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TRANSIENT HEAT CONDUCTION PROBLEM FOR A COMPOSITE LAYER ON A HOMOGENEOUS SUBSTRATE

The paper deals with the transient heat conduction problems of a periodic composite layer joined with a homogeneous half-space. The layer is composed of periodically repeated cells with rectangular cross-sections. The composite solid is heated on its boundary by a normal heat flux with uniform intensity. From the results, some solutions of the heat conduction problems for the particular cases of the composite structure are also derived. The influence of thermal and geometric properties of the composite components on the temperature distributions is presented in the form of graphs.

1. Introduction. Composite materials with periodic structures are widely utilized in building engineering, machine elements, and aviation structures. Composite components possess different thermal characteristics. The investigations connected with modeling and solving heat conduction problems in periodic composites are important and are the subject of many papers [1–11, 13–15]. A variety of methods exact, approximate and purely numerical are available for deriving mathematical solutions for periodic composites. However, in cases when the number of repeated cells is large, it seems useful to employ homogenization procedures and description of heat conductions with approximate models. One of the methods is the homogenized model with microlocal parameters [14]. The homogenized model enables determining of mean and local temperature gradients and heat fluxes in each material component of the composite.

This paper is a continuation of our previous studies [7, 8]. The composite structure permits to obtain some special cases of the periodically homogeneous strip: a layered body, a composite with periodically distributed inclusions, a chessboard structure and a homogeneous body. Thus, from the derived solution the distributions of temperature and heat flux for the particular cases of the composite strip structure are obtained.

2. The homogenized composite model. We consider the nonstationary heat conduction problem of a periodic composite layer resting on a homogeneous half-space. Both structures are assumed to be nondeformable. The nonhomogeneous layer is composed of periodically repeated cells with rectangular cross-section, see Fig. 1. Let a, b be the cross-sectional dimensions of the fundamental composite cell and a_1, b_1 the dimensions of the cross-section of the first composite component. Perfect thermal contact on the interfaces is assumed. The composite body is heated on its boundary by a normal heat flux with constant intensity q at time $t > 0$. Let

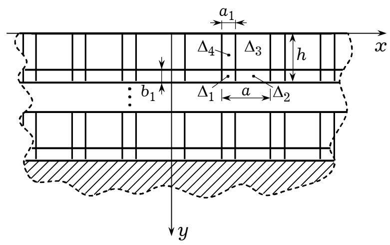

Fig. 1

$\Delta = \{(x, y) \in \mathbb{R}^2;\ 0 < x < a,\ 0 < y < b\}$ represent the cross-section of the fundamental cell, see Fig. 1. Denote by Δ_i the cross-section of the i-th composite component with thermal conductivity K_i, thermal capacity c_i, $i = 1, 2, 3, 4$, and density $\rho_1 = \rho_2 = \rho_3 = \rho_4 = \rho$.

The nonstationary heat conduction problem of the composite strip will be described within the framework of the homogenized model with microlocal parameters [7, 9, 14]. According to the results of [7] the temperature T_s in the composite layer is written in the following form (the summation convention holds with respect to all repeated indices):

$$T_s(x,y,t) = \theta(x,y,t) + \underline{\ell_i(x,y)\gamma_i(x,y,t)}, \qquad i = 1,2,3,4\,, \tag{1}$$

where $\ell_i : \mathbb{R}^2 \to \mathbb{R}$ are known *a priori* Δ-periodic functions called shape functions which satisfy the conditions

$$\int_\Delta \ell_i(x,y)\,dx\,dy = 0, \qquad\qquad |\ell_i(x,y)| < \max(a,b)\,.$$

The function θ is an unknown function interpreted as the macrotemperature, and the functions γ_i stand for extra unknown functions called the thermal microlocal parameters, which are related to the periodic structure of the body. The shape functions are chosen so that the continuity conditions on the interfaces are satisfied. Let the shape functions be given in the form [7]:

$$\ell_{1,3}(x,y) = \begin{cases} x - 0.5a_1, & (x,y) \in \Delta_{1,4}, \\ (a_1 - \eta_1)/(1-\eta_1) - 0.5a_1, & (x,y) \in \Delta_{2,3}, \\ 0, & (x,y) \in \Delta_{3,1} \cup \Delta_{4,2}, \end{cases}$$

