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MULTIPOINT FORMULA BASED ON ASSOCIATED CONTINUED FRACTION

The m-point formula for a function approximation has been constructed by using a function expansion into an associated continued function of special type at m, $m \ge 2$, points and properties of functions being the unit factorization. The properties of a such associated continued fraction have been also investigated.

1. Preliminary investigation. It is known that generalizations of the classic Taylor formula have some advantages in comparison with the Taylor formula. O. M. Lytvyn and V. L. Rvachov [3] proposed to construct an approximate Taylor-like polynom, using the Taylor expansion at several points and connected them with special real functions being the unit factorization, i.e. the polynom of the form

$$T(x) = \sum_{m=1}^{k} h_m(x) \sum_{s=0}^{n} \frac{d^s f(x, x_m)}{s!}, \tag{1}$$

where $x \in \mathbb{R}$, $x_m \in [a, b] \subset \mathbb{R}$ and functions $h_m(x) \in \mathrm{C}^\infty$ are non-negative, $\sum_{m=1}^{k} h_m(x) \equiv 1$, and applied it for approximate solving of boundary problems for systems of differential equations. They also proposed several possibilities for a choice of the functions $h_m(x)$. For instance, if in (1) $k = 2$, $x_1 = a$, $x_2 = b$ it is convenient in practice to exploit functions $h_m(x)$ in the form

$$h_m(x) = \begin{cases} 0, & x \le x_{m-1}, \quad x \ge x_{m+1}, \\ g_p\left(\dfrac{x - x_{m-1}}{x_m - x_{m-1}}\right), & x_{m-1} \le x \le x_m, \\ g_p\left(\dfrac{x_{m+1} - x}{x_{m+1} - x_m}\right), & x_m \le x \le x_{m+1}, \end{cases}$$

where

$$g_p(x) = \frac{(2p+1)!}{(p!)^2} \frac{\int_0^x t^p (1-t)^p \, dt}{\int_0^1 t^p (1-t)^p \, dt}, \qquad p \ge n .$$

It was interesting to construct multipoint approximate formula, using rational polynomials, in particular, the Thiele polynomial [4], closely connected with the Taylor expansion. In [1] the formula like (1) was constructed

$$R(x) = \sum_{m=1}^{k} h_m(x) \frac{P_n(x, x_m)}{Q_n(x, x_m)}, \tag{2}$$

where $\dfrac{P_n(x, x_m)}{Q_n(x, x_m)}$ are the n-th approximants of the Thiele expansions at the points x_m respectively, and used for the function approximation.

2. Investigation purpose. To construct the multipoint Thile formula the corresponding continued fraction to the Taylor expansion was used [4]. It is naturally to continue previous investigations, using the different type of continued fraction expansions. Since an associated continued fraction is one of the important type of continued fractions we will consider the formula (2) with the n-th approximants of the associated continued fractions of the special type. It is known that a continued fraction of the form

$$1+\cfrac{k_1 z}{1+\ell_1 z-\cfrac{k_2 z^2}{1+\ell_2 z-\cfrac{k_3 z^2}{1+\ell_3 z+\ddots}}}, \tag{3}$$

where $k_n \neq 0$, k_n and ℓ_n, $n = 1,2,\ldots$, are complex constants, $z \in \mathbb{C}$, is called an associated continued fraction and it was also shown that the even part of a regular C-fraction is an associated continued fraction [4, 5]. We will consider a continued fraction of the following form:

$$\omega_0 + \omega_1(z-z_0) + \mathop{\mathrm{D}}_{k=1}^{\infty} \frac{(z-z_0)^2}{\omega_{2k} + (z-z_0)\omega_{2k+1}}, \tag{4}$$

where $\omega_{2k} \neq 0$, ω_{2k}, ω_{2k+1}, $k = 0,1,\ldots$, are complex constants, obtained from the interpolating problem [2] in the one-dimensional case, investigate its properties and apply it for the construction of the multipoint formula.

3. Main results. At first we show that the fraction (4) is the associated continued fraction with the order of correspondence of its n-th approximant $\frac{P_n(z)}{Q_n(z)}$ equals $2n+2$ (the order of correspondence of the n-th approximant of the continued fraction (3) is equal to $2n+1$ [5]).

