, \tag{13}$$

where the constant $C_1 > 0$ does not depend on $\tilde{u}$, ω, u_0, F, b,

$$\mathbf{F}(\tau,\omega) = \int_\Omega |u_0(x,\omega)|^2\,dx + \int_{Q_{0,\tau}} \Big[|F(x,t,\omega)|^2 + |\Delta b(x,t,\omega)|^2 + + \sum_{i=1}^n |b_{x_i}(x,t,\omega)|^{p(x,t)} + |b(x,t,\omega)|^{q(x,t)} + + |b(x,t,\omega)|^2 \Big] dx\,dt\,,$$

$\tau \in (0,T]$, $\omega \in \mathbb{S}$. Let's take the maximum for τ in (12) and $\tau = T$ in (13). Since the function $\mathbf{F}(T,\cdot)$ belongs to $L_1(\mathbb{S})$, we can obtain (by integrating the obtained equalities with respect to the variable $\omega \in \mathbb{S}$) the existence of integrals, that guarantee that (8) holds.

We prove the uniqueness of the solution to the problem by contradiction. Suppose that $\tilde{u}^1$ and $\tilde{u}^2$ are two solutions to problem (5)–(7), $\tilde{u} = \tilde{u}^1 - \tilde{u}^2$. Then we have

$$\frac{1}{2}\int_\Omega |\tilde{u}(x,\tau,\omega)|^2\,dx + \int_{Q_{0,\tau}} \sum_{k=1}^N \Big[\alpha\,|\Delta\tilde{u}_k|^2 + + \sum_{i=1}^n a_{ik}(x,t)\big(|\tilde{u}^1_{x_i} + b_{x_i}|^{p(x,t)-2}\,(\tilde{u}^1_{k,x_i} + b_{k,x_i}) - - |\tilde{u}^2_{x_i} + b_{x_i}|^{p(x,t)-2}\,(\tilde{u}^2_{k,x_i} + b_{k,x_i})\big) \times \times \big((\tilde{u}^1_{k,x_i} + b_{k,x_i}) - (\tilde{u}^2_{k,x_i} + b_{k,x_i})\big) + + g_k(x,t)\big(|\tilde{u}^1 + b|^{q(x,t)-2}\,(\tilde{u}^1_k + b_k) - - |\tilde{u}^2 + b|^{p(x,t)-2}\,(\tilde{u}^2_k + b_k)\big) \times \times \big((\tilde{u}^1_k + b_k) - (\tilde{u}^2_k + b_k)\big) + \Big\{ \varphi_k\big((\mathrm{E}(\tilde{u}^1 + b))_k\big) -$$

$$- \varphi_k\big((\mathrm{E}(\tilde{u}^2 + b))_k\big) \Big\} \tilde{u}_k \Big] dx\, dt = 0\,,$$

$$\tau \in (0, T], \quad \omega \in \mathbb{S}\,.$$

Discarding the monotonous components, we obtain the following estimate

$$\frac{1}{2} \int_{\Omega} |\tilde{u}(x, \tau, \omega)|^2\, dx + \int_{Q_{0,\tau}} \alpha\, |\Delta \tilde{u}|^2\, dx\, dt \le \int_{Q_{0,\tau}} \sum_{k=1}^{N} |\varphi_k\big((\mathrm{E}(\tilde{u}^1 + b))_k\big) -$$

$$- \varphi_k\big((\mathrm{E}(\tilde{u}^2 + b))_k\big)||\tilde{u}_k|\, dx\, dt\,. \tag{14}$$

From (14), conditions **(Φ)−(E)**, and the Gronwall − Belman lemma, we get, in particular, that

$$\int_{\Omega} |\tilde{u}(x, \tau, \omega)|^2\, dx \le 0, \qquad \tau \in (0, T), \qquad \omega \in \mathbb{S}\,.$$

Integrating by $\tau \in (0, T)$ and $\omega \in \mathbb{S}$, we obtain that $\tilde{u} = 0$, i.e, $\tilde{u}^1 = \tilde{u}^2$, for example, in the sense of the space $L_2(\Pi_{0,T})$. Theorem 1 is proved. ♦

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СТОХАСТИЧНІ ПАРАБОЛІЧНІ СИСТЕМИ ЗІ ЗМІННИМИ ПОКАЗНИКАМИ НЕЛІНІЙНОСТІ

Розглянуто деякі нелінійні параболічні системи диференціальних рівнянь з частинними похідними з білим шумом. Досліджено початково-крайову задачу для системи та доведено існування та єдиність слабкого розв'язку задачі.

Ключові слова: *диференціальні рівняння з частинними похідними, стохастична параболічна система, змінний показник нелінійності, білий шум, слабкий розв'язок.*

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