$$\ell_{2,4}(x,y) = \begin{cases} y - 0.5b_1, & (x,y) \in \Delta_{1,2}, \\ (b_1 - \eta_2)/(1-\eta_2) - 0.5b_1, & (x,y) \in \Delta_{4,3}, \\ 0, & (x,y) \in \Delta_{2,1} \cup \Delta_{3,2}, \end{cases} \tag{2}$$

where

$$\eta_1 = a_1/a, \qquad\qquad \eta_2 = b_1/b\,. \tag{3}$$

Denote by $\langle f(\cdot)\rangle$ the mean value of Δ-periodic integrable function taking constant values f_i in Δ_i:

$$\langle f \rangle \equiv \frac{1}{ab}\sum_{i=1}^{4}\int_{\Delta_i} f_i\,dx\,dy = \frac{1}{ab}\sum_{i=1}^{4} f_i\,|\Delta_i|\,, \tag{4}$$

where

$$\Delta_1 = (0,a_1)\times(0,b_1), \qquad |\Delta_1| = a_1 b_1\,,$$

$$\Delta_2 = (a_1,a)\times(0,b_1), \qquad |\Delta_2| = (a-a_1)b_1\,,$$

$$\Delta_3 = (a_1,a)\times(b_1,b), \qquad |\Delta_3| = (a-a_1)(b-b_1)\,,$$

$$\Delta_4 = (0,a_1)\times(b_1,b), \qquad |\Delta_4| = a_1(b-b_1)\,. \tag{5}$$

By using equations (2)–(5) and according to the results of papers [7, 14], the governing equations of the homogenized model with microlocal parameters can be written in the form

$$\langle K\rangle(\theta_{,xx} + \theta_{,yy}) + \langle K\ell_{i,x}\rangle\gamma_{i,x} + \langle K\ell_{i,y}\rangle\gamma_{i,y} - \rho\langle c\rangle\theta_{,t} = 0\,, \tag{6}$$

$$\langle K\ell_{1,x}\rangle\theta_{,x} + \langle K(\ell_{1,x})^2\rangle\gamma_1 = 0, \qquad \langle K\ell_{2,y}\rangle\theta_{,y} + \langle K(\ell_{2,y})^2\rangle\gamma_2 = 0\,, \tag{7}$$

$$\langle K\ell_{3,x}\rangle\theta_{,x} + \langle K(\ell_{3,x})^2\rangle\gamma_3 = 0\,, \qquad \langle K\ell_{4,y}\rangle\theta_{,y} + \langle K(\ell_{4,y})^2\rangle\gamma_4 = 0\,, \tag{8}$$

where the following set of material constants is obtained

$$\langle K\rangle = \eta_1\eta_2 K_1 + (1-\eta_1)\eta_2 K_2 + (1-\eta_1)(1-\eta_2)K_3 + \eta_1(1-\eta_2)K_4\,,$$

$$\langle c\rangle = \eta_1\eta_2 c_1 + (1-\eta_1)\eta_2 c_2 + (1-\eta_1)(1-\eta_2)c_3 + \eta_1(1-\eta_2)c_4\,,$$

$$\left\langle K\ell_{1,x}\right\rangle = \eta_1\eta_2(K_1 - K_2), \qquad \left\langle K\ell_{2,y}\right\rangle = \eta_1\eta_2(K_1 - K_4),$$
$$\left\langle K\ell_{3,x}\right\rangle = \eta_1(1-\eta_2)(K_4 - K_3), \qquad \left\langle K\ell_{4,y}\right\rangle = (1-\eta_1)\eta_2(K_2 - K_3),$$
$$\left\langle K\ell_{1,y}\right\rangle = \left\langle K\ell_{2,x}\right\rangle = \left\langle K\ell_{3,y}\right\rangle = \left\langle K\ell_{4,x}\right\rangle = 0,$$
$$\left\langle K(\ell_{1,x})^2\right\rangle = \eta_1\eta_2[K_1 + \eta_1 K_2/(1-\eta_1)],$$
$$\left\langle K(\ell_{2,y})^2\right\rangle = \eta_1\eta_2[K_1 + \eta_2 K_4/(1-\eta_2)],$$
$$\left\langle K(\ell_{3,x})^2\right\rangle = \eta_1(1-\eta_2)[K_4 + \eta_1 K_3/(1-\eta_1)],$$
$$\left\langle K(\ell_{4,y})^2\right\rangle = (1-\eta_1)\eta_2[K_2 + \eta_2 K_3/(1-\eta_2)]. \tag{9}$$