By equivalent transformations the continued fraction (4) can be reduced to the following form:

$$\omega_0\left(1+\frac{\omega_1}{\omega_0}(z-z_0)+\mathop{\mathrm{D}}_{k=1}^{\infty}\frac{\frac{1}{\omega_{2k-2}\omega_{2k}}(z-z_0)^2}{1+(z-z_0)\frac{\omega_{2k+1}}{\omega_{2k}}}\right)$$

and using the difference equations [(2.1.6), [5]], we have that the n-th numerator $P_n(z)$ and denominator $Q_n(z)$ of (4) are polynomials in $z-z_0$ of the form

$$P_n(z) = \omega_0 + a_{n,1}(z-z_0) + a_{n,2}(z-z_0)^2 + \ldots + a_{n,n+1}(z-z_0)^{n+1},$$

$$Q_n(z) = 1 + b_{n,1}(z-z_0) + b_{n,2}(z-z_0)^2 + \ldots + b_{n,n}(z-z_0)^n,$$

where $n = 0,1,2,\ldots$; $a_{i,j}$, $1 \le i \le n$; $1 \le j \le n+1$, $b_{i,j}$, $1 \le i$, $j \le n$, depend on ω_{ij}, $0 \le i$, $j \le 2n+1$. Thus the n-th approximant $f_n(z) = \frac{P_n(z)}{Q_n(z)}$ is holomorphic at the $z = z_0$ and its Taylor series at $z = z_0$

$$\frac{P_n(z)}{Q_n(z)} = c_0 + c_1^{(n)}(z-z_0) + c_2^{(n)}(z-z_0)^2 + \ldots, \qquad c_0 = \omega_0,$$

has a positive radius of convergence.

Theorem 1. *1°. A continued fraction* (4) *corresponds to a uniquely determined formal power series*

$$f(z) = c_0 + c_1(z - z_0) + c_2(z - z_0)^2 + \dots . \tag{5}$$

The order of correspondence of the n*-th approximant* $\dfrac{P_n(z)}{Q_n(z)}$ *is* $2n+2$ *and hence the Taylor series at* $z = z_0$ *of* $\dfrac{P_n(z)}{Q_n(z)}$ *has the form*

$$\frac{P_n(z)}{Q_n(z)} = c_0 + c_1(z - z_0) + c_2(z - z_0)^2 + \dots +$$

$$+ c_{2n+1}(z - z_0)^{2n+1} + c_{2n+2}^{(n)}(z - z_0)^{2n+2} + \dots .$$

2°. If two continued fractions of the same form (4) *correspond to the same formal power series* (5)*, then their corresponding coefficients coincide.*

P r o o f. By using the determinant formula (2.1.9) from [5] we obtain

$$\frac{P_{n+1}(z)}{Q_{n+1}(z)} - \frac{P_n(z)}{Q_n(z)} = \frac{1}{Q_{n+1}(z)Q_n(z)} \frac{(-1)^n (z - z_0)^{2n+2}}{\prod\limits_{k=1}^{n+1} \omega_{2k-2}\omega_{2k}} =$$

$$= (-1)^n \frac{(z - z_0)^{2n+2}}{\prod\limits_{k=1}^{n+1} \omega_{2k-2}\omega_{2k}} + (c_{2n+3}^{(n+1)} - c_{2n+3}^{(n)})(z - z_0)^{2n+3} + \dots$$

and hence the first part of the theorem follows from Theorem 5.1 [5]. The second part of the theorem is proved by induction on n. ◊

Theorem 2. *For a given formal power series* (5) *there exists an unique associated continued fraction of the form*

$$c_0 + c_1(z - z_0) + \cfrac{k_1(z - z_0)^2}{1 + \ell_1(z - z_0) - \cfrac{k_2(z - z_0)^2}{1 + \ell_2(z - z_0) - \ddots}}, \tag{6}$$

where $k_n \neq 0,\ k_n = \dfrac{\varphi_{n+1}\varphi_{n-1}}{\varphi_n^2},\ n = 1, 2, \dots,$

$$\varphi_0 = \varphi_1 = 1, \qquad \varphi_m = \begin{vmatrix} c_2 & c_3 & \dots & c_m \\ c_3 & c_4 & \dots & c_{m+1} \\ \dots & \dots & \dots & \dots \\ c_m & \dots & \dots & c_{2m-2} \end{vmatrix}, \qquad m = 2, 3, \dots,$$

