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WEIGHT-VIBRATION PARETO OPTIMIZATION OF A DUAL MASS FLYWHEEL*

By using the methodology of the multi-objective optimal design of engineering systems, the problem of weight-vibration Pareto optimization of a dual mass flywheel is considered with the aim to study the feasibility of its application in heavy-duty truck powertrains. The results obtained show the following: the solution of the considered optimization problem does exist; the mass inertia, stiffness and damping parameters of the absorber optimized in an operating engine speed range of 600−2000 *rpm do exist, providing the best attenuation of the torque oscillation at the transmission input shaft. Finally, the results show the feasibility evidence for the application of weight-vibration optimized dual mass flywheel in heavy-duty truck drivetrain systems.*

Key words: *torsional vibration absorber, dual mass flywheel, drivetrain system of a heavy-duty truck, global sensitivity analysis, weight-vibration Pareto optimization.*

Introduction. An engineering system must meet a plenty of requirements, e. g. system's quickness and accuracy, safety and user friendliness, noiseless and low level of vibrations, environmental friendliness and cost efficiency. These are some of the constraints to be satisfied during the design process of modern engineering products, which make the design process of engineering systems to be very complicative.

In this paper, the methodology of multi-objective optimal design of engineering systems is presented. The methodology is based on the global sensitivity analysis (GSA) and Pareto optimization techniques. It has been implemented in the computer toolbox SAMO, developed at Mechanical Systems, Division of Dynamics, Chalmers University of Technology [4]. The methodology and toolbox SAMO were successfully used for optimal design of engineering systems with different applications [5−7]. Herein, we apply the methodology for solving the weight-vibration Pareto optimization of the design of dual mass flywheels for application in torsional vibration attenuation in heavy-duty truck powertrains. A dual mass flywheel (DMF) is a well-known design of torsional vibration absorbers and was a subject for intensive research [1, 3, 8−10]. The research is ongoing to understand whether this concept of absorber is suitable for the attenuation of torsional vibrations in the powertrain of heavy-duty trucks [1, 9, 10].

The outline of the paper is as follows. In Section 1, the global sensitivity analysis and Pareto optimization problems are formulated for the mathematical model of a generic engineering system. These problems' formulations, together with outline of the algorithm of the GSA and the structure of the toolbox SAMO, constitute the basis of the methodology for designing optimal engineering products. The results of weight-vibration Pareto optimal design of a torsional vibration absorber for application in a heavy-duty truck powertrain are presented in Sections 2 and 3. The paper is finalized with conclusions and outline of future research.

Some results of the paper were presented at the 10th International Conference on Mathematical Problems of Mechanics of Nonhomogeneous Structures, September 17−20, 2019, Lviv, Ukraine [2].

1. Sensitivity analysis and Pareto optimization. Consider an engineering system that consists of a number of functional components, representing mass

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inertia, stiffness and damping system's characteristics. Let $\mathbf{q} = [q_1, q_2, \ldots, q_n]^\top$ is the vector of generalized coordinates, $\mathbf{T} = [T_1, T_2, \ldots, T_m]^\top$ is the vector of external loads, e.g. forces or/and torques, acting on the system, and $\mathbf{d} = [d_1, d_2, \ldots, d_k]^\top$ is the vector of design parameters representing the mass inertia, stiffness and damping characteristics of all functional components of the system.

The following expression will be used to represent the set of operational scenarios ($\mathbf{OS}s$) of the generic engineering system in question:

$$\mathbf{OS}s = \{\mathbf{T}(t),\ \mathbf{q}(t),\ \mathbf{d},\ t \in [t_0, t_f],\ \mathbf{d} \in \Omega\}. \tag{1}$$

In expression (1), t_0, t_f are the initial and final instants of time and Ω is the domain of feasible values for the vector of design parameters.

For any feasible vector of design parameters $\mathbf{d} = [d_1, d_2, \ldots, d_k]^\top \in \Omega$, and the given external loads $\mathbf{T} = [T_1, T_2, \ldots, T_m]^\top$, the vector of generalized coordinates $\mathbf{q} = [q_1, q_2, \ldots, q_n]^\top$ satisfies the equation

$$\mathbf{L}\big[\mathbf{q}(t), \dot{\mathbf{q}}(t), \ddot{\mathbf{q}}(t), \mathbf{T}(t), \mathbf{d}\big] = \mathbf{0}. \tag{2}$$

Here, $\mathbf{L}$ is an operator that together with given initial state of the system

$$\mathbf{q}(0) = \mathbf{q}^0, \quad \dot{\mathbf{q}}(0) = \dot{\mathbf{q}}^0 \tag{3}$$

determine the system performance (response), i. e. vector $\mathbf{q}[t, t_0, \mathbf{q}^0, \dot{\mathbf{q}}^0, \mathbf{T}(t), \mathbf{d}]$ for all $t \in [t_0, t_f]$.

Equation (2) along with the initial state (3) constitute the mathematical model of a generic engineering system and allow to obtain all its feasible operational scenarios.

As an example of the mathematical model (2), (3), the following matrix equation

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{U}[t, \mathbf{T}(t)] \tag{4}$$

together with the initial state (3) govern the motion of an n-degree-of-freedom mechanical system with linear stiffness and damping functional components. Here, $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are the mass inertia, the damping and the stiffness matrices and $\mathbf{U}$ is the vector of generalized forces.