By using relations (7)–(9) the microlocal parameters γ_i can be eliminated from equation (6). Thus, we obtain

$$A_1\theta_{,xx} + A_2\theta_{,yy} - \rho\left\langle c\right\rangle\theta_{,t} = 0, \tag{10}$$

where

$$A_1 = \left\langle K\right\rangle - \frac{\left\langle K\ell_{1,x}\right\rangle^2}{\left\langle K(\ell_{1,x})^2\right\rangle} - \frac{\left\langle K\ell_{3,x}\right\rangle^2}{\left\langle K(\ell_{3,x})^2\right\rangle},$$

$$A_2 = \left\langle K\right\rangle - \frac{\left\langle K\ell_{2,y}\right\rangle^2}{\left\langle K(\ell_{2,y})^2\right\rangle} - \frac{\left\langle K\ell_{4,y}\right\rangle^2}{\left\langle K(\ell_{4,y})^2\right\rangle}. \tag{11}$$

Since $|\ell_i(x,y)| < \max(a,b)$ for every (x,y), then for small $\max(a,b)$ the underlined terms in equation (1) are small and will be neglected (functions $\ell_i(\cdot)$ take infinitesimal values during the homogenization procedure, see [14]). However, the derivatives $\ell_{i,x}$ and $\ell_{i,y}$ are not small and they cannot be neglected, so we have:

$$T_s \approx \theta, \qquad T_{s,x} \approx \theta_{,x} + \ell_{i,x}\gamma_i, \qquad T_{s,y} \approx \theta_{,y} + \ell_{i,y}\gamma_i. \tag{12}$$

Thus, by using equations (7)–(9) and (12) the heat flux vector $\mathbf{q}_i$ takes the form

$$\mathbf{q}_1 = [B_1\theta_{,x},\ B_2\theta_{,y}], \qquad \mathbf{q}_2 = [B_1\theta_{,x},\ B_3\theta_{,y}], \tag{13}$$
$$\mathbf{q}_3 = [B_4\theta_{,x},\ B_3\theta_{,y}], \qquad \mathbf{q}_4 = [B_4\theta_{,x},\ B_2\theta_{,y}],$$

where

$$B_1 = \frac{K_1K_2}{(1-\eta_1)K_1 + \eta_1K_2}, \qquad B_2 = \frac{K_1K_4}{(1-\eta_2)K_1 + \eta_2K_4},$$

$$B_3 = \frac{K_2K_3}{(1-\eta_2)K_2 + \eta_2K_3}, \qquad B_4 = \frac{K_3K_4}{(1-\eta_1)K_4 + \eta_1K_3}. \tag{14}$$

From equations (13) and (14) it follows that the continuity conditions for the heat flux on the composite interfaces are satisfied.

3. Problem formulation. Consider the composite strip heated by a normal heat flux with constant intensity q at time $t > 0$ on the upper boundary plane. Let $d = nb$ be the thickness of the composite layer, where n is a sufficiently large natural number (Fig. 1). The strip is assumed to be perfectly joined with the homogenous half-space $y \geq d$. From the assumptions given above it follows that the problem is independent with respect of the variable x. So, we consider the following equations of heat conduction:

$$\theta_{,yy} - k_s^{-1}\theta_{,t} = 0, \qquad 0 < y \le d, \qquad t > 0\,, \tag{15}$$

and

$$T_{f,yy} - k_f^{-1}T_{f,t} = 0, \qquad y \ge d, \qquad t > 0\,, \tag{16}$$

where $k_s = A_2/(\rho\langle c\rangle)$ and k_f are the thermal diffusivities of the composite strip and foundation, respectively.

