UDK 539.3

B. Rogowski

TORSION OF FIBER-REINFORCED COMPOSITE WITH CYLINDRICAL INTERFACE CRACK

The mode III axisymmetric cylindrical interface crack problem for circular cylinder composite structure is considered. The crack between two different homogeneous orthotropic materials in the form of a composite cylinder is subjected to internal loading in circumferential direction. The displacements and stresses are written in terms of inverse Fourier transforms with respect to the direction of cylinder axis. The elasticity boundary-value problem is reduced to the integral equations, which next are inverted to two uncoupled infinite systems of simultaneous algebraic equations. The main results of the study are the stress intensity factors, the displacement and stress fields and the crack opening displacements obtained as functions of composite geometric and material parameters.

Introduction. The fracture and fatigue of composites are very important from engineering practice point of view and are still being extensively developed (see S. L. Bai and G. K. Hu [1], for instance). The initiation and growth of a cylindrically shaped crack along an interface is important for the failure analysis for composite structures containing the interface cracks of cylindrical shape. The elasticity problem of interface crack between two isotropic solids was analyzed first by F. Erdogan and T. Ozbek [2, 5]. Later on, a cylindrical crack in a homogeneous transversally isotropic solid was analyzed by H. Kasano et al. [3]. The cylindrical interface crack model between two anisotropic solids (H. Kasano et al. [4]) has been improved by the author (B. Rogowski [6]) concerning the solution for the problem of a cylindrical crack on bimaterial interface in an infinite elastic solid under torsion.

The cylindrical interface crack solution for a two-phase composite cylinder is obtained in this paper. The single elastic orthotropic solid cylinder (bar or fiber) is considered. It is bonded through a cylindrical interface $r = a$, $|z| > c$, and debonded on a remaining part of the interface $r = a$, $|z| < c$ (cf. Fig. 1). Both constituents of the composite cylinder are modelled as three-dimensional linear elastic orthotropic bodies. It is assumed that the length of the cylindrically shaped crack is small compared to the length of the cylinder and the geometry of the medium, and the loads applied in circumferential direction are axisymmetric. Then, through a proper superposition the problem has been reduced to a perturbation problem, in which the crack surface tractions are the only external loads and the cylinder can be assumed to be infinitely long. Needless to say, from the viewpoint of fracture mechanics, the perturbation problem would contain all the relevant information such as the stress intensity factors and the crack-opening displacements.

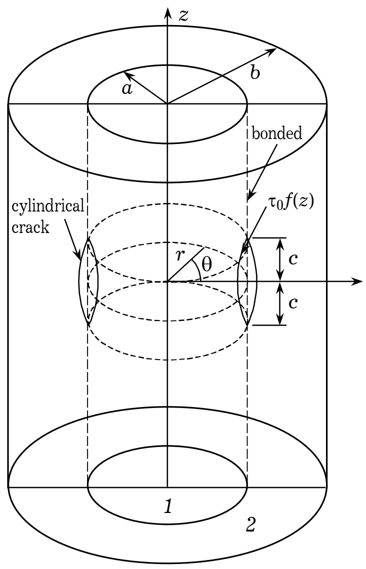

Fig. 1

Linear elasticity problem formulation. Let us consider a cylinder specimen of the composite material made of fiber with the radius denoted by a and surrounding matrix with the external radius being equal to b. The composite components are linear elastic and orthotropic with the radial shear moduli taken as $G_r^{(1)}$ for fiber and $G_r^{(2)}$ for matrix while the longitudinal ones are equal to $G_z^{(1)}$ and $G_z^{(2)}$, respectively. The composite so defined contains a cylindrically shaped interface crack of radius a and the length equal to 2c. The crack faces are loaded by internal tangential traction equal to $\tau_0 f(z)$ resulting in torsional axisymmetric deformation (cf. Fig. 1).

