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THE MAXIMUM TENSILE STRESS IN A MICRO-PERIODIC COMPOSITE HALF-SPACE WITH SLANT LAYERING UNDER FRICTIONAL CONTACT ON ITS SURFACE[*]

The paper deals with a plane elasticity problem for a composite half-space subjected to frictional contact. The half-space is regarded as a micro-periodic layered composite consisting of two components. The layer interfaces are parallel to one another and oblique to the boundary of half-space. The homogenized model with micro-local parameters is used to solve the considered problem. The solution is obtained in a general form and specifically for the case of elliptical loads [4]. *On the basis of obtained results, the maximum tensile stress is analyzed on the boundary of half-space and presented graphically.*

Key words: *micro-periodic composite, maximum tensile stress, slant lamination, frictional contact, homogenized model.*

Introduction. Micro-periodic composites or strata are often encountered in the modern engineering structures (e.g., thin-layer laminates, sandwich panels, etc.) and natural formations (e.g., soil, sediments, etc.). A number of important applications (e.g., tunnel boring in layered rocks) are concerned with the analysis of mechanical performance of the solids of such kind which implies the solution of relevant boundary value problems of the elasticity theory, particularly, when the surface of solids is subjected to mechanical contact. The most typical cases studied in the relevant literature (see, e.g., [2, 3] for the overview) deal with the situation when the stratification is either parallel or perpendicular to the surface of a considered solid. In extension to the previous results obtained by Sebestianiuk et al. [6], this paper deals with a contact problem for a half-space made of a micro-periodic material for the case when the layering or stratification is inclined towards the boundary at an arbitrary angle. Comparison with existing results for layering perpendicular to the boundary is implemented for partial validation.

1. Formulation of the problem. Consider a two-dimensional contact problem for a composite half-space with micro-periodic slant layering. A rigid cylinder is indented into the half-space through the limiting plane along its generatrix (Fig. 1***a***). Assume the cylinder–half-space system to be in the state of limit equilibrium under the normal and shearing forces P and $Q = fP$, respectively. Here, f is the friction coefficient. A periodic cell of the half-space material is composed of two (1st and 2nd) alternating isotropic layers. The Lamé constants of the 1st and 2nd material components are denoted as λ_1, μ_1 and λ_2, μ_2, respectively. For the considered plane strain problem, two coordinate systems are introduced: (x, y, z) related to the composite layering structure, and (ξ, η, z) related to the half-space surface $\eta = 0$ and rotated about z-axis by an angle α. The considered kind of contact at the limit state of equilibrium is basically modelled [1] by a boundary value problem with appropriate elliptical loads (Fig. 1***b***)

$$p(\xi) = p_{\max}\sqrt{a^2 - \xi^2}, \qquad \xi \in [-a, a],$$

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$$\tau(\xi) = p_{\max} f \sqrt{a^2 - \xi^2}, \qquad \xi \in [-a, a], \tag{1}$$

where $2a$ is the alleged width of contact zone and $p_{\max}$ stands for the maximum normal loading occurring in the middle of this zone.

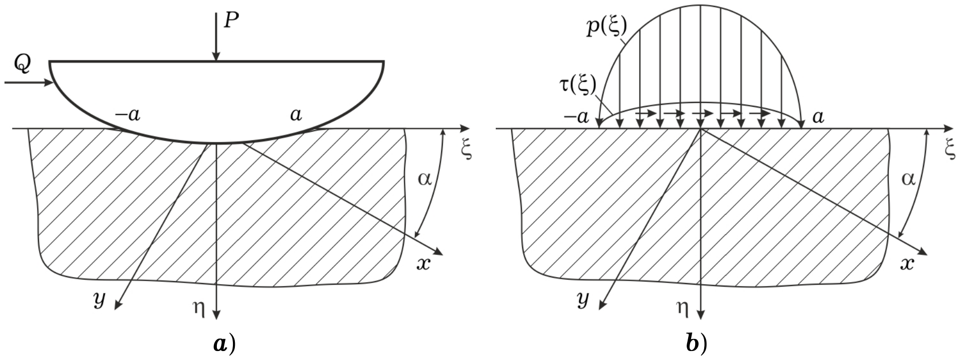

Fig.1. Scheme of the considered plane boundary value problem for a composite half-space: ***a***) the actual contact problem at the limit equilibrium, ***b***) the corresponding elliptical-load problem.