The boundary conditions will be taken into account in the following approximate form

$$A_2\theta_{,y}(y,t) = -q, \qquad y = 0, \qquad t > 0\,, \tag{17}$$

$$\theta(d,t) = T_f(d,t), \qquad A_2\theta_{,y}(y,t) = K_f T_{f,y}(y,t), \qquad y = 0, \qquad t > 0\,, \tag{18}$$

where K_f is the thermal conductivity of the foundation. The initial conditions are assumed as

$$\theta(y,0) = 0, \qquad 0 \le y \le d, \qquad T_f(y,0) = 0, \qquad d \le y < \infty\,, \tag{19}$$

and the condition of regularity at infinity is

$$T_f(y,t) \to 0 \qquad \text{for} \qquad y \to \infty\,. \tag{20}$$

4. Solution of the problem. Denoting the Laplace transforms by [12]

$$\bar\theta(y,p) = \int_0^\infty \theta(y,t)\exp(-pt)\,dt, \qquad \bar T_f(y,p) = \int_0^\infty T_f(y,t)\exp(-pt)\,dt\,,$$

the solution of the nonstationary heat conductivity problem (15)–(20) can be written as

$$\bar\theta(y,p) = \frac{q}{p\sqrt{p}\,N(p)}\operatorname{ch}\left[(d-y)\sqrt{p/Ak_1}\,\right] + \varepsilon\operatorname{sh}\left[(d-y)\sqrt{p/Ak_1}\,\right], \tag{21}$$

$$\bar T_f(y,p) = \frac{q}{p\sqrt{p}\,N(p)}\exp\left[-(y-d)\sqrt{p/k_f}\,\right], \tag{22}$$

where

$$N(p) = \frac{A_2}{\sqrt{Ak_1}}\operatorname{sh}\left(d\sqrt{\frac{p}{Ak_1}}\right) + \frac{K_f}{\sqrt{k_f}}\operatorname{ch}\left(d\sqrt{\frac{p}{Ak_1}}\right), \tag{23}$$

$$\varepsilon = \frac{K_f}{A_2}\sqrt{\frac{Ak_1}{k_f}}\,, \tag{24}$$

$$A = \frac{K^*}{c^*}, \qquad K^* = \frac{\eta_1 K_4^*}{(1-\eta_2)+\eta_2 K_4^*} + \frac{(1-\eta_1)K_2^* K_3^*}{(1-\eta_2)K_2^* + \eta_2 K_3^*}\,, \tag{25}$$

$$c^* = \eta_1\eta_2 + (1-\eta_1)\,c_2^* + (1-\eta_1)(1-\eta_2)c_3^* + \eta_1(1-\eta_2)c_4^*)\,, \tag{26}$$

$$K_2^* = \frac{K_2}{K_1}, \qquad K_3^* = \frac{K_3}{K_1}, \qquad K_4^* = \frac{K_4}{K_1}\,,$$

$$c_2^* = \frac{c_2}{c_1}, \qquad c_3^* = \frac{c_3}{c_1}, \qquad c_4^* = \frac{c_4}{c_1}\,, \tag{27}$$

$k_1 = K_1/(\rho c_1)$ is the thermal diffusivity of the first composite component. We note that the parameter ε (24) refers to the coefficient of thermal activity of the foundation concerning a composite strip.

Decomposing the denominator $N(p)$ (23) in a series by the parameter $\beta = (1-\varepsilon)/(1+\varepsilon)$, the solutions in (21) and (22) can be rewritten as

$$\bar{\theta}(y,p) = \frac{q\sqrt{k_1}}{K_1\sqrt{K^*c^*}}\left\{\sum_{n=0}^{\infty}\frac{\gamma^n}{p\sqrt{p}}\exp\left[-(2dn+y)\sqrt{\frac{p}{Ak_1}}\right]+\right.$$

$$\left.+\sum_{n=1}^{\infty}\frac{\gamma^n}{p\sqrt{p}}\exp\left[-(2dn-y)\sqrt{\frac{p}{Ak_1}}\right]\right\}, \qquad (28)$$

$$\bar{T}_f(y,p) =$$

$$= \frac{2q\sqrt{k_1}}{K_1\sqrt{K^*c^*}(1+\varepsilon)}\sum_{n=0}^{\infty}\frac{\gamma^n}{p\sqrt{p}}\exp\left[-(2dn+d)\sqrt{\frac{p}{Ak_1}}-(y-d)\sqrt{\frac{p}{k_f}}\right], \qquad (29)$$

where

$$\gamma = \begin{cases}\beta, & 0 \le \beta < 1,\\ (-1)^n\,|\beta|, & -1 < \beta < 0.\end{cases}$$