1.1. Global sensitivity analysis and Pareto optimization problems formulations. As the first step in optimal design of an engineering system, it is important to study the sensitivity of the system's response with respect to variation of its design parameters. Sensitivity analysis of an engineering system with respect to varying parameter d_i can be carried out either locally or globally. In local sensitivity analysis, the effects of design input d_i on the system response is approximated as partial derivative of an objective function used as a measure of the system response with respect to design parameter d_i which is taken around a fixed point d_i^0. Such an approach only considers variation of an objective function with respect to a single design parameter at a time. Furthermore, the domain of input design variables might not be appropriately scanned using local method.

The global sensitivity analysis is one of the most prominent approaches in the design of engineering systems that can provide informative insight into the design process. To determine global sensitivity indices, multilayer integrals must be evaluated. This process demands a heavy computational effort. Below, the multiplicative dimensional reduction method proposed in [11], is briefly described. This method is used in the computer toolbox SAMO [4] and can approximate global sensitivity indices in an efficient and accurate manner.

An objective function can be express as function of a set of independent random variables, i.e., design parameters $\mathbf{d} = [d_1, d_2, \ldots, d_k]^\top \in \Omega$, through respective deterministic functional relationship $F = F(\mathbf{d})$. It is proposed to approximate the function F as

$$F(\mathbf{d}) \approx [F(\mathbf{c})]^{1-k} \prod_{i=1}^{k} F(d_i, \mathbf{c}_{-i}), \tag{5}$$

where $F(\mathbf{c})$ is a constant, and $F(d_i, \mathbf{c}_{-i})$ denotes the function value for the case that all inputs except d_i are fixed at their respective cut point coordinates, $\mathbf{c} = [c_1, \ldots, c_k]^\top$. Expression (5) is capable to approximate the function F with a satisfactory level of accuracy and is particularly useful for approximating the integrals required for calculating sensitivity indices [11]. Using this approach, primary and higher order sensitivity indices can be approximated as follows

$$S_i \approx \frac{\left(\dfrac{\beta_i}{\alpha_i^2} - 1\right)}{\left(\displaystyle\prod_{j=1}^{k} \dfrac{\beta_j}{\alpha_j^2}\right) - 1}, \quad S_{i_1 \ldots i_s} \approx \frac{\displaystyle\prod_{j=1}^{s}\left(\dfrac{\beta_{ij}}{\alpha_{ij}^2} - 1\right)}{\left(\displaystyle\prod_{j=1}^{k} \dfrac{\beta_j}{\alpha_j^2}\right) - 1}. \tag{6}$$

The coefficients α_j, and β_j are defined as mean and mean square of the j-th univariate function, respectively, and are represented as

$$\alpha_j \approx \sum_{\ell=1}^{N} w_{j\ell} F(d_{j\ell}, \mathbf{c}_{-j\ell}), \quad \beta_j \approx \sum_{\ell=1}^{N} w_{j\ell} F^2(d_{j\ell}, \mathbf{c}_{-j\ell}). \tag{7}$$

Here, N is the total number of integration points, $d_{j\ell}$, and $w_{j\ell}$ are the ℓ-th Gaussian integration abscissas and corresponding weight, respectively.

Finally, total sensitivity index corresponding to the parameter d_i can be expressed as

$$S_i^{\mathrm{T}} \approx \frac{1 - \dfrac{\alpha_i^2}{\beta_i}}{1 - \left(\displaystyle\prod_{j=1}^{k} \dfrac{\alpha_j^2}{\beta_j}\right)}. \tag{8}$$

It should be noted that total number of objective function evaluations required for calculating sensitivity indices using this method is only $k \times N$, where k is the number of design parameters.

To accomplish sensitivity analysis of a system output, a suitable cut point together with a probability distribution must be chosen. Equations (6)−(8) were then utilized to attain sensitivity indices. More details on multiplicative dimensional reduction method for global sensitivity analysis can be found in [11].

Let the following functionals are chosen to measure quality of performance of the engineering system in question

$$F_1[\mathbf{q}(t), \mathbf{d}], \ldots, F_{nF}[\mathbf{q}(t), \mathbf{d}]. \tag{9}$$

The following problem of the global sensitivity analysis for a generic engineering system is formulated.

Problem GSA. Let $\mathbf{d} = [d_1, d_2, \ldots, d_k]^{\top}$ be the vector of the design parameters of the generic engineering system in question. It is required for a given feasible operational scenario

$$\tilde{\mathbf{OS}} \in \mathbf{OS}s \tag{10}$$

to determine, by making use of equation (8), the total sensitivity indices

$$S_i^{\mathrm{T}}(F_j), \quad i = 1, \ldots, k, \;\; j = 1, \ldots, nF , \tag{11}$$

of the functionals (9) for all varying design parameters d_i, subject to equation (2), initial state (3) and the restriction

$$\mathbf{d} = [d_1, d_2, \ldots, d_k]^{\top} \in \Omega . \tag{12}$$

The solution of the *problem GSA* provides mapping between the values of the total sensitivity indices (11) and the design parameters (12) of the generic engineering system.