2. Solution technique. In order to solve the formulated problem within the framework of the classical theory of elasticity, a homogenized model with micro-local parameters in the plane strain state can be implemented with respect to the stratification-related coordinate system (x, y, z). This model is governed [9] by the system of equations

$$A_1 \frac{\partial^2 U}{\partial x^2} + (B + C)\frac{\partial^2 V}{\partial x \partial y} + C\frac{\partial^2 U}{\partial y^2} = 0,$$

$$A_2 \frac{\partial^2 V}{\partial y^2} + (B + C)\frac{\partial^2 U}{\partial x \partial y} + C\frac{\partial^2 V}{\partial x^2} = 0, \tag{2}$$

and the stresses within the m-th composite component can be given as [3]

$$\sigma_{xx}^{(m)} = A_1 \frac{\partial U}{\partial x} + B\frac{\partial V}{\partial y}, \qquad \sigma_{xy}^{(m)} = C\left(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x}\right),$$

$$\sigma_{yy}^{(m)} = D_m \frac{\partial U}{\partial x} + E_m \frac{\partial V}{\partial y}, \qquad \sigma_{zz}^{(m)} = \frac{\lambda_m}{\lambda_m + 2\mu_m}\left(\sigma_{xx}^{(m)} + \sigma_{yy}^{(m)}\right). \tag{3}$$

Here, U and V are the components of macro-displacement vector $\mathbf{U} = (U, V, 0)$, $\sigma_{xx}^{(m)}$, $\sigma_{xy}^{(m)}$, $\sigma_{yy}^{(m)}$, and $\sigma_{zz}^{(m)}$ are the stress tensor components within the m-th material component of a periodic cell and A_m, B, C, D_m, E_m, $m = 1, 2$, are constants computed within the framework of the homogenized model as

$$A_1 = \frac{(\lambda_1 + 2\mu_1)(\lambda_2 + 2\mu_2)}{(1 - \chi)(\lambda_1 + 2\mu_1) + \chi(\lambda_2 + 2\mu_2)} > 0,$$

$$A_2 = A_1 + \frac{4\chi(1 - \chi)(\mu_1 - \mu_2)(\lambda_1 - \lambda_2 + \mu_1 - \mu_2)}{(1 - \chi)(\lambda_1 + 2\mu_1) + \chi(\lambda_2 + 2\mu_2)} > 0,$$

$$B = \frac{(1 - \chi)\lambda_2(\lambda_1 + 2\mu_1) + \chi\lambda_1(\lambda_2 + 2\mu_2)}{(1 - \chi)(\lambda_1 + 2\mu_1) + \chi(\lambda_2 + 2\mu_2)} > 0,$$

$$C = \frac{\mu_1\mu_2}{(1 - \chi)\mu_1 + \chi\mu_2} > 0, \qquad D_m = \frac{\lambda_m}{\lambda_m + 2\mu_m} A_1 > 0,$$

$$E_m = \frac{4\mu_m(\lambda_m + \mu_m)}{\lambda_m + 2\mu_m} + \frac{\lambda_m}{\lambda_m + 2\mu_m} B > 0, \qquad m = 1, 2, \tag{4}$$

where χ is a periodic cell saturation within the 1st material: $\chi = \ell_1 / \ell$, $\ell = \ell_1 + \ell_2$ and ℓ_m is the thickness of the m-th alternating layer.

Basing on (1)–(4), the averaged boundary conditions

$$\sigma_{\eta\eta}(\xi, 0) = \chi\sigma_{\eta\eta}^{(1)} + (1-\chi)\sigma_{\eta\eta}^{(2)} = -p(\xi)H(a - |\xi|), \qquad \xi \in \mathbb{R},$$

$$\sigma_{\xi\eta}(\xi, 0) = \chi\sigma_{\xi\eta}^{(1)} + (1-\chi)\sigma_{\xi\eta}^{(2)} = -\tau(\xi)H(a - |\xi|), \qquad \xi \in \mathbb{R}, \tag{5}$$

can be imposed for stress vector components on the surface of the half-space for the homogenized problem, where $H(\xi)$ is the Heaviside step-function and $\sigma_{\xi\xi}$ and $\sigma_{\eta\eta}$ are averaged stresses within a composite periodic cell.