The forms (28) and (29) of the solutions permit finding readily their inverse Laplace transforms [12]

$$T_s(y,t) \approx \theta(y,t) = \frac{2q\sqrt{k_1 t}}{K_1\sqrt{K^*c^*}}\left[\sum_{n=0}^{\infty}\gamma^n \operatorname{ierfc}\left(\frac{2dn+y}{2\sqrt{Ak_1 t}}\right)+\sum_{n=1}^{\infty}\gamma^n \operatorname{ierfc}\left(\frac{2dn-y}{2\sqrt{Ak_1 t}}\right)\right],$$

$$0 \le y \le d\,, \qquad (30)$$

$$T_f(y,t) = \frac{4q\sqrt{k_1 t}}{K_1\sqrt{K^*c^*}(1+\varepsilon)}\sum_{n=0}^{\infty}\gamma^n \operatorname{ierfc}\left(\frac{d(2n+1)}{2\sqrt{Ak_1 t}}+\frac{y-a}{2\sqrt{k_f t}}\right), \quad y \ge d\,, \qquad (31)$$

where $\operatorname{ierfc}(x) = \exp(-x^2)/\sqrt{\pi} - x\operatorname{erfc}(x)$, $\operatorname{erfc}(x) = 1 - \operatorname{erf}(x)$ and $\operatorname{erf}(x)$ is the error function. The solutions (30) and (31) are the basis for our subsequent numerical investigations.

5. Numerical results and final remarks. The temperature distributions in the composite strip and the homogeneous substrate given by equations (30) and (31) depend on many dimensionless parameters. These include thermal parameters, which are related to the thermal properties of the first component of composite K_2^*, K_3^*, K_4^*, c_2^*, c_3^*, c_4^* (27), and two geometrical parameters η_1, η_2 (3). Introduce the following dimensionless variables

$$d^* = \frac{d}{h}, \qquad y^* = \frac{y}{h}, \qquad t^* = \frac{k_1 t}{h^2}, \qquad T^* = \frac{T}{\Lambda}, \qquad \Lambda = \frac{qh}{K_1},$$

where $T = T_s$ for $0 \le y \le d$ and $T = T_f$ for $y > d$ and h is a characteristic dimension (for example, the thermal penetration depth [10]).

The distributions of dimensionless temperature in the composite layer and the homogeneous half-space are presented in Figs 2–11. The structure of the composite strip permits us to consider some particular cases of the body. Figs 2–11 are performed for the dimensionless time moment $t^* = 0.5$ and the width of the strip $d^* = 0.2$ in the case when $c_1 = c_2 = c_3 = c_4 = c$.

The dimensionless temperature T^* as a function of dimensionless variable y^* for the layered structure of the strip is shown in Figs 2 and 3. The thermal conductivity of the substrate is taken to be the same as the thermal conductivity of the upper layer ($K_3 = K_4 = K_f$). The temperature T^* referred to the homogeneous half-space with the conductivity coefficient K_f is presented by curves *1*; the temperature in the homogeneous strip-substrate system for $K_f/K_1 = 2$ in Fig. 2 and for $K_f/K_1 = 0.5$ in Fig. 3 is presented by curves *2*.

The solid curves refer to the composite strip – substrate system for heat conduction ratio $K_f/K_1 = K_3/K_1 = K_4/K = 2$, $K_2/K_1 = 1$ (Fig. 2) and for $K_f/K_1 = K_3/K_1 = K_4/K = 0.5$, $K_2/K_1 = 1$ (Fig. 3) at fixed geometric parameter $\eta_1 = 1$ and for values of the parameter $\eta_2 = 0.3, 0.5, 0.7$.