After the *problem GSA* is solved, the vector of the most important design parameters

$$\mathbf{d}_s = [d_{s1}, d_{s2}, \ldots, d_{sk}]^{\top} \in \Omega, \quad 1 \le sk \le k , \tag{13}$$

as well as the most sensitive functionals $F_j[\mathbf{q}(t), \mathbf{d}]$, $1 \le j \le nF_1 \le nF$, are identified. Then, the Pareto optimization problem is now stated as follows.

Problem PO. For given feasible operational scenario (10), it is required to determine the design parameters

$$\mathbf{d}_s = \mathbf{d}_s^* = [d_{s1}^*, d_{s2}^*, \ldots, d_{sk}^*]^{\top}, \quad sk \in [1, \ldots, k] ,$$

and the vector of generalized coordinates $\mathbf{q}(t) = \mathbf{q}^*(t)$ that altogether satisfy the system of variational equations

$$\min_{\mathbf{d}_s \in \Omega} \left(F_j[\mathbf{q}(t), \mathbf{d}_s] \right) = F_j[\mathbf{q}^*(t), \mathbf{d}_s^*], \quad j = 1, \ldots, nF_1 ,$$

subject to the mathematical model (2)–(3) and restriction (13).

In [4], the computer code SAMO, developed at Chalmers University of Technology, is presented as an efficient toolbox for optimal design of engineering systems. At this stage, the toolbox SAMO includes two modules: SAMO-GSA and SAMO-PO. The module SAMO-GSA is based on the multiplicative version of the dimensional reduction method [11] to solve the above formulated *problem GSA*. In the SAMO-GSA an efficient approximation is employed to simplify the computation of variance-based sensitivity indices associated with a general function of n-random varying parameters. Then, the results of the solution of *problem GSA* might be used as an input to the SAMO-PO module for multi-objective optimization (the above formulated *problem PO*). The module SAMO-PO works based on genetic algorithm (GA). The GA settings include lower and upper bounds for variation of the design parameters, population size, number of generations, elite count, and Pareto fraction settings. The results of SAMO-PO module are presented in terms of Pareto fronts and corresponding Pareto sets for further analysis and decision making by the user. More details on toolbox SAMO and the link to the corresponding computer codes for different examples can be found in [4].

2. Weight-vibration Pareto optimization of a dual mass flywheel. Here, we apply the methodology presented in Section 1 for solving the weight-vibration Pareto optimization of the design of a dual mass flywheel for application in torsional vibration attenuation in heavy-duty truck powertrains.

2.1. A drivetrain system equipped with a dual mass flywheel. Consider a system depicted in Fig. 1. The system comprises an engine, (E), a torsional vibration absorber, (DMF), and the load transmission system, (LTS). Assume that vibration absorber (Fig. 1) consists of two rigid bodies called the primary flywheel (PFW) and the secondary flywheel (SFW). The wheels are connected by a massless linear torsional spring and a massless linear torsional viscous damper. The engine output shaft AB and the transmission input shaft CD are assumed to be rigid and connected rigidly to the PFW and to the SFW, respectively. The torque $T_e(t)$ rotates the primary flywheel about the shaft AB.

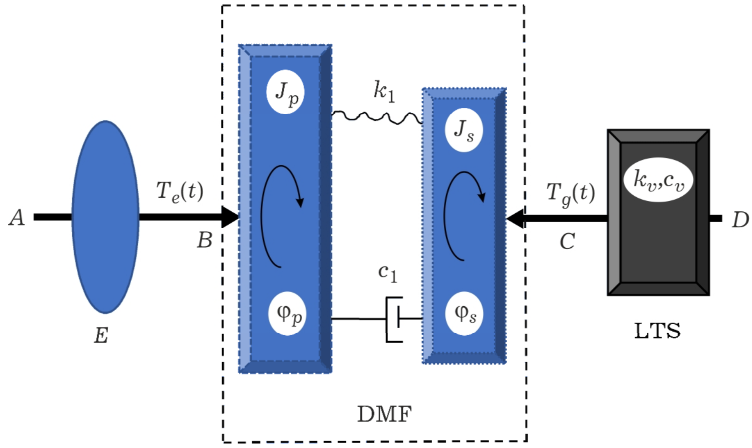

Fig. 1. Sketch of a generic drivetrain system equipped with a dual mass flywheel.

In Fig. 1, φ_p and φ_s are the absolute angles of rotation of the PFW and the SFW, respectively, J_p and J_s are torsional moments of inertia of the PFW and the SFW, respectively, k_1 and c_1 are coefficients of torsional stiffness and torsional damping.

The equations of torsional vibration dynamics of the drivetrain system equipped with a DMF can be written in matrix form (4) with

$$\mathbf{q} = [\varphi_p, \varphi_s]^\top, \quad \dot{\mathbf{q}} = [\dot\varphi_p, \dot\varphi_s]^\top, \quad \ddot{\mathbf{q}} = [\ddot\varphi_p, \ddot\varphi_s]^\top,$$

$$\mathbf{U}[t, \mathbf{T}(t)] = [T_e(t), -T_g(t)]^\top,$$

$$\mathbf{M} = \begin{pmatrix} J_p & 0 \\ 0 & J_s \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} c_1 & -c_1 \\ -c_1 & c_1 \end{pmatrix}, \quad \mathbf{K} = \begin{pmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{pmatrix}. \tag{14}$$

Equations (4) and (14), together with the following initial state

$$\varphi_p(t_0) = \varphi_p^0, \quad \varphi_s(t_0) = \varphi_s^0, \quad \dot\varphi_p(t_0) = \dot\varphi_p^0, \quad \dot\varphi_s(t_0) = \dot\varphi_s^0, \tag{15}$$

constitute the mathematical model of a drivetrain system having a DMF.