The following boundary conditions are imposed:

$$\sigma_{r\theta}^{(1)}(a,z) = \sigma_{r\theta}^{(2)}(a,z), \qquad |z| < \infty, \tag{1}$$

$$\sigma_{r\theta}^{(1)}(a,z) = -\tau_0 f(z), \qquad |z| < c, \tag{2}$$

$$v^{(1)}(a,z) = v^{(2)}(a,z), \qquad |z| > c \tag{3}$$

$$\sigma_{r\theta}^{(2)}(b,z) = 0, \qquad |z| < \infty, \tag{4}$$

where σ represents the components of stress, v is the circumferential displacement, while the upper indices refer to the fiber (*1*) and matrix (*2*).

In the cylindrical co-ordinates (r,θ,z) the torsional displacement $v(r,z)$ of a problem with axial symmetry satisfies the following equilibrium equation:

$$G_r\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} - \frac{v}{r^2}\right) + G_z\frac{\partial^2 v}{\partial z^2} = 0\,. \tag{5}$$

The displacement $v(r,z)$ may be expressed in the form

$$v^{(1)}(r,z) = \int_0^\infty p^{-1} J_1\left(\frac{pr}{s_1}\right)\left[A(p)\sin(pz) + B(p)\cos(pz)\right]dp, \quad 0 \le r \le a\,, \tag{6}$$

$$v^{(2)}(r,z) = \int_0^\infty p^{-1} C_1\left(\frac{pr}{s_2}\right)\left[C(p)\sin(pz) + D(p)\cos(pz)\right]dp, \quad a \le r \le b\,, \tag{7}$$

and yields the stresses

$$\sigma_{r\theta}^{(1)} = G_z^{(1)}\, s_1 \int_0^\infty J_2\left(\frac{pr}{s_1}\right)\left[A(p)\sin(pz) + B(p)\cos(pz)\right]dp\,, \tag{8}$$

$$\sigma_{\theta z}^{(1)} = G_z^{(1)} \int_0^\infty J_1\left(\frac{pr}{s_1}\right)\left[A(p)\cos(pz) - B(p)\sin(pz)\right]dp, \quad 0 \le r \le a\,, \tag{9}$$

$$\sigma_{r\theta}^{(2)} = -\,G_z^{(2)}\, s_2 \int_0^\infty C_2\left(\frac{pr}{s_2}\right)\left[C(p)\sin(pz) + D(p)\cos(pz)\right]dp\,, \tag{10}$$

$$\sigma_{\theta z}^{(2)} = G_z^{(2)} \int_0^\infty C_1\left(\frac{pr}{s_2}\right)\left[C(p)\cos(pz) - D(p)\sin(pz)\right]dp, \quad a \le r \le b\,. \tag{11}$$

In above Eqs (6)–(11) we use the following notations:

$$C_n\left(\frac{pr}{s_2}\right) = Y_2\left(\frac{pb}{s_2}\right)J_n\left(\frac{pr}{s_2}\right) - J_2\left(\frac{pb}{s_2}\right)Y_n\left(\frac{pr}{s_2}\right), \qquad n = 1,2\,, \tag{12}$$

$J_n(\cdot)$, $Y_n(\cdot)$ are the Bessel functions of the first and second kinds of order n such that

$$J_n(x) = \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{x}{2}\right)^{n+2k}}{k!\,\Gamma(n+k+1)}, \qquad Y_n(x) = \lim_{m\to n} \frac{J_m(x)\cos m\pi - J_{-m}(x)}{\sin m\pi}.$$

The orthotropy measures for both of the composite components are

$$s_1 = \sqrt{\frac{G_r^{(1)}}{G_z^{(1)}}}, \qquad s_2 = \sqrt{\frac{G_r^{(2)}}{G_z^{(2)}}}.$$

The continuity condition (1) leads to the following relation:

$$\left\{ \begin{array}{c} C(p) \\ D(p) \end{array} \right\} = -\mu \frac{J_2\left(\frac{pa}{s_1}\right)}{C_2\left(\frac{pa}{s_2}\right)} \left\{ \begin{array}{c} A(p) \\ B(p) \end{array} \right\},$$

where

$$\mu = \frac{G_z^{(1)}\, s_1}{G_z^{(2)}\, s_2},$$

and condition (4) is identically satisfied by Eqs (10), (12).