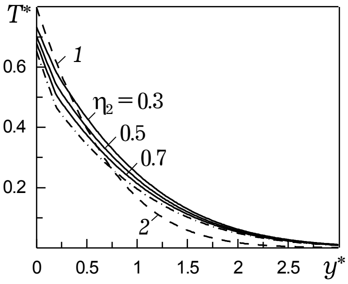

Fig. 2

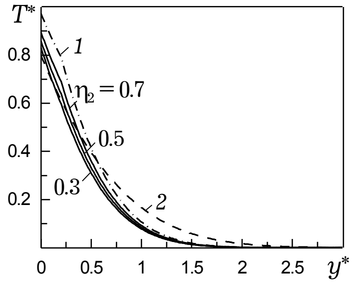

Fig. 3

The dimensionless temperature T^* as a function of dimensionless variable y^* in the non-homogeneous structure composed of the layered strip with the layering normal to the boundary and the homogeneous substrate is presented in Figs 4 and 5. The temperature T^* referred to the homogeneous half-space is presented by curves *1*; the temperature in the homogeneous strip-substrate system for $K_f / K_1 = 2$ in Fig. 4 and for $K_f / K_1 = 0.5$ in Fig. 5 is presented by curves *2*.

The solid curves refer to the composite strip – substrate system for heat conduction ratio $K_f/K_1 = K_2/K_1 = K_3/K_1 = 2$, $K_4/K_1 = 1$ (Fig. 4) and for $K_f/K_1 = K_2/K_1 = K_3/K_1 = 0.5$, $K_4/K_1 = 1$ (Fig. 5) at fixed geometric parameter $\eta_2 = 1$ and for values of the parameter $\eta_1 = 0.3, 0.5, 0.7$.

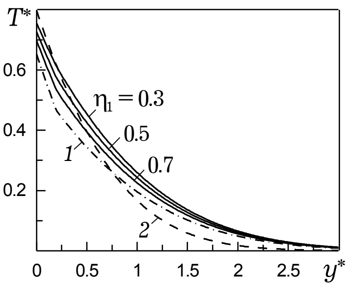

Fig. 4

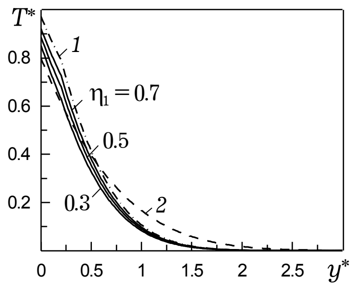

Fig. 5

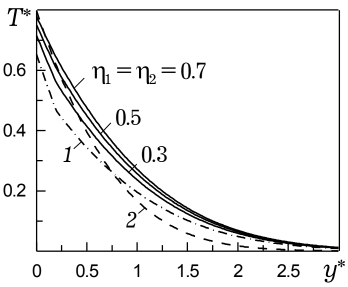

Fig. 6

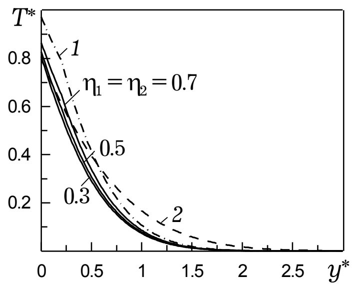

Fig. 7

The temperature T^* as a function of dimensionless variable y^* for composite with periodically distributed rectangular inclusions at fixed values of the parameters η_1, η_2 is shown in Figs. 6 and 7. The temperature T^* referred to the homogeneous half-space is presented by curves *1*; the temperature in the homogeneous strip-substrate system for $K_f / K_1 = 2$ in Fig. 6 and for $K_f / K_1 = 0.5$ in Fig. 7 is presented by curves *2*.

The solid curves refer to the composite strip − substrate system for heat conduction ratio $K_f/K_1 = K_2/K_1 = K_3/K_1 = K_4/K_1 = 2$ (Fig. 6) and for $K_f/K_1 = K_2/K_1 = K_3/K_1 = K_4/K_1 = 0.5$ (Fig. 7) at several values of the geometric parameters η_1 and η_2.