2.2. Global sensitivity analysis of a drivetrain system equipped with a DMF. The set of operational scenarios (1) for the system in question will be defined by the expressions

$$\mathbf{OS}s = \{\mathbf{T}(t) = [T_e(t), -T_g(t)]^\top, \quad \mathbf{q}(t) = [\varphi_p(t), \varphi_s(t)]^\top,$$
$$\mathbf{d} = [J_p, J_s, k_1, c_1]^\mathrm{T}, \quad t \in [t_0, t_f], \quad \mathbf{d} \in \Omega\}, \tag{16}$$

where

$$T_e(t) = T_m + a_e \sin(\omega_{n_0} t), \quad \omega_{n_0} = n_0 \omega, \quad \omega = 2\pi n_e / 60, \tag{17}$$

$$T_g(t) = k_v(\varphi_s - \varphi_v) + c_v(\dot\varphi_s - \dot\varphi_v), \quad \varphi_v(t) = \omega_v t. \tag{18}$$

Here, in expressions (17) the engine input torque $T_e(t)$ is modelled by the constant torque T_m plus harmonic function, ω_{n_0} is the n_0-engine order vibration frequency, that is n_0 times the angular velocity ω, and n_e is the engine speed in rpm. The torque at the transmission input shaft $T_g(t)$ is modelled by the expressions (18) and k_v, c_v are equivalent torsional stiffness and damping coefficients of the load transmission system, φ_v, ω_v are absolute angle of rotation and angular velocity of the transmission input shaft.

Consider the vector

$$\mathbf{d} = [d_1, d_2, d_3, d_4]^\top = [J_p, J_s, k_1, c_1]^\top \in \Omega \tag{19}$$

and the functionals

$$F_1(\mathbf{d}) = \int_{600}^{2000} \operatorname{std}\left(T_g[\mathbf{q}(t), \mathbf{d}, n_e]\right) dn_e\,, \tag{20}$$

$$F_2(\mathbf{d}) = J_p + J_s\,, \tag{21}$$

$$F_3(\mathbf{d}) = \int_{600}^{2000} \operatorname{std}\left(T_f[\mathbf{q}(t), \mathbf{d}, n_e]\right) dn_e \tag{22}$$

as the vector of design parameters and the quality measures of the performance of the drivetrain system equipped with a DMF. Here, the function T_f represents the friction torque in the stiffness-damping interface of a DMF and is defined as

$$T_f = k_1(\varphi_p - \varphi_s) + c_1(\dot{\varphi}_p - \dot{\varphi}_s)\,. \tag{23}$$

The functionals $F_1(\mathbf{d})$ and $F_3(\mathbf{d})$ characterize the oscillations of the torque at the transmission input shaft and the energy dissipating in a DMF in the operating engine speed range $600\,\text{rpm} \le n_e \le 2000\,\text{rpm}$, respectively. The functional $F_2(\mathbf{d})$ characterizes the mass inertia properties of the DMF and is well relevant for estimating the total weight of the absorber.

The global sensitivity analysis problem, formulated in Section 1, was solved for the drivetrain system equipped with a DMF by using the differential equation of motion (4) with (14), the initial state (15), the vector of the design parameters (19) and the functionals (20)−(22). The feasible operational scenario (10) was given by the torques $T_e(t)$ and $T_g(t)$ that are determined by expressions (17), (18).

The third engine order vibration harmonic is in focus of analysis as one of the most significant contribution to the oscillatory response [1, 9], i. e. in all simulations the engine order vibration frequency n_0 is chosen to be equal to 3. The rest values of the parameters for the torque $T_e(t)$ are the following: the mean value of engine input torque $T_m = 300\,\text{Nm}$; the amplitude of engine torque harmonic excitation $a_e = 500\,\text{Nm}$; and the engine speed n_e was chosen in the range of $600 - 2000\,\text{rpm}$. The values for the parameters of the torque $T_g(t)$ at the transmission input shaft are: $k_v = 10^5\,\text{Nm/rad}$, $c_v = 0.1\,\text{Nms/rad}$, and $\omega_v = \omega_{n_0}/3$.

The results of the GSA of the drivetrain system with respect to variation of the design parameters (19) have been obtained for engine speeds in the range of $600 - 2000\,\text{rpm}$ by using the computer code SAMO with settings given in Table 1. Here, the nominal values of design parameters of the DMF are chosen to be feasible for application in heavy-duty truck drivetrain systems. The analysis was performed with normal distribution of varying parameters and coefficient of variation equal to 0.15.

Table 1. Settings for GSA and Pareto optimization of a drivetrain system with DMF.