and $\ell_0 = c_1,\ \ell_n = \dfrac{\chi_n}{\varphi_n} - \dfrac{\chi_{n+1}}{\varphi_{n+1}},\ n = 1, 2, \dots,$

$$\chi_1 = 0, \quad \chi_2 = c_3, \quad \chi_m = \begin{vmatrix} c_2 & c_3 & \dots & c_{m-1} & c_{m+1} \\ c_3 & c_4 & \dots & c_m & c_{m+2} \\ \dots & \dots & \dots & \dots & \dots \\ c_m & c_{m+1} & \dots & c_{2m-3} & c_{2m-1} \end{vmatrix}, \quad m = 3, 4, \dots,$$

if $\varphi_n \neq 0,\ n = 1, 2, \dots .$

The p r o o f of this theorem is similar to the proof of the theorem 7.14 [5] and therefore is omitted. ◊

Remark 1. One can note that putting in (4)

$$\omega_{4n} = -\frac{k_1k_2\dots k_{2n-1}}{k_2k_4\dots k_{2n}}, \qquad \omega_{4n-1} = \frac{\ell_{2n-1}k_2k_4\dots k_{2n-2}}{k_1k_3\dots k_{2n-1}},$$

$$\omega_{4n-2} = \frac{k_2k_4\dots k_{2n-2}}{k_1k_3\dots k_{2n-1}}, \qquad \omega_{4n+1} = -\frac{\ell_{2n}k_1k_3\dots k_{2n-1}}{k_2k_4\dots k_{2n}},$$

where $n = 1,2,\dots$, $k_0 = 1$, after elementary transformations, the fraction (6) is obtained.

Let us assume that a function $f(z)$ given in the bounded domain G, $G \subset \mathbb{C}$, expands into the associated continued fraction (4) at the point $z = z_0$, $z_0 \in G$ and find the remainder term of the function approximation by the $n-$th approximant of the continued fraction (4).

Theorem 3. *Let a function* $f(z)$ *given in the bounded domain* $G \subset \mathbb{C}$, *being* $2n+2$ *continuously differentiable on* G, *be approximated by the continued fraction* (4). *Then the remainder term* $R_{2n+2}(f,z) = f(z) - \frac{P_n(z)}{Q_n(z)}$ *at any point* $z \in G$, *different from zeros of* $Q_n(z)$, *can be represented in the following form*:

$$R_{2n+2}(f,z) = \frac{(z-z_0)^{2n+2}}{(2n+2)!\,Q_n(z)}\ \frac{d^{2n+2}}{dz^{2n+2}}\left[f(z)\,Q_n(z)\right]_{z=\xi}, \quad \xi \in G, \quad z_0 \in G. \qquad (7)$$

P r o o f. We expand the function $\psi(z) = f(z)Q_n(z) - P_n(z)$, equals zero at $z = z_0$, into the Taylor series at $z = z_0$:

$$\psi(z) = \sum_{k=0}^{2n+1}\frac{d^k\psi(z_0)}{dz^k}\,\frac{(z-z_0)^k}{k!} + R_{2n+2}(z),$$

where $R_{2n+2}(z)$ is the remainder term of the Taylor formula. Taking into account the fraction (4) is the associated fraction for the Taylor series, we have at once

$$\frac{d^p}{dz^p}\left[f(z) - \frac{P_n(z)}{Q_n(z)}\right] = 0 \qquad \text{at} \qquad z = z_0, \quad p = 0,1,\dots,2n+1,$$

and thus $\frac{d^p}{dz^p}\left[\psi(z)\right] = 0$ at $z = z_0$, $p = 0,1,\dots,2n+1$. Using the Rolle theorem for the real function $\varphi(t)$, $0 \le t \le 1$,

$$\varphi(t) = \\ = \psi(z) - \sum_{k=0}^{2n+1}\frac{d^k\psi\big(z_0+(z-z_0)t\big)}{dz^k}\,\frac{(z-z_0)^k}{k!}(1-t)^k - R_{2n+2}(z)(1-t)^{2n+2},$$

we obtain $\varphi'(\theta) = 0$, $0 < \theta < 1$. It means that

$$R_{2n+2}(z) = \frac{d^{2n+2}}{dz^{2n+2}}\,\psi\big(z_0+(z-z_0)\theta\big)\,\frac{(z-z_0)^{2n+2}}{(2n+2)!},$$