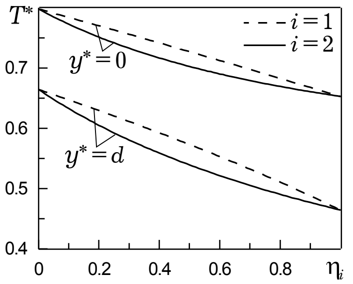

Fig. 8

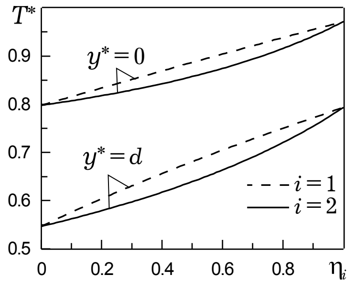

Fig. 9

Figs 8 and 9 show the dimensionless temperature on the boundary surfaces of the strip $y^* = 0$ and $y^* = d$ as functions of the parameter η_1 (the dashed curves) and η_2 (the solid curves) for different values of thermal conductivities. The solid curves refer to the composite strip − homogeneous substrate system for heat conduction ratio $K_f/K_1 = K_3/K_1 = K_4/K_1 = 2$, $K_2/K_1 = 1$ (Fig. 8) and for $K_f/K_1 = K_3/K_1 = K_4/K_1 = 0.5$, $K_2/K_1 = 1$ (Fig. 9) at fixed geometric parameter $\eta_1 = 1$. The dashed curves refer to the case $K_f/K_1 = K_2/K_1 = K_3/K_1 = 2$, $K_4/K_1 = 1$ (Fig. 8) and $K_f/K_1 = K_2/K_1 = K_3/K_1 = 0.5$, $K_4/K_1 = 1$ (Fig. 9) at fixed value $\eta_2 = 1$.

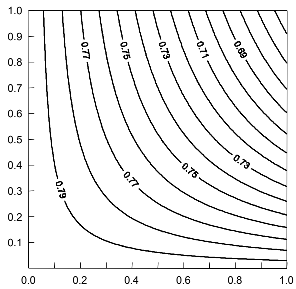

Fig. 10

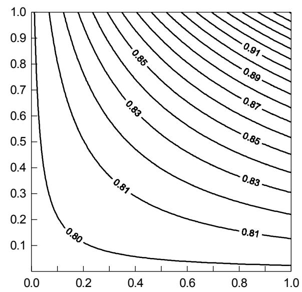

Fig. 11

The lines of constant dimensionless temperature on the boundary surface $y^* = 0$ are presented in Figs 10 and 11 in the case of a composite strip with periodically distributed inclusions for thermal conductivities $K_f/K_1 = K_2/K_1 = K_3/K_1 = K_4/K_1 = 2$ (Fig. 10) and $K_f/K_1 = K_2/K_1 = K_3/K_1 = K_4/K_1 = 0.5$ (Fig. 11) at different values of the geometrical parameters η_i, $i = 1,2$.

The results obtained for the temperature distributions in the composite strip attached to a homogeneous half-space given by equations (30) and (31) have permitted us to obtain the solutions of heat conduction problems for the layered strip, chessboard structures, the composite with periodically distributed inclusions. Assuming that $K_1 = K_2 = K_3 = K_4$ and $c_1 = c_2 = c_3 = c_4$ from equation (30) we obtain the solution for the homogeneous layer. Moreover, the above case under assumption $K_f = K_i$, $k_f = k_i$, $i = 1,2,3,4$, leads to the case of homogeneous semi-space.

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НЕСТАЦІОНАРНА ЗАДАЧА ТЕПЛОПРОВІДНОСТІ ДЛЯ КОМПОЗИТНОЇ СМУГИ НА ОДНОРІДНІЙ ОСНОВІ

Розглянуто нестаціонарну задачу теплопровідності для смуги, нанесеної на однорідну основу. Смуга складається із періодичної системи комірок прямокутного поперечного перерізу. Таке кусково-однорідне напівбезмежне тіло нагрівається на вільній поверхні тепловим потоком сталої інтенсивності. Числові розрахунки проведено для деяких часткових форм композиту. Вплив теплофізичних і геометричних параметрів задачі на розподіл температури подано у вигляді графіків.

НЕСТАЦИОНАРНАЯ ЗАДАЧА ТЕПЛОПРОВОДНОСТИ ДЛЯ КОМПОЗИЦИОННОЙ ПОЛОСЫ НА ОДНОРОДНОМ ОСНОВАНИИ

Рассмотрена нестационарная задача теплопроводности для полосы, нанесенной на однородное основание. Полоса состоит из периодической системы ячеек прямоугольного поперечного сечения. На свободной поверхности составное тело нагревается тепловым потоком постоянной интенсивности. Числовые расчеты представлены для некоторых частных случаев формы композита. Влияние теплофизических и геометрических параметров задачи на распределение температуры показано в виде графиков.

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