Design parameter, $\mathbf{d}$	J_p, kgm^2	J_s, kgm^2	k_1, Nm/rad	c_1, Nms/rad
Nominal values, $\mathbf{d}$	1.8	0.9	12 732	30
Lower bounds, $\mathbf{d}$	0.2	0.1	2 000	0
Upper bounds, $\mathbf{d}$	2.4	1.2	26 242	150

The solution of the global sensitivity problem for engine speeds in the range of $600-2000$ rpm is depicted in Fig. 2. The solution is presented by means of mapping between the design parameters $d_1 = J_p$, $d_2 = J_s$, $d_3 = k_1$, $d_4 = c_1$ and the values of total sensitivity indices of the objective functions (20)–(22).

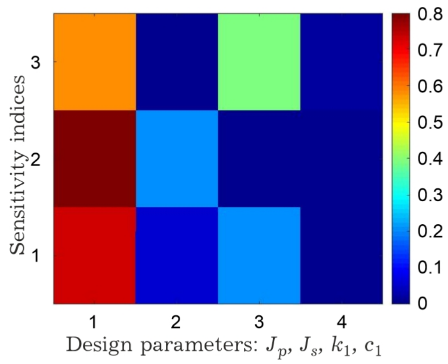

Fig. 2. Sensitivity indices of the objective functions $F_1(\mathbf{d})$, $F_2(\mathbf{d})$, $F_3(\mathbf{d})$ for the DMF in the operating engine speed range 600 rpm $\le n_e \le 2000$ rpm.

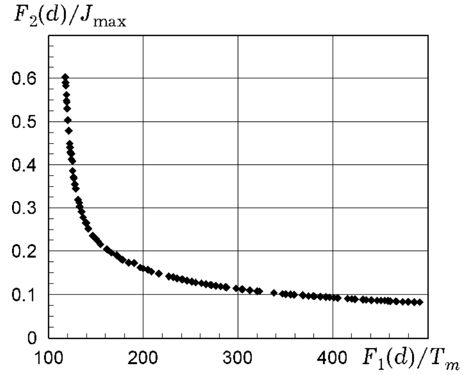

Fig. 3. Pareto front of weigh-vibration Pareto optimization for the DMF in operating speed range 600 rpm $\le n_e \le 2000$ rpm, $T_m = 300$ Nm, $J_{\max} = 4.05$ kgm^2.

2.3. Pareto optimization of a drivetrain system equipped with a DMF. The multi-objective optimization problem, formulated in Section 1, is considered now for the drivetrain system equipped with a DMF. The problem is stated as follows: for the feasible operational scenario that is given by expressions (16)–(18), it is required to determine the vector of the design parameters of the DMF

$$\mathbf{d} = [J_p^*,\ J_s^*, k_1^*,\ c_1^*]^\top = \mathbf{d}^* \in \Omega$$

and the torsional vibration dynamics $\mathbf{q}(t) = \mathbf{q}^*(t)$ that satisfy the variational equations

$$\min_{\mathbf{d}\in\Omega} \left\{ \int_{600}^{2000} \mathrm{std}\left(T_g[\mathbf{q}(t), \mathbf{d}, n_e]\right) dn_e \right\} = \int_{600}^{2000} \mathrm{std}\left(T_g[\mathbf{q}^*(t), \mathbf{d}^*, n_e]\right) dn_e \ ,$$

$$\min_{\mathbf{d}\in\Omega} \{J_p + J_s\} = J_p^* + J_s^*,$$

subject to the differential equation (4) with (14), the initial state (15), and the restrictions on the design parameters provided by the lower and upper bounds in Table 1.

This problem was solved by the computer code SAMO for the same operational scenarios as the problem of the global sensitivity analysis. The corresponding system of the differential equations was solved by using a MATLAB subroutine ode45 with absolute and relative tolerances equal to $1\mathrm{e}-5$. The setting of the genetic algorithm was as follows: population size $= 100$; number of generations $= 100$; elite count $= 4$; and Pareto fraction $= 1$.

The Pareto front, i.e., the best trade-off relationship between (20) and (21) obtained for engine speeds in the range of $600 - 2000\,\text{rpm}$, is shown in Fig. 3.

Every point of the Pareto front corresponds to the set of values of the design parameters of the DMF. The values of the design parameters J_p^*, J_s^*, k_1^*, c_1^* that minimize the objective function (20) are

$$[J_p^*, J_s^*, k_1^*, c_1^*]^\top = [2.34,\ 0.1,\ 3938,\ 30]^\top. \tag{24}$$

These values correspond to the highest point of the Pareto front. The DMF with the design parameters (24) performs the best attenuation of the torsional oscillation of the torque at the transmission input shaft with the value of the objective function (20) equal to $35250\,\text{Nm}$. The obtained design of the DMF is characterized by the feasible total mass inertia $J_p^* + J_s^* = 2.44\,\text{kgm}^2$.

The values of the design parameters J_p^*, J_s^*, k_1^*, c_1^* that minimize the objective function (21) are

$$[J_p^*, J_s^*, k_1^*, c_1^*]^\top = [0.23,\ 0.1,\ 4010,\ 14]^\top. \tag{25}$$

These values correspond to the lowest point of the Pareto front in Fig. 3. The DMF with the design parameters (25) is characterized by lowest feasible total mass inertia $J_p^* + J_s^* = 0.33\,\text{kgm}^2$ but attenuation of the torsional oscillation of the torque at the transmission input shaft with this design of the DMF much worse than in case of using the design parameters (24). The value of objective function (20) for the obtained design parameters (25) is about $150000\,\text{Nm}$.