There remain two unknown functions $A(p)$ and $B(p)$, for which the following dual-integral equation is obtained

$$\int_0^{\infty} J_2\left(\frac{pa}{s_1}\right)\left[A(p)\sin(pz) + B(p)\cos(pz)\right] dp = -\frac{\tau_0 f(z)}{G_z^{(1)}\, s_1}, \quad |z| < c, \tag{13}$$

$$\int_0^{\infty} p^{-1} J_2\left(\frac{pa}{s_1}\right)\left[\frac{J_1\left(\frac{pa}{s_1}\right)}{J_2\left(\frac{pa}{s_1}\right)} + \mu \frac{C_1\left(\frac{pa}{s_2}\right)}{C_2\left(\frac{pa}{s_2}\right)} \right] \times$$

$$\times\left[A(p)\sin(pz) + B(p)\cos(pz)\right] dp = 0, \quad |z| > c. \tag{14}$$

The series solution. We introduce the following dimensionless quantities:

$$r = \rho c, \qquad z = \xi c, \qquad c = \lambda_0 a, \qquad c = \lambda_1 b, \qquad q = pc. \tag{15}$$

In order to solve Eqs (13), (14), the following series representation is introduced identically satisfying Eq. (14):

$$\left\{ \begin{array}{c} A(q) \\ B(q) \end{array} \right\} J_2\left(\frac{q}{s_1\lambda_0}\right)\left[\frac{J_1\left(\frac{q}{s_1\lambda_0}\right)}{J_2\left(\frac{q}{s_1\lambda_0}\right)} + \mu \frac{C_1\left(\frac{q}{s_1\lambda_0}\right)}{C_2\left(\frac{q}{s_1\lambda_0}\right)} \right] =$$

$$= -\frac{\tau_0\, c}{G_z^{(1)}\, s_1} \left\{ \begin{array}{c} \sum_{n=1}^{\infty} y_n J_{2n}(q) \\ \sum_{n=0}^{\infty} x_n J_{2n+1}(q) \end{array} \right\}, \tag{16}$$

where $J_\nu(q)$ is the Bessel function of the first kind of order ν.

Substituting Eq. (16) into (13) and by using notation (15) together with the following series representation:

$$\cos(q\xi) = J_0(q) + 2\sum_{m=1}^{\infty} J_{2m}(q)\cos(2m\alpha),$$

$$\sin(q\xi) = 2\sum_{m=1}^{\infty} J_{2m-1}(q)\sin[(2m-1)\alpha], \qquad \xi = \sin\alpha\,,$$

$$f(\xi) = a_0 + 2\sum_{m=1}^{\infty}\left(a_m \cos(2m\alpha) + b_m \sin[(2m-1)\alpha]\right),$$

$$\left\{\begin{matrix} a_m \\ b_m \end{matrix}\right\} = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} f(\alpha)\left\{\begin{matrix} \cos(2m\alpha) \\ \sin[(2m-1)\alpha] \end{matrix}\right\} d\alpha, \qquad |\,\xi\,| \le 1, \qquad |\,\alpha\,| \le \frac{\pi}{2}\,,$$

the following uncoupled infinite systems of simultaneous algebraic equations for the determination of the unknown coefficients x_n and y_n are obtained as

$$\sum_{n=0}^{\infty} x_n \int_0^{\infty} \frac{J_{2n+1}(q)\, J_{2m}(q)}{\dfrac{J_1}{J_2} + \mu\dfrac{C_1}{C_2}}\, dq = a_m, \qquad m = 0,1,2,\ldots,$$

$$\sum_{n=1}^{\infty} y_n \int_0^{\infty} \frac{J_{2n}(q)\, J_{2m-1}(q)}{\dfrac{J_1}{J_2} + \mu\dfrac{C_1}{C_2}}\, dq = b_m, \qquad m = 1,2,\ldots,$$

where

$$J_i = J_i\left(\frac{q}{s_i\,\lambda_0}\right), \qquad C_i = C_i\left(\frac{q}{s_i\,\lambda_0}\right), \qquad i = 1,2\,.$$