$$\frac{d^{2n+2}}{dz^{2n+2}}\,\psi\big(z_0+(z-z_0)\theta\big) = \frac{d^{2n+2}}{dz^{2n+2}}\left[f(z_0+(z-z_0)\theta)Q_n\big(z_0+(z-z_0)\theta\big)\right],$$

where $0 < \theta < 1$, taking into account the degree of $P_n(z)$, and $R_{2n+2}(f,z) = \frac{R_{2n+2}(z)}{Q_n(z)}$. From this, the statement of the theorem follows. ◊

In the next results we will consider the continued fraction (4) with real coefficients and use it for the approximation of a real function.

Theorem 4. *Let $f(x)$ given on $[0,1]$ be expanded into the continued fraction* (4) *at the point $x = x_0$, $x_0 \in (0,1)$, and all partial denominators of* (4) *are positive and satisfy the following conditions*

$$\omega_{2k} + (x - x_0)\omega_{2k+1} \geq 1 + (x - x_0)^2, \qquad k = 1, 2, \ldots. \tag{8}$$

Then the continued fraction (4) *converges uniformly on $[0,1]$ and for all $x \in (0,1)$*

$$| R_{2n+2}(f,x) | \leq \frac{M_n (x - x_0)^{2n+2}}{(2n+2)!\left(1 + (x - x_0)^2\right)^n}, \tag{9}$$

where

$$M_n = \max_{\xi \in (0,1)} \left| \frac{d^{2n+2}}{dx^{2n+2}} \left[f(x) Q_n(x) \right]_{x=\xi} \right|. \tag{10}$$

P r o o f. Because of all partial numerators of (4) are positive and taking into account the recurrence relations for $Q_n(x)$, we have by induction, $Q_n(x) \geq \left(1 + (x - x_0)^2\right)^n$ then using (7), we receive (9). The uniform convergence of (4) follows immediately from the Worpitzky theorem [4, 5]. ◊

Remark 2. If in the theorem 4 we put $| \omega_{2k} + \omega_{2k+1}(x - x_0) | \geq 2 + (x - x_0)^2$, $k = 1, 2, \ldots$, instead of (8) and will not demand positiveness of the partial denominators, then the continued fraction (4) converges uniformly on $[0,1]$ and for all $x \in (0,1)$

$$| R_{2n+2}(f,x) | \leq \frac{M_n (x - x_0)^{2n+2}}{(2n+2)!2\left(1 + (x - x_0)^2\right)^n},$$

where M_n is defined by (10).

Now we use the associated continued fraction (4) and its properties for the function approximation by the formula looks like the formula (2). Let a function $f(x)$ given on $[a,b]$ be a $2n+2$ continuously differentiate function at least on (a,b) and represented by the formula for all $x \in (a,b)$

$$f(x) = h_1(x) \left\{ \frac{P_n(x, x_1)}{Q_n(x, x_1)} + r_n(x, x_1) \right\} + h_2(x) \left\{ \frac{P_n(x, x_2)}{Q_n(x, x_2)} + r_n(x, x_2) \right\}, \tag{11}$$

where $x_1, x_2 \in (a,b)$; $\frac{P_n(x, x_1)}{Q_n(x, x_1)}$ and $\frac{P_n(x, x_2)}{Q_n(x, x_2)}$ are the n-th approximants of the function expansions into the associated continued fraction (4) at the points $x = x_1$ and $x = x_2$ respectively; $r_n(x, x_1)$ and $r_n(x, x_2)$ are the corresponding reminders, i. e.

$$r_n(x, x_1) = f(x) - \frac{P_n(x, x_1)}{Q_n(x, x_1)}, \qquad r_n(x, x_2) = f(x) - \frac{P_n(x, x_2)}{Q_n(x, x_2)}.$$

Functions $h_i(x) \in C^\infty$, $i = 1, 2$, are non-negative and can be used in the form

$$h_i(x) = \begin{cases} 0, & x \le x_{i-1}, \quad x \ge x_{i+1}, \\ g_p\left(\dfrac{x - x_{i-1}}{x_i - x_{i-1}}\right), & x_{i-1} \le x \le x_i, \\ g_p\left(\dfrac{x_{i+1} - x}{x_{i+1} - x_i}\right), & x_i \le x \le x_{i+1}, \end{cases} \tag{12}$$

where

$$g_p(x) = \frac{(2p+1)!}{(p!)^2} \frac{\int_0^x t^p (1-t)^p \, dt}{\int_0^1 t^p (1-t)^p \, dt}, \qquad p \ge 2n + 2.$$