At infinitely distant points, the stresses are assumed to be vanishing:

$$\sigma_{\xi\xi}^{(m)},\ \sigma_{\xi\eta}^{(m)},\ \sigma_{\eta\eta}^{(m)} \to 0, \qquad \xi^2 + \eta^2 \to \infty, \qquad m = 1, 2. \tag{6}$$

By making use of the solution method [6] and introducing the dimensionless coordinates $\breve{\xi} = \xi / a$ and $\breve{\eta} = \eta / a$ with account for (5) and (6), the dimensionless components of the stress tensor for m-th material constituent $\breve{\sigma}_{ij}^{(m)} = \sigma_{ij}^{(m)} / p_{\max}$, $\{i, j\} = \{x, y\}$, in the coordinate system (x, y, z) related to the layering direction take the form:

$$\begin{aligned}\breve{\sigma}_{ij}^{(m)}(\breve{\xi}, \breve{\eta}) = \sum_{k=1}^{2}\int_0^\infty \frac{J_1(\breve{s})}{\breve{s}} \exp(-a_1^k \breve{s}\breve{\eta}) \times \\ \times\Big(\big(P_{ijm}^{(1)k} + (-1)^k Q_{ijm}^{(1)k} f\big) \cos\big((\breve{\xi} - a_2^k\breve{\eta})\breve{s}\big) - \\ -\big(P_{ijm}^{(2)k} + (-1)^k Q_{ijm}^{(1)k} f\big) \sin\big((\breve{\xi} - a_2^k\breve{\eta})\breve{s}\big)\Big) d\breve{s}, \\ \{i, j\} = \{x, y\}, \qquad m = 1, 2,\end{aligned} \tag{7}$$

where $J_1(\breve{s})$ is the first-kind Bessel function of order 1 and parameters $P_{ijm}^{(1)k}$, $P_{ijm}^{(2)k}$, $Q_{ijm}^{(1)k}$, $Q_{ijm}^{(2)k}$, a_1^k, a_2^k depend on the elastic properties of the layers and the lamina angle α [6] as given in Appendix **A**.

On the surface of the half-space, stresses (7) appear as

$$\breve{\sigma}_{ij}^{(m)}(\breve{\xi}) = \begin{cases} -\sum_{k=1}^{2}\big(P_{ijm}^{(2)k} + (-1)^k Q_{ijm}^{(2)k} f\big)\big(\breve{\xi} + \sqrt{\breve{\xi}^2 - 1}\big), & \breve{\xi} < -1, \\ \sum_{k=1}^{2}\Big(\big(P_{ijm}^{(1)k} + (-1)^k Q_{ijm}^{(1)k} f\big)\sqrt{1 - \breve{\xi}^2} - \\ \qquad - \big(P_{ijm}^{(2)k} + (-1)^k Q_{ijm}^{(2)k} f\big)\breve{\xi}\Big), & |\breve{\xi}| < 1, \\ -\sum_{k=1}^{2}\big(P_{ijm}^{(2)k} + (-1)^k Q_{ijm}^{(2)k} f\big)\big(\breve{\xi} - \sqrt{\breve{\xi}^2 - 1}\big), & \breve{\xi} > 1. \end{cases} \tag{8}$$

3. Numerical results and discussion. The numerical evaluation of formulas (8) have shown that the maximum tensile stress distribution on the surface of the considered half-space resemble ones obtained for the analogous problem for an anisotropic half-space [5, 7].