Using the following relation

$$\frac{J_{2m}(q)}{q} = \frac{J_{2m-1}(q) + J_{2m+1}(q)}{4m}\,,$$

adding the m-th equation to the $(m+1)$-th equation and dividing by $2(2m+1)$ or $4m$, we obtain

$$\sum_{n=0}^{\infty} x_n A_{mn} = \frac{a_m + a_{m+1}}{2(2m+1)}\,, \qquad m = 0,1,2,\ldots,$$

$$\sum_{n=1}^{\infty} y_n B_{mn} = \frac{b_m + b_{m+1}}{4m}\,, \qquad m = 1,2,\ldots,$$

where

$$A_{mn} = \int_0^{\infty} \frac{J_{2n+1}(q)\, J_{2m+1}(q)}{q\left(\dfrac{J_1}{J_2} + \mu\dfrac{C_1}{C_2}\right)}\, dq, \qquad m,\, n = 0,1,2,\ldots,$$

$$B_{mn} = \int_0^{\infty} \frac{J_{2n}(q)\, J_{2m}(q)}{q\left(\dfrac{J_1}{J_2} + \mu\dfrac{C_1}{C_2}\right)}\, dq, \qquad m,\, n = 1,2,\ldots,$$

are the matrices symmetric with respect to m and n.

The displacement on the cylindrical interface $r = a$ is expressed by

$$v^{(1)}(a,\xi) = -\frac{\tau_0 c}{G_z^{(1)}\, s_1}\,\frac{1}{1+\mu}\left\{\sum_{n=0}^{\infty} x_n\left[\frac{T_{2n+1}\left(\sqrt{1-\xi^2}\right)}{2n+1}\, H(1-|\,\xi\,|) + \right.\right.$$

$$+\mu\int_0^{q_0} q^{-1}J_{2n+1}(q)\,f(q)\cos(q\xi)\,dq+\mu\sum_{i=1}^{\infty}a_i\int_{q_0}^{\infty}q^{-(i+1)}J_{2n+1}(q)\cos(q\xi)\,dq\Bigg]+$$

$$+\sum_{n=1}^{\infty}y_n\Bigg[\xi\frac{U_{2n-1}\left(\sqrt{1-\xi^2}\right)}{2n}H(1-|\xi|)+\mu\int_0^{q_0}q^{-1}J_{2n}(q)f(q)\sin(q\xi)\,dq+$$

$$+\mu\sum_{i=1}^{\infty}a_i\int_{q_0}^{\infty}q^{-(i+1)}J_{2n}(q)\sin(q\xi)\,dq\Bigg]\Bigg\}, \tag{17}$$

$$v^{(2)}(a,\xi)=-\frac{\tau_0 c}{G_z^{(2)}\,s_1}\,\frac{\mu}{1+\mu}\Bigg\{\sum_{n=0}^{\infty}x_n\Bigg[\frac{T_{2n+1}\left(\sqrt{1-\xi^2}\right)}{2n+1}H(1-|\xi|)-$$

$$-\int_0^{q_0}q^{-1}J_{2n+1}(q)\,f(q)\cos(q\xi)\,dq-\sum_{i=1}^{\infty}a_i\int_{q_0}^{\infty}q^{-(i+1)}J_{2n+1}(q)\cos(q\xi)\,dq\Bigg]+$$

$$+\sum_{n=1}^{\infty}y_n\Bigg[\xi\frac{U_{2n-1}\left(\sqrt{1-\xi^2}\right)}{2n}H(1-|\xi|)-\int_0^{q_0}q^{-1}J_{2n}(q)\,f(q)\sin(q\xi)\,dq-$$

$$-\sum_{i=1}^{\infty}a_i\int_{q_0}^{\infty}q^{-(i+1)}J_{2n}(q)\sin(q\xi)\,dq\Bigg]\Bigg\}. \tag{18}$$