Taking into account (7), (10), for the reminder form of (11) $r_n(x, x_1, x_2) = h_1(x) r_n(x, x_1) + h_2(x) r_n(x, x_2)$ we can write the inequality for all $x \in (a, b)$

$$| r_n(x, x_1, x_2) | \le \frac{M_n}{(2n+2)!} \left[h_1(x) \frac{(x - x_1)^{2n+2}}{Q_n(x, x_1)} + h_2(x) \frac{(x - x_2)^{2n+2}}{Q_n(x, x_2)} \right],$$

where $x_1, x_2 \in (a, b)$, and

$$M_n = \max \left\{ \max_{\xi_1 \in (a,b)} \left| \left[f(x) Q_n(x, x_1) \right]^{(2n+2)}_{x = \xi_1} \right|, \max_{\xi_2 \in (a,b)} \left| \left[f(x) Q_n(x, x_2) \right]^{(2n+2)}_{x = \xi_2} \right| \right\}. \tag{13}$$

Theorem 5. *Let the function* $f(x)$ *given on* $[a, b]$, $b - a = 1$, *being* $2n + 2$ *continuously differentiable at least on* (a, b), *expands into the associated continued fraction* (4) *at the points* $x = x_1$ *and* $x = x_2$, $x_1, x_2 \in (a, b)$.

Then,

(*i*) *the formula* (11) *is valid on* (a, b);

(*ii*) *if all partial denominators in the both terms of* (11) *are positive and*

$$\omega_{2k, x_1} + (x - x_1) \omega_{2k+1, x_1} \ge 1 + (x - x_1)^2,$$

$$\omega_{2k, x_2} + (x - x_2) \omega_{2k+1, x_2} \ge 1 + (x - x_2)^2,$$

where $k = 1, 2, \dots, n$, *and* ω_{2k, x_1}, ω_{2k+1, x_1}, ω_{2k, x_2}, ω_{2k+1, x_2} *are the coefficients in the partial denominators of the expansions into* (4) *at* $x = x_1$ *and* $x = x_2$ *respectively, then for all* $x \in (a, b)$

$$| r_n(x, x_1, x_2) | \le \frac{M_n}{(2n+2)! \, L^n \, 2^{2n+2}},$$

where M_n *is defined by* (13), *and* $L = \min\left\{ (1 + (x - x_1)^2), \ (1 + (x - x_2)^2) \right\}$.

P r o o f. For the sake of simplicity we put $a = 0$, $b = 1$ and using the theorem 4 we receive that

$$Q_n(x, x_1) \geq \left(1 + (x - x_1)^2\right)^n, \qquad Q_n(x, x_2) \geq \left(1 + (x - x_2)^2\right)^n .$$

Thus

$$|\, r_n(x, 0, 1)\,| \leq \frac{M_n}{(2n+2)!\, L^n} \left|\, h_1(x)(x - x_1)^{2n+2} + h_2(x)(x - x_2)^{2n+2} \,\right| .$$

Having used properties of functions (12), as it was done in [3], we conclude that

$$\left|\, h_1(x)(x - x_1)^{2n+2} + h_2(x)(x - x_2)^{2n+2} \,\right| \leq \frac{1}{2^{2n+2}}$$

and then

$$|\, r_n(x, 0, 1)\,| \leq \frac{M_n}{(2n+2)!\, L^n\, 2^{2n+2}} . \ \Diamond$$

Theorem 6. *Let the function $f(x)$ given on $[a,b]$, $b - a = 1$, being $2n + 2$ continuously differentiable at least on (a,b), expands into the associated continued fraction* (4) *at the points $x = x_1$ and $x = x_2$, $x_1, x_2 \in (a,b)$.*