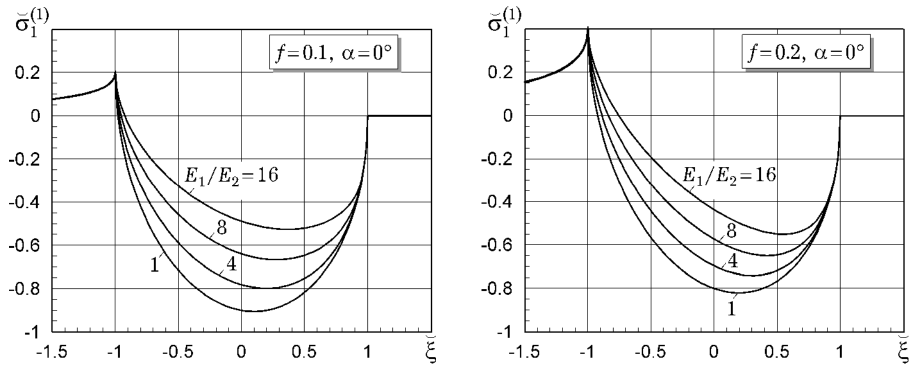

Fig. 2. Maximum tensile stress for different values of the Young modules ratio $E_1 \,/\, E_2$ at lamination perpendicular to the boundary.

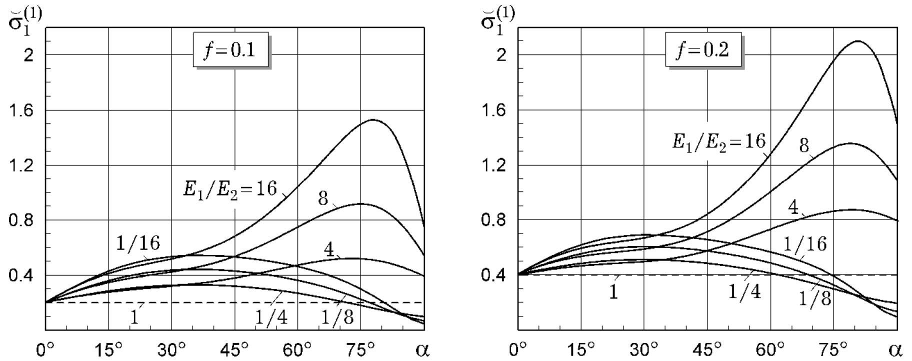

Fig. 3. Maximum tensile stress at $\breve{\xi} = -1$, $\breve{\eta} = 0$ for different values of the Young modules ratio $E_1 \,/\, E_2$ and lamina angle α.

Next, we analyze a composite component of higher stiffness due to its higher load transfer. The maximum tensile stress $\breve{\sigma}_1^{(1)}$ can be expressed as [8]

$$\breve{\sigma}_1^{(1)} = \frac{1}{2}\left(\breve{\sigma}_{xx}^{(1)} + \breve{\sigma}_{yy}^{(1)}\right) + \frac{1}{2}\sqrt{\left(\breve{\sigma}_{xx}^{(1)} - \breve{\sigma}_{yy}^{(1)}\right)^2 + 4\left(\breve{\tau}_{xx}^{(1)}\right)^2}\,.$$

By making use of the constructed solution (8), the typical maximum tensile stress distribution on the surface for different values of the Young modules ratio $E_1 \,/\, E_2$ is shown in Fig. 2. As it can be seen from this figure, the maximum values of tensile stress occur at the point $\breve{\xi} = -1$, $\breve{\eta} = 0$, where

$$\breve{\sigma}_1^{(1)}(-1,0) = \sum_{k=1}^{2}\left(P_{ijm}^{(2)k} + (-1)^k Q_{ijm}^{(2)k} f\right),$$

which for $\alpha = 0^\circ$ yields a simple formula

$$\breve{\sigma}_1^{(1)}(-1,0) = (\gamma_1 + \gamma_2) f\,.$$

Here, γ_k are parameters directly related to the homogenized model parameters [4, 6] and given in Appendix **A** by formulae (**A.**10).