By the analogous way the stress field components within the composite are derived as

$$\sigma_{r\theta}^{(1)}(a,\xi)=-\frac{\tau_0}{1+\mu}\Bigg\{\sum_{n=0}^{\infty}x_n\Bigg[\frac{T_{2n+1}\left(\sqrt{1-\xi^2}\right)}{\sqrt{1-\xi^2}}H(1-|\xi|)-$$

$$-(-1)^n\frac{R_{2n+1}(\xi)}{\sqrt{\xi^2-1}}H(|\xi|-1)+\int_0^{q_0}J_{2n+1}(q)\cos(q\xi)\,dq+$$

$$+\sum_{i=1}^{\infty}b_i\int_{q_0}^{\infty}q^{-i}J_{2n+1}(q)\cos(q\xi)\,dq\Bigg]+\sum_{n=1}^{\infty}y_n\Bigg[\xi\frac{U_{2n-1}\left(\sqrt{1-\xi^2}\right)}{\sqrt{1-\xi^2}}H(1-|\xi|)+$$

$$+(-1)^n\,\mathrm{sgn}(\xi)\frac{R_{2n}(\xi)}{\sqrt{\xi^2-1}}H(|\xi|-1)+\int_0^{q_0}J_{2n}(q)\,g(q)\sin(q\xi)\,dq+$$

$$+\sum_{i=1}^{\infty}b_i\int_{q_0}^{\infty}q^{-i}J_{2n}(q)\sin(q\xi)\,dq\Bigg]\Bigg\}=\sigma_{r\theta}^{(2)}(a,\xi), \tag{19}$$

$$\sigma_{\theta z}^{(1)}(a,\xi) = -\frac{\tau_0}{(1+\mu)\,s_1}\left\{ -\sum_{n=0}^{\infty} x_n \left[\xi \frac{U_{2n}\left(\sqrt{1-\xi^2}\right)}{\sqrt{1-\xi^2}} H(1-|\xi|) - \right.\right.$$

$$\left. + \mu \int_0^{q_0} J_{2n+1}(q)\, f(q) \sin(q\xi)\, dq + \mu \sum_{i=1}^{\infty} a_i \int_{q_0}^{\infty} q^{-i} J_{2n+1}(q) f(q) \sin(q\xi)\, dq \right] +$$

$$+ \sum_{n=1}^{\infty} y_n \left[(-1)^n \frac{T_{2n}(\xi)}{\sqrt{1-\xi^2}} H(1-|\xi|) + \mu \int_0^{q_0} J_{2n}(q)\, f(q) \cos(q\xi)\, dq + \right.$$

$$\left.\left. + \mu \sum_{i=1}^{\infty} a_i \int_{q_0}^{\infty} q^{-i} J_{2n}(q) \cos(q\xi)\, dq \right]\right\}, \tag{20}$$

$$\sigma_{\theta z}^{(2)}(a,\xi) = \frac{\tau_0}{(1+\mu)\,s_2}\left\{ -\sum_{n=0}^{\infty} x_n \left[\xi \frac{U_{2n}\left(\sqrt{1-\xi^2}\right)}{\sqrt{1-\xi^2}} H(1-|\xi|) - \right.\right.$$

$$\left. + \int_0^{q_0} J_{2n+1}(q)\, f(q) \sin(q\xi)\, dq - \sum_{i=1}^{\infty} a_i \int_{q_0}^{\infty} q^{-i} J_{2n+1}(q) f(q) \sin(q\xi)\, dq \right] +$$

$$+ \sum_{n=1}^{\infty} y_n \left[(-1)^n \frac{T_{2n}(\xi)}{\sqrt{1-\xi^2}} H(1-|\xi|) - \int_0^{q_0} J_{2n}(q)\, f(q) \cos(q\xi)\, dq + \right.$$

$$\left.\left. - \sum_{i=1}^{\infty} a_i \int_{q_0}^{\infty} q^{-i} J_{2n}(q) \cos(q\xi)\, dq \right]\right\}, \tag{21}$$

where $T_n(x)$, $U_n(x)$ are the Chebyshev polynomials of the first and second kind, respectively, and

$$R_n(\xi) = \left(|\xi| - \sqrt{\xi^2 - 1} \right)^n, \qquad |\xi| > 1,$$

$$f(q) = \frac{\dfrac{J_1}{J_2} - \dfrac{C_1}{C_2}}{\dfrac{J_1}{J_2} + \mu \dfrac{C_1}{C_2}}, \qquad g(q) = \frac{1 - \dfrac{J_1}{J_2} + \mu\left(1 - \dfrac{C_1}{C_2}\right)}{\dfrac{J_1}{J_2} + \mu \dfrac{C_1}{C_2}}.$$