Then,

(i) *the formula* (11) *is valid on (a,b);*

(ii) *if all partial denominators in the both terms of* (11) *satisfy the following conditions*

$$\left|\, \omega_{2k,x_1} + (x - x_1)\omega_{2k+1,x_1} \,\right| \geq 2 + (x - x_1)^2 ,$$

$$\left|\, \omega_{2k,x_2} + (x - x_2)\omega_{2k+1,x_2} \,\right| \geq 2 + (x - x_2)^2 ,$$

where $k = 1, 2, \ldots, n$, then for all $x \in (a,b)$

$$|\, r_n(x, x_1, x_2)\,| \leq \frac{M_n}{(2n+2)!\, L^n\, 2^{2n+3}} ,$$

where M_n is defined by (13) *and* $L = \min\left\{ \left(1 + (x - x_1)^2\right),\ \left(1 + (x - x_2)^2\right) \right\}$.

P r o o f. We can assume that all partial denominators are positive, if not multiplying the numerator and denominator by (-1), we change the denominator sign. If all partial numerators are positive then using recurrent relations for $Q_n(x, x_1)$, $Q_n(x, x_2)$ we obtain $Q_n(x, x_1) \geq \left(2 + (x - x_1)^2\right)^n$, $Q_n(x, x_2) \geq$ $\geq \left(2 + (x - x_2)^2\right)^n$ by induction. If some of partial numerators or all of them are negative, then (4) will be transformed to the continued fraction with positive partial numerators, excluding, may be the first partial numerator as it was done in Lemma 3.1 [4]. It is not difficult to show that in this case $Q_n(x, x_1) \geq 2\left(1 + (x - x_1)^2\right)^n$, $Q_n(x, x_2) \geq 2\left(1 + (x - x_2)^2\right)^n$ and the result of the theorem follows by analogy with the theorem 5. $\Diamond$

An m-point formula can be constructed by analogy with the two-point formula. One denotes by x_i, $i = 1, 2, \ldots, m$, a sequence of values of x on the interval (a,b), $b - a = 1$, such that $a < x_1 < x_2 < \ldots < x_m < b$. The length of the interval $x_i - x_{i-1}$ is equal to $\dfrac{b-a}{m}$. Let a function $f(x)$ given on $[a,b]$,

$b - a = 1$, be a $2n + 2$ continuously differentiable function at least on (a, b), then an m-point formula, based on an associated continued fraction (4), is the following formula for all $x \in (a, b)$:

$$f(x) = \sum_{i=1}^{m} h_i(x) \left[\frac{P_n(x, x_i)}{Q_n(x, x_i)} \right] + r_{mn}(x, x_1, x_2, \ldots, x_m),$$

where $x_1, x_2, \ldots, x_m \in (a, b)$, $h_i(x)$ are defined by (12), $\frac{P_n(x, x_i)}{Q_n(x, x_i)}$ are the n-th approximants of the function expansions into the associated continued fraction (4) at points x_i respectively,

$$r_{mn}(x, x_1, x_2, \ldots, x_m) = \sum_{i=1}^{m} h_i(x) r_n(x, x_i), \qquad r_n(x, x_i) = f(x) - \frac{P_n(x, x_i)}{Q_n(x, x_i)}.$$

Reminders $r_n(x, x_i)$ for this formula are investigated in similar way as for the formula (11).

4. Conclusion. Proposed formulae give possibility to approximate functions by rational polynomials. However, only the form of remainders of considered formulae have been proposed and it will be interesting to obtain the error estimations for some class of functions and investigate the possibilities of different continued fractions be applied to a such problem in future.

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БАГАТОТОЧКОВА ФОРМУЛА, ЯКА БАЗУЄТЬСЯ НА ПРИЄДНАНОМУ НЕПЕРЕРВНОМУ ДРОБІ

Для наближення функції однієї змінної побудовано m-точкову формулу, в якій використано розвинення функції у приєднаний неперервний дріб спеціального вигляду в m, $m \geq 2$, точках і властивості функцій, які є розвиненням одиниці. Досліджено також властивості такого дробу.

МНОГОТОЧЕЧНАЯ ФОРМУЛА, ОСНОВАННАЯ НА ПРИСОЕДИНЕННОЙ НЕПРЕРЫВНОЙ ДРОБИ

Для приближения функции одной переменной построена m-точечная формула, использующая разложение функции в присоединенную непрерывную дробь специального вида в m, $m \geq 2$, точках и свойства функций, являющихся разложением единицы. Исследуются также свойства такой дроби.

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