In the special case of the stratification being perpendicular to the boundary of the half-plane, $\alpha = 0^\circ$, when $\nu_1 = \nu_2 = 0.3$ and $\chi = 0.5$ (Fig. 2), the maximum tensile stress can be given in the form:

$$\breve{\sigma}_1^{(1)}(-1,0) = \left(\sqrt{\frac{7}{5E_1 \,/\, E_2 + 2}} + \sqrt{\frac{7}{5E_2 \,/\, E_1 + 2}}\right) f \approx 2f\,. \qquad (9)$$

It is worth noting that the error of the approximation (9) for the value of maximum tensile stress, i.e. $\breve{\sigma}_1^{(1)} \approx 2f$, falls within few percent. This can be regarded as an important tribological conclusion: at the point $\breve{\xi} = -1$, $\breve{\eta} = 0$ when the stratification is perpendicular to the surface, the maximum tensile stress $\breve{\sigma}_1^{(1)}$ is independent of the composite components stiffness ratio E_1 / E_2.

Next, we analyze this critical point $\breve{\xi} = -1$, $\breve{\eta} = 0$ on the surface and calculate the characteristics of the maximum tensile stress for different values of the Young modules ratio E_1 / E_2 and the lamina angle α (Fig. 3). As it can be seen in this figure, the maximum tensile stresses for $\alpha \neq 90^\circ$ are strongly dependent on material properties and lamination angle.

Conclusions. The pressure on the microperiodic half-space with slant layering was considered and the maximum tensile stress on the surface was analyzed, with special attention given to the point, where these stress peaks its maximum. Basing on the analysis of the results obtained, the following conclusions can be drawn:

1. If the stratification is perpendicular to the boundary of the half-space, the value of maximum tensile stress at the critical point depends very little on the ratio of the Young modules of the composite material components and can be expressed as $\breve{\sigma}_1^{(1)} \approx 2f$ within the engineering accuracy.
2. The maximum value of tensile stress for a composite component with higher stiffness at different angles of layering varies and reaches different maxima at different angles depending on the ratio of component stiffness.

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***Appendix* A**

Parameters used in the paper, $m, \ell, k = 1, 2$:

$$P_{xxm}^{(\ell)k} = A_1 æ_k D_{1k}^{(\ell)p} + B D_{3k}^{(\ell)p}, \qquad Q_{xxm}^{(\ell)k} = A_1 æ_k D_{1k}^{(\ell)\tau} + B D_{3k}^{(\ell)\tau},$$

$$P_{xym}^{(\ell)k} = C(1 + æ_k) D_{2k}^{(\ell)p}, \qquad Q_{xym}^{(\ell)k} = C(1 + æ_k) D_{2k}^{(\ell)\tau},$$

$$P_{yym}^{(\ell)k} = D_m æ_k D_{1k}^{(\ell)p} + E_m D_{3k}^{(\ell)p}, \quad Q_{yym}^{(\ell)k} = D_m æ_k D_{1k}^{(\ell)\tau} + E_m D_{3k}^{(\ell)\tau}; \qquad (\mathbf{A}.1)$$

$$D_{1k}^{(1)X} = R_k^{(1)X} C_1^k - R_k^{(2)X} C_2^k, \quad C_{1k}^{(2)X} = R_k^{(2)X} C_1^k + R_k^{(1)X} C_2^k,$$

$$D_{2k}^{(1)X} = R_k^{(1)X} C_3^k - R_k^{(2)X} C_4^k, \quad C_{2k}^{(2)X} = R_k^{(2)X} C_3^k + R_k^{(1)X} C_4^k,$$

$$D_{3k}^{(1)X} = R_k^{(1)X} C_5^k - R_k^{(2)X} C_6^k, \quad C_{3k}^{(2)X} = R_k^{(2)X} C_5^k + R_k^{(1)X} C_6^k, \; X = \{p, \tau\}; \quad (\mathbf{A}.2)$$

$$H_{1k}^{(1)} = \left((a_1^k)^2 - (a_2^k)^2\right) E_{11}^{(k)} + a_2^k E_{12}^{(k)} - E_{13}^{(k)}, \quad H_{1k}^{(2)} = 2 a_1^k a_2^k E_{11}^{(k)} - a_1^k E_{12}^{(k)},$$