The a_i and b_i are coefficients of the asymptotic expansions

$$f(q) = \sum_{i=1}^{\infty} a_i q^{-i}, \qquad g(q) = \sum_{i=1}^{\infty} b_i q^{-i}, \qquad q \ge q_0,$$

where

$$a_1 = \frac{3}{2}\lambda_0 \frac{s_1 + s_2}{1+\mu},$$

$$a_2 = \frac{3}{4}\lambda_0^2 \frac{s_1 + s_2}{1+\mu}\left[\frac{5}{2}(s_1 - s_2) - 3\frac{s_1 - \mu s_2}{1+\mu}\right],$$

$$b_1 = -\frac{3}{2}\lambda_0 \frac{s_1 - \mu s_2}{1+\mu},$$

$$b_2 = -\frac{3}{4}\lambda_0^2 \frac{1}{1+\mu}\left[\frac{3}{2}(s_1^2 + s_2^2) - 9\frac{(s_1 - \mu s_2)^2}{1+\mu}\right].$$

In the solution (19) the following notation is used: $\mathrm{sgn}(\xi) = 1$ for $\xi > 1$, $\mathrm{sgn}(\xi) = -1$ for $\xi < -1$; $H(\cdot)$ is the Heaviside unit step function equal to unity for positive argument and zero for negative argument and q_0 is some large value of q.

From Eqs (17), (18) it is clear that the crack-opening displacement is expressed by

$$v^{(1)}(a,\xi) - v^{(2)}(a,\xi) =$$

$$= -\frac{\tau_0\, c}{G_z^{(1)}\, s_1}\left[\sum_{n=0}^{\infty} \frac{x_n}{2n+1} T_{2n+1}\left(\sqrt{1-\xi^2}\right) + \xi \sum_{n=1}^{\infty} \frac{y_n}{2n} U_{2n-1}\left(\sqrt{1-\xi^2}\right)\right],$$

and at $\xi = 0$ assumes the value

$$v^{(1)}(a,0) - v^{(2)}(a,0) = -\frac{\tau_0\, c}{G_z^{(1)}\, s_1} \sum_{n=0}^{\infty} \frac{x_n}{2n+1}.$$

We observe in Eqs (19)–(21) that the stress field $\sigma_{r\theta}$ has a singularity at $\xi \to 1+0$ and $\xi \to -1-0$ of order $(|\xi|-1)^{-1/2}$, while the stresses $\sigma_{\theta z}^{(i)}$, $i = 1,2$, have singularity at $\xi \to 1-0$ and $\xi \to -1+0$ of order $(1-|\xi|)^{-1/2}$. To evaluate the strength of these stress singularities, the stress intensity factors are defined as follows:

$$K_{r\theta}^{\pm} = \lim_{\xi \to \pm 1 \pm 0} \sqrt{2c(|\xi|-1)}\, \left[\sigma_{r\theta}^{\pm}\right]_{|\xi|>1}, \tag{22}$$

$$K_{\theta z}^{\pm} = \lim_{\xi \to \pm 1 \mp 0} \sqrt{2c(1-|\xi|)}\, \left[\sigma_{\theta z}^{\pm}\right]_{|\xi|<1}. \tag{23}$$

Starting from Eqs (19)–(21) and (22), (23) we obtain

$$K_{r\theta}^{\pm} = -\frac{\tau_0 \sqrt{c}}{1+\mu}\left[-\sum_{n=0}^{\infty}(-1)^n x_n \pm \sum_{n=1}^{\infty}(-1)^n y_n\right], \qquad \text{in} \quad \Omega, \tag{24}$$

$$K_{\theta z}^{\pm} = -\frac{\tau_0 \sqrt{c}}{(1+\mu)s_1}\left[\mp \sum_{n=0}^{\infty}(-1)^n x_n + \sum_{n=1}^{\infty}(-1)^n y_n\right], \qquad \text{in} \quad \Omega_1, \tag{25}$$

$$K_{\theta z}^{\pm} = \frac{\tau_0 \sqrt{c}}{(1+\mu)s_2}\left[\mp \sum_{n=0}^{\infty}(-1)^n x_n + \sum_{n=1}^{\infty}(-1)^n y_n\right], \qquad \text{in} \quad \Omega_2, \tag{26}$$

where Ω_1 is the fiber domain, Ω_2 is the matrix domain and $\Omega = \Omega_1 \cup \Omega_2$.