3. Results and discussion. The application of the global sensitivity analysis and the Pareto optimization provides deep insight into torsional vibration dynamics of a generic drivetrain system with a DMF. The chosen functionals (20)–(22) are appropriate to focus the design process for the vibration absorber on the best attenuation of the oscillation of the torque at the transmission input shaft, to minimize its weight, as well as decrease the energy dissipation in the stiffness-damping interface of the absorber.

The results of the global sensitivity analysis of the drivetrain system with respect to the design parameters of the DMF, presented in Section 2 (Fig. 2), make it possible to conclude the following. For the drivetrain system equipped with the DMF in the operating engine speed range $600\,\text{rpm} \le n_e \le 2000\,\text{rpm}$ the moment of inertia of the primary flywheel, J_p, as well as the stiffness of the absorber, k_1, mostly affect the vibration attenuation and the energy efficiency of the design of the vibration absorber. The weight of the absorber, as it is expected, depends on mass inertia parameters only.

The solution of the Pareto optimization problem, presented in Section 2, shows that there exist a clear trade-off between the measure of the oscillation attenuation of the torque at the transmission input shaft and the total mass inertia characteristics (the weight) of the optimized DMF of the drivetrain system in the operating engine speed range $600\,\text{rpm} \le n_e \le 2000\,\text{rpm}$ (Fig. 3).

The standard deviation of the torques at the transmission input shaft as a function of the engine speed for the DMF with nominal design parameters

$$[J_p, J_s, k_1, c_1]^\top = [1.8,\ 0.9,\ 12732,\ 30]^\top \tag{26}$$

(curve ***1***) and for the absorber with the weight-vibration optimized design parameters (24) (curve ***3***) are depicted in Figs. 4 and 5 for different ranges of the engine speeds. Analysis of Figs. 4, 5 shows that the efficiency of the attenuation of the oscillation of the torque at the transmission input shaft by using weight-vibration Pareto optimized DMF has significantly increased in comparison to the performance of the DMF with nominal design parameters. For instance, in case of engine speed $n_e = 1200$ rpm, the standard deviation of the torque at the transmission input shaft ($\mathrm{std}[T_g(t)]$) in drivetrain system with nominal design parameters of the DMF is equal to 114 Nm, and it is decreased to the value of $\mathrm{std}[T_g(t)] = 24$ Nm in case of using the DMF with obtained weight-vibration optimized parameters (24). As it follows from Figs. 4, 5, both resonance peaks of the curves ***1*** significantly reduced in case of using the DMF with weight-vibration optimized parameters.

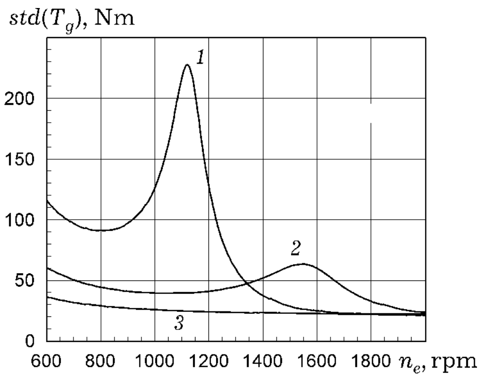

Fig. 4. Standard deviation of the torques at the transmission input shaft in the operating engine speed range 600 rpm $\le n_e \le 2000$ rpm for the DMF with nominal design parameters (26) (curve ***1***) and with weight-vibration optimized parameter (24) (curve ***3***), as well as with energy-vibration optimized parameters (27) for the DMF (curve ***2***).

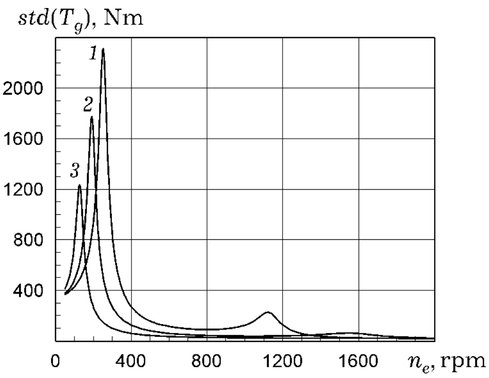

Fig. 5. Standard deviation of the torques at the transmission input shaft in the operating engine speed range 50 rpm $\le n_e \le 2000$ rpm for the DMF with nominal design parameters (26) (curve ***1***) and with weight-vibration optimized parameter (24) (curve ***3***), as well as with energy-vibration optimized parameters (27) for the DMF (curve ***2***).

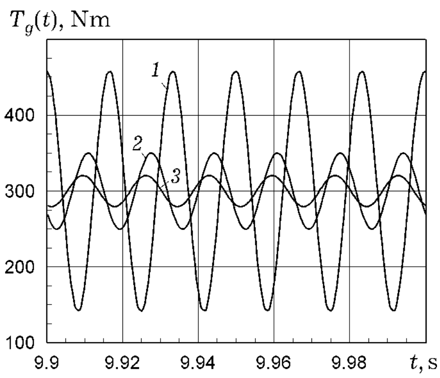

Fig. 6. The torques at the transmission input shaft for the engine speed 1200 rpm for the DMF with nominal design parameters (26) (curve ***1***) and with weight-vibration optimized parameters (24) (curve ***3***), as well as with energy-vibration optimized parameters (27) for the DMF (curve ***2***).