$$H_{2k}^{(1)} = \left((a_1^k)^2 - (a_2^k)^2\right) E_{21}^{(k)} + a_2^k E_{22}^{(k)} - E_{23}^{(k)}, \quad H_{2k}^{(2)} = 2 a_1^k a_2^k E_{21}^{(k)} - a_1^k E_{22}^{(k)}; \; (\mathbf{A}.3)$$

$$E_{11}^{(k)} = \left(A_1 æ_k \sin^2\alpha + B\cos^2\alpha + C(1 + æ_k)\cos^2\alpha\right) \sin\alpha,$$

$$E_{12}^{(k)} = \left(2(A_1 æ_k - B)\sin^2\alpha + C(1 + æ_k)(\cos^2\alpha - \sin^2\alpha)\right) \cos\alpha,$$

$$E_{13}^{(k)} = \left(A_1 æ_k \cos^2\alpha + B\sin^2\alpha - C(1+æ_k)\cos^2\alpha\right)\sin\alpha\,,$$

$$E_{21}^{(k)} = \left(Bæ_k \sin^2\alpha + A_2\cos^2\alpha + C(1+æ_k)\sin^2\alpha\right)\cos\alpha\,,$$

$$E_{22}^{(k)} = \left(2(Bæ_k - A_2)\cos^2\alpha + C(1+æ_k)(\cos^2\alpha - \sin^2\alpha)\right)\sin\alpha\,,$$

$$E_{23}^{(k)} = \left(Bæ_k \cos^2\alpha + A_2\sin^2\alpha - C(1+æ_k)\sin^2\alpha\right)\cos\alpha\,; \qquad (\mathbf{A}.4)$$

$$R_k^{(1)p} = \frac{F_1 F_{11}^{(k)} + F_2 F_{12}^{(k)}}{F_1^{\,2} + F_2^{\,2}}, \qquad R_k^{(2)p} = \frac{F_1 F_{12}^{(k)} - F_2 F_{11}^{(k)}}{F_1^{\,2} + F_2^{\,2}},$$

$$R_k^{(1)\tau} = \frac{F_1 F_{21}^{(k)} + F_2 F_{22}^{(k)}}{F_1^{\,2} + F_2^{\,2}}, \qquad R_k^{(2)\tau} = \frac{F_1 F_{22}^{(k)} - F_2 F_{21}^{(k)}}{F_1^{\,2} + F_2^{\,2}}\,; \qquad (\mathbf{A}.5)$$

$$C_1^k = (a_1^k)^2 \sin^2\alpha - \left(\cos\alpha - a_2^k \sin\alpha\right)^2,$$

$$C_2^k = -2a_1^k(\cos\alpha - a_2^k\sin\alpha)\sin\alpha\,,$$

$$C_3^k = (a_1^k)^2 \sin\alpha\cos\alpha + \left(\cos\alpha - a_2^k\sin\alpha\right)\left(\sin\alpha + a_2^k\cos\alpha\right),$$

$$C_4^k = a_1^k\left(\sin\alpha + a_2^k\cos\alpha\right)\sin\alpha - a_1^k\left(\cos\alpha + a_2^k\sin\alpha\right)\cos\alpha\,,$$

$$C_5^k = (a_1^k)^2\cos^2\alpha - \left(\sin\alpha + a_2^k\cos\alpha\right)^2,$$

$$C_6^k = 2a_1^k(\sin\alpha + a_2^k\cos\alpha)\cos\alpha\,; \qquad (\mathbf{A}.6)$$

$$F_1 = H_{11}^{(1)}H_{22}^{(1)} + H_{12}^{(2)}H_{21}^{(2)} - H_{11}^{(2)}H_{22}^{(2)} - H_{12}^{(1)}H_{21}^{(1)}\,,$$

$$F_2 = H_{11}^{(1)}H_{22}^{(2)} + H_{11}^{(2)}H_{22}^{(1)} - H_{12}^{(2)}H_{21}^{(1)} - H_{12}^{(1)}H_{21}^{(2)}\,; \qquad (\mathbf{A}.7)$$

$$F_{11}^{(1)} = H_{12}^{(1)}\cos\alpha - H_{22}^{(1)}\sin\alpha, \qquad F_{12}^{(1)} = H_{12}^{(2)}\cos\alpha - H_{22}^{(2)}\sin\alpha\,,$$