The K^+ and K^- denote the stress intensity factors for the crack tips $\xi = 1$ and $\xi = -1$, respectively. It is noted, that $\sigma_{\theta z}$ undergoes a jump across the cylindrical interface, since

$$s_1\,\sigma_{\theta z}^{(1)}(a,\xi)-s_2\,\mu\,\sigma_{\theta z}^{(2)}(a,\xi)=$$

$$=\begin{cases}-\tau_0\left[-\sum\limits_{n=0}^{\infty}x_n\,\dfrac{\xi U_{2n}\left(\sqrt{1-\xi^2}\right)}{\sqrt{1-\xi^2}}+\sum\limits_{n=1}^{\infty}(-1)^n\,y_n\,\dfrac{T_{2n}(\xi)}{\sqrt{1-\xi^2}}\right], & |\,\xi\,|<1,\\ 0, & |\,\xi\,|>1.\end{cases}$$

This equation yields

$$\frac{\sigma_{\theta z}^{(1)}(a,\xi)}{G_z^{(1)}}=\frac{\sigma_{\theta z}^{(2)}(a,\xi)}{G_z^{(2)}},\qquad |\,\xi\,|>1\,,$$

$$\frac{K_{\theta z}^{(1)}}{G_z^{(1)}}-\frac{K_{\theta z}^{(2)}}{G_z^{(2)}}=-\,\frac{\tau_0\sqrt{c}}{G_z^{(1)}\,s_1}\left[-\sum_{n=0}^{\infty}(-1)^n\,x_n+\sum_{n=1}^{\infty}(-1)^n\,y_n\right],\qquad \xi\to 1-0\,.$$

For the crack tip $\xi=-1+0$ in the second of these equations the sign minus before the first series should be replaced by the sign plus.

The main problem arising in calculation of the particular engineering examples is the convergence of the coefficients x_n and y_n together with the series (24)–(26) giving the stress intensity factors. The matrices appearing in the algebraic equations must be integrated over the infinite interval $0\le q<\infty$. Convergence of the integrands as $q\to\infty$ and the numerical integration must be treated. Next, the infinite interval is separated into two subintervals $0\le q\le q_0$ and $q_0\le q<\infty$.

As a result, it is obtained that

$$A_{mn}=A'_{mn}+A''_{mn},\qquad B_{mn}=B'_{mn}+B''_{mn}\,,$$

where the first order coefficients are equal to

$$A'_{mn}=\int_0^{q_0}\frac{J_{2n+1}(q)\,J_{2m+1}(q)}{q\left(\dfrac{J_1}{J_2}+\mu\,\dfrac{C_1}{C_2}\right)}\,dq\,,$$

$$B'_{mn}=\int_0^{q_0}\frac{J_{2n}(q)\,J_{2m}(q)}{q\left(\dfrac{J_1}{J_2}+\mu\,\dfrac{C_1}{C_2}\right)}\,dq\,,$$

while the second order coefficients are

$$A''_{mn}=\frac{(-1)^{n+m}}{\pi(1+\mu)}\left[\frac{1-\sin(2q_0)}{q_0}+2\,\mathrm{Ci}\,(q_0)-\right.$$

$$\left.-\,\frac{3}{2}\,\lambda_0\,\frac{s_1-\mu s_2}{1+\mu}\left(\frac{1-\sin(2q_0)}{2q_0^2}-\frac{\cos(2q_0)}{q_0}-2\mathrm{si}\,(2q_0)\right)\right]+O\!\left(q_0^{-3}\right),$$

$$B''_{mn}=\frac{(-1)^{n+m}}{\pi(1+\mu)}\left[\frac{1+\sin(2q_0)}{q_0}-2\,\mathrm{Ci}\,(q_0)-\right.$$

$$\left.-\,\frac{3}{2}\,\lambda_0\,\frac{s_1-\mu s_2}{1+\mu}\left(\frac{1+\sin(2q_0)}{2q_0^2}+\frac{\cos(2q_0)}{q_0}+2\mathrm{si}\,(2q_0)\right)\right]+O\!\left(q_0^{-3}\right),$$

$O\!\left(q_0^{-3}\right)$ denotes the Landau symbol; $\mathrm{si}\,(q_0)$, $\mathrm{Ci}\,(q_0)$ are recognized as the sine and cosine integrals.