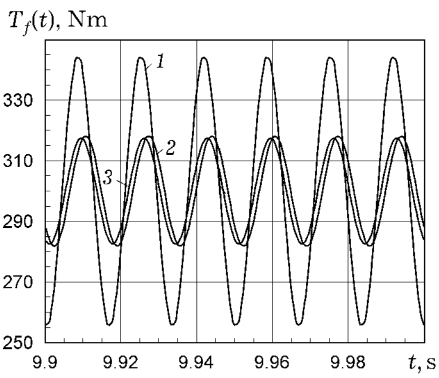

Fig. 7. The friction torques at the stiffness-damping interface of the DMF for the engine speed 1200 rpm for the nominal design parameters (26) (curve ***1***) and with weight-vibration optimized parameters (24) (curve ***3***), as well as with energy-vibration optimized parameters (27) for the DMF (curve ***2***).

Figures 6 and 7 present the time history of the torques at the transmission input shaft (18), as well as the time history of the friction torques (23), illustrating how much the DMF with optimized design parameters (24) can enhance the attenuation of the torques' oscillations in comparison to the DMF with nominal design parameters (26).

The choice of objective functions is an important step in the design optimization of an engineering system. Earlier in [1], different functionals were proposed for design optimization of vibration absorbers for heavy-duty truck drivetrain systems. The same as in Section 2.3, the Pareto optimization problem was formulated and solved for the drivetrain system equipped with the DMF in case of minimizing the objective functions (20), (22) subject to the differential equation (4) with (14), the initial state (15), and the restrictions on the design parameters provided by the lower and upper bounds in Table 1.

The functionals (20), (22) characterize the energy of the oscillations of the torque at the transmission input shaft and the energy dissipating in a DMF in the operating engine speed range $600\ \text{rpm} \le n_e \le 2000\ \text{rpm}$. It is believed that by minimizing these functionals at the same time, the obtained design parameters increase the energy efficiency of a vibration absorber.

The obtained values of the design parameters of the DMF which minimize the objective function (20) in the Pareto optimization problem (20), (22) are as follows [1]:

$$[J_p^*, J_s^*, k_1^*, c_1^*]^\top = [2.7,\ 0.45,\ 10967,\ 41]^\top . \tag{27}$$

These values correspond to the highest point of the obtained Pareto front in the bi-objective optimization problem that was solved by using the functionals (20), (22). The DMF with the design parameters (27) performs the best attenuation of the torsional oscillation of the torque at the transmission input shaft of the drivetrain system in the operating engine speed range $600\ \text{rpm} \le n_e \le 2000\ \text{rpm}$.

In Figs. 4–7, curve ***2*** represents the corresponding characteristics obtained by solving the energy-vibration Pareto optimization problem (20), (22) for the drivetrain system equipped with the DMF [1]. Analysis of Figs. 4–7 shows that within the framework of considered assumptions, the weight-vibration Pareto optimized DMF attenuate the oscillations of the torques at the transmission input shaft much better in comparison to the performance of the DMF with energy-vibration optimized design parameters (27) obtained earlier in [1].

The quantitative analysis of the values of the nominal design parameters (26), the weight-vibration optimized parameters (24), and the energy-vibration optimized parameters (27) of the DMF shows that the solution of the weight-vibration Pareto optimization problem is resulted in the lowest total mass inertia of the vibration absorber. This can be a significant advantage of the weight-vibration optimized DMF for its implementation in real drivetrain systems.

Conclusions and outlook. The following concluding remarks can be drawn.

- The methodology of multi-objective optimal design of engineering systems based on global sensitivity analysis and Pareto optimization has been proven to be efficient for advanced analysis and designing of torsional vibration absorbers for drivetrain systems.
- There exists a clear trade-off between the measure of oscillation attenuation of the torque at the transmission input shaft and the measure of the total weight in designing of the DMF for heavy-duty truck drivetrain systems.
- For a heavy-duty truck drivetrain system equipped with a DMF there exists the weight-vibration bi-objective optimized mass inertia, stiffness

and damping parameters providing the best attenuation of oscillation of the torque at the transmission input shaft in the operating engine speed range $600 - 2000\,\text{rpm}$, when the third engine order vibration harmonic is in focus.

- The results obtained show evidence of feasibility of application of the weight-vibration optimized dual mass flywheels in heavy-duty truck drivetrain systems.

Verification and validation of the results obtained using a complete model of a drivetrain system of a heavy-duty truck [9], as well as experimental data are important next steps of the study [10].