$$F_{11}^{(2)} = H_{21}^{(1)}\sin\alpha - H_{11}^{(1)}\cos\alpha, \qquad F_{12}^{(2)} = H_{21}^{(2)}\sin\alpha - H_{11}^{(2)}\cos\alpha\,,$$

$$F_{21}^{(1)} = H_{22}^{(1)}\cos\alpha + H_{12}^{(1)}\sin\alpha, \qquad F_{22}^{(1)} = H_{22}^{(2)}\cos\alpha + H_{12}^{(2)}\sin\alpha\,,$$

$$F_{21}^{(2)} = H_{11}^{(1)}\sin\alpha + H_{21}^{(1)}\cos\alpha, \qquad F_{22}^{(2)} = H_{11}^{(2)}\sin\alpha + H_{21}^{(2)}\cos\alpha\,; \qquad (\mathbf{A}.8)$$

$$a_1^k = \frac{\gamma_k}{\gamma_k^2\sin^2\alpha + \cos^2\alpha}, \qquad a_2^k = \frac{(\gamma_k^2 - 1)\sin\alpha\cos\alpha}{\gamma_k^2\sin^2\alpha + \cos^2\alpha}\,; \qquad (\mathbf{A}.9)$$

$$\gamma_k = \sqrt{\frac{A_1A_2 - 2BC - B^2 + (-1)^k\sqrt{\Delta}}{2A_2C}}, \qquad æ_k = \frac{A_2\gamma_k^2 - C}{B + C},$$

$$\Delta = (B^2 + 2BC - A_1A_2)^2 - 4A_1A_2C^2 > 0\,. \qquad (\mathbf{A}.10)$$

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МЕЖА МІЦНОСТІ ПРИ РОЗТЯГУ У МІКРОПЕРІОДИЧНОМУ КОМПОЗИТНОМУ ПІВПРОСТОРІ З НАХИЛЕНИМ ЛАМІНУВАННЯМ ЗА ФРИКЦІЙНОГО КОНТАКТУ НА ЙОГО ПОВЕРХНІ

У статті розглянуто плоску задачу теорії пружності для композитного півпростору за умов фрикційного контакту. Півпростір виготовлено з мікроперіодичного шаруватого матеріалу, скомпонованого з двох складників. Межі розділу шарів є взаємно паралельними і нахиленими до обмежуючої поверхні півпростору. Задачу розв'язано з використанням усередненої моделі з мікролокальними параметрами. Знайдено загальний розв'язок задачі та проаналізовано окремий випадок еліптичного навантаження [4]. На основі отриманого розв'язку проаналізовано максимальні розтягувальні напруження на поверхні півпростору з поданням результатів у графічній формі.

Ключові слова: *мікроперіодичний композит, максимальні розтягувальні напруження, нахилене ламінування, контактна взаємодія з тертям, усереднена модель.*

ПРЕДЕЛ ПРОЧНОСТИ ПРИ РАСТЯЖЕНИИ В МИКРОПЕРИОДИЧЕСКОМ КОМПОЗИТНОМ ПОЛУПРОСТРАНСТВЕ С НАКЛОННЫМ ЛАМИНИРОВАНИЕМ ПРИ ФРИКЦИННОМ КОНТАКТЕ НА ЕГО ПОВЕРХНОСТИ

В статье рассмотрена плоская задача теории упругости для композитного полупространства в условиях фрикционного контакта. Полупространство изготовлено из микропериодического слоистого материала, состоящего из двух компонент. Границы раздела слоев являются взаимно параллельными и наклонены к ограничивающей поверхности полупространства. Задача решена с использованием усредненной модели с микролокальными параметрами. Найдено общее решение задачи, а также проанализирован частный случай эллиптического нагружения [4]. На основе полученного решения проанализированы максимальные растягивающие напряжения на поверхности полупространства с представлением результатов в графическом виде.

Ключевые слова: *микроперидический композит, максимальные растягивающие напряжения, наклонное ламинирование, контактное взаимодействие с трением, усреднённая модель.*

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