The integrals A'_{mn} and B'_{mn} may be easily integrated numerically, e.g. utilizing Simpson rule or using symbolic operations built-up in some mathematical computational packages, since the integrands are bounded and well-behaved. It is observed that the coefficients A''_{mn} and B''_{mn} represent small contributions to the total integrals A_{mn} and B_{mn} for a prescribed large value of q_0, therefore an appropriate choice of q_0 will give a prescribed accuracy of the solution.

Note that the coefficients x_n and y_n are associated with even and odd parts of the function $f(z)$, respectively, namely with

$$f_1(z) = \frac{f(z) + f(-z)}{2} \qquad \text{and} \qquad f_2(z) = \frac{f(z) - f(-z)}{2}.$$

If is $f(z)$ an even function, then $y_n = 0$, if $f(z)$ is an odd function, then $x_n = 0$.

1. *Bai S. L., Hu G. K.* A simple transverse damage model for unidirectional composites // Engng. Fract. Mech. – 1997. – **56**. – P. 57–64.
2. *Erdogan F., Ozbek T.* Stresses in fiber-reinforced composites with imperfect bonding // Trans. ASME. J. Appl. Mech. – 1969. – **16**. – P. 865–869.
3. *Kasano H., Matsumoto H., Nakahara I.* A cylindrical interface crack in a nonhomogeneous anisotropic elastic body // Bull. JSME. – 1986. – **29**. – P. 1973–1981.
4. *Kasano H., Matsumoto H., Nakahara I.* A torsion-free axisymmetric problem of a cylindrical crack in a transversally isotropic body // Bull. JSME. – 1984. – **27**. – P. 1323–1332.
5. *Ozbek T., Erdogan F.* Some elasticity problems in fiber-reinforced composites with imperfect bonds // Int. J. Engng. Sci. – 1969. – **7**. – P. 931–946.
6. *Rogowski B.* The cylindrical crack on bimaterial interface under torsion // Sc. Bull. TU of Kielce, 3rd Conf. Fracture Mechanics. – Poland, 1991.

КРУЧЕННЯ АРМОВАНОГО ВОЛОКНОМ КОМПОЗИТА З ЦИЛІНДРИЧНОЮ МІЖФАЗНОЮ ТРІЩИНОЮ

Розглянуто осесиметричну задачу кручення для кругового в плані циліндра з композитного матеріалу, послабленого міжфазною тріщиною. Тріщина на межі розділу двох ортотропних матеріалів піддана дії внутрішніх дотичних зусиль вздовж інтерфейсу. Переміщення і напруження записано через обернені перетворення Фур'є відносно осьової координати. Крайова задача пружності зведена до інтегральних рівнянь, з яких отримано дві незалежні нескінченні системи алгебричних рівнянь. Визначено коефіцієнти інтенсивності напружень, переміщення і напруження, а також розкриття тріщини залежно від геометричних і фізичних параметрів матеріалу композиту.

КРУЧЕНИЕ АРМИРОВАННОГО ВОЛОКНОМ КОМПОЗИТА С ЦИЛИНДРИЧЕСКОЙ МЕЖФАЗНОЙ ТРЕЩИНОЙ

Рассмотрена осесимметричная задача кручения для кругового в плане цилиндра из композитного материала, ослабленного межфазной трещиной. Трещина на границе раздела двух ортотропных материалов подвержена действию внутренних касательных усилий вдоль интерфейса. Перемещения и напряжения записаны через обратные преобразования Фурье относительно осевой координаты. Краевая задача упругости сведена к интегральным уравнениям, из которых получены две независимые бесконечные системы алгебраических уравнений. Определены коэффициенты интенсивности напряжений, перемещения и напряжения, а также раскрытие трещины в зависимости от геометрических и физических параметров материала композита.

Techn. Univ. of Lodz, Lodz, Poland

Received
16.04.03