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1. *Berbyuk V*. Design optimization of torsional vibration absorbers for heavy-duty truck drivetrain systems // Vibration. – 2019. – **2**, No. 3. – P. 240–264. – https://doi.org/10.3390/vibration2030015.
2. *Berbyuk V*. Towards optimal design of engineering systems // In: R. M. Kushnir, H. S. Kit (eds.). Mathematical Problems of Mechanics of Nonhomogeneous Structures: Proc. of the 10th Intern. Sci. Conf. – L'viv: Pidstryhach Institute for Applied Problems of Mechanics and Mathematics of NAS of Ukraine, 2019. – **5**. – P. 278–279.
3. *Mall P., Fidlin A., Krüger A., Groß H*. Simulation based optimization of torsional vibration dampers in automotive powertrains // Mech. Mach. Theory. – 2017. – **115**. – P. 244–266. – https://doi.org/10.1016/j.mechmachtheory.2017.05.010.
4. *Mousavi Bideleh S. M., Berbyuk V*. A computer code for sensitivity analysis and multiobjective optimization: SAMO tutorial. – Research report 2017:01. – Göteborg: Department of Mechanics and Maritime Sciences, Chalmers University of Technology, Sweden, 2017. – 49 p.
5. *Mousavi Bideleh S. M., Berbyuk V*. Global sensitivity analysis of bogie dynamics with respect to suspension components // Multibody Syst. Dyn. – 2016. – **37**, No. 2. – P. 145–174. – https://doi.org/10.1007/s11044-015-9497-0.
6. *Mousavi Bideleh S. M., Berbyuk V*. Multiobjective optimisation of bogie suspension to boost speed on curves // Vehicle Syst. Dyn. – 2016. – **54,** No. 1. – P. 58–85. – https://doi.org/10.1080/00423114.2015.1114655.
7. *Mousavi Bideleh S. M., Berbyuk V*. Pareto optimization of a nonlinear tuned mass damper to control vibrations in hand held impact machines // In G. Kerschen (ed.) Nonlinear Dynamics / Proc. of the 36th IMAC, A Conference and Exposition on Structural Dynamics 2018. – Vol. 1. – Cham: Springer, 2019. – P. 27–44. – https://doi.org/10.1007/978-3-319-74280-9_4.
8. *Song L. Q., Zeng L. P., Zhang S. P., Zhou J. D., Niu H. E*. Design and analysis of a dual mass flywheel with continuously variable stiffness based on compensation principle // Mech. Mach. Theory. – 2014. – **79**. – P. 124–140. – https://doi.org/10.1016/j.mechmachtheory.2014.04.004.
9. *Wramner L*. Dual mass flywheels in truck powertrains: Modelling, simulations and validation // Thesis for the degree of Licentiate, Department of Mechanics and Maritime Sciences, Chalmers University of Technology, Göteborg, Sweden. – 2018. – 22 p.
10. *Wramner L., Berbyuk V., Johansson H*. Vibration dynamics in non-linear dual mass flywheels for heavy-duty trucks // Proc. of International Conference on Noise and Vibration Engineering (ISMA 2018), Leuven, Belgium, 2018. – P. 1863–1876.
11. *Zhang X., Pandey M. D*. An effective approximation for variance-based global sensitivity analysis //Reliab. Eng. Syst. Saf. – 2014. – **121**. – P. 164–174. – https://doi.org/10.1016/j.ress.2013.07.010.

ПАРЕТО ОПТИМІЗАЦІЯ ЗА ВАГОЮ ТА ВІБРАЦІЄЮ МАХОВИКА ПОДВІЙНОЇ МАСИ

З використанням методології багатоцільового оптимального проектування інженерних систем розглянуто проблему Парето оптимізації за вагою та вібрацією маховика з подвійною масою з метою вивчення доцільності його застосування у силових агрегатах вантажних автомобілів. Отримані результати показують таке: розв'язок розглянутої задачі оптимізації існує; параметри маси, жорст-

кості та в'язкості вібродемпфера, оптимізовані в робочому діапазоні частот обертів двигуна 600–2000 об/хв, існують та забезпечують найкраще гасіння коливань крутного моменту на вхідному валу передачі. Результати свідчать про доцільність застосування маховика з подвійною масою, оптимізованого за вагою та вібрації, в системах приводу важких вантажних автомобілів.

Ключові слова: *крутний вібраційний поглинач, маховик подвійної маси, система приводу грузовика великої вантажопідйомності, аналіз глобальної чутливості, Парето оптимізація за вагою та вібрації.*

ПАРЕТО ОПТИМИЗАЦИЯ ПО ВЕСУ И ВИБРАЦИИ МАХОВИКА ДВОЙНОЙ МАССЫ

С использованием методологии многоцелевого оптимального проектирования инженерных систем рассмотрена проблема Парето оптимизации по весу и вибрации маховика с двойной массой с целью изучения целесообразности его применения в силовых агрегатах грузовых автомобилей. Полученные результаты показывают следующее: решение рассматриваемой задачи оптимизации существует; параметры массы, жесткости и вязкости вибродемпфера, оптимизированные в рабочем диапазоне частот вращения двигателя 600–2000 об/мин, существуют и обеспечивают наилучшее гашение колебаний крутящего момента на входном валу передачи. Результаты свидетельствуют о целесообразности применения маховика с двойной массой, оптимизированного по весу и вибрации, в системах привода тяжелых грузовых автомобилей.

Ключевые слова: *крутящий вибрационный поглотитель, маховик двойной массы, система привода грузовика большой грузоподъемности, анализ глобальной чувствительности, Парето оптимизация по весу и вибрации.*

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