

THE INFLUENCE OF TEMPERATURE ON THE CYCLIC PROPERTIES OF THE TRANSVERSELY ISOTROPIC NANOCOMPOSITE SYSTEM UNDER KINEMATIC HARMONIC LOADING

A micromechanical model is developed to determine effective inelastic properties of nanocomposite under monoharmonic deformation by taking into account detailed micro-structural geometries and constitutive models of the constituents. By using the Correspondence Principle in Viscoelasticity and the modified Mori-Tanaka method, the effects of interface between inclusion and matrix is taken into account. By applying the presently developed model, a numerical analysis for determination of complex moduli for polymeric nanocomposite reinforced by nanofibers composed from carbon nanotubes (CNTs) is conducted at the isothermal conditions. Analysis of the complex moduli dependence on temperature and amplitude of strain intensity is performed. Composites reinforced with unidirectionally aligned nanofibers are considered. Results demonstrate a significant dependence of storage and loss moduli on the temperature within the wide range of it. The storage and loss moduli are found to increase monotonically with the increase of the nanofiber volume fraction while decrease with increasing temperature. The results show that, the strength of material decreases with increasing temperature in elastic and inelastic regions and the inelastic behavior occurs at lower strain amplitude with increase of temperature.

Introduction. The wide application of polymeric composites in engineering solutions, especially in aircraft, automotive, medical and ship-building industry, could be observed in the last decades. The desire to replace the structures made of metal alloys by nanofibre-reinforced polymeric composites was caused mainly by their high specific strength and stiffness properties with a simultaneous significant mass reduction. These properties of nanocomposite allow for the application of such materials in the most responsible engineering constructions like bodies of vehicles, turbine blades etc.

A lot of structures and elements made of the composite materials are subjected to cyclic loading and vibrate with high stress/strain amplitudes, which may cause the time dependent or inelastic behavior and consequently the self-heating effect occurrence. Therefore, it is necessary to carry out a detailed analysis of an influence of the self-heating effect on evolution of operational properties and consequently fatigue of polymeric nanocomposites. Temperature has a significant influence on rates of inelastic strain response, and in practical work it is often necessary to adjust an inelastic analysis for varying temperature. To study the mechanical energy dissipation phenomenon, it is necessary to characterize inelastic and time dependent behavior of material under different conditions such as temperature, amplitude of loading and frequency. This paper is devoted to survey the effects of temperature on complex moduli of transversely isotropic nanocomposite system with unidirectionally aligned nanofibres at the wide range of amplitude of loading at constant frequency. It is important to notice that, only the thermal effects on the complex moduli of pure matrix need to be determined since these effects on the nanofibers properties are negligible. Currently, there are two approaches to address the characterization problem for inelastic behavior of matrix (polymer). In the frame of the first approach, the complex set of constitutive equations governing response of numerous internal parameters is introduced while within the second approach, the approximate amplitude relations are used to characterize the cyclic response of the material, i.e. the relations between amplitudes of the main mechanical field parameters over the cycle are established [3, 6]. The key point of the amplitude theories is concept of complex moduli [3]. For an inelastic (particularly viscoelastic) material, the relaxa-

tion and creep compliance modulus which govern the relation between strain and stress amplitudes is represented by a complex quantity with real and imaginary parts referred to as storage and loss modulus, respectively [1, 2]. To accurately predict an overall performance and lifetime of polymer nanocomposites, it is necessary to model time dependent and inelastic responses of the constituents such as inclusions, polymer and interface by taking to account the micro-structural characteristics of nano-inclusion, such as size, shape, and compositions of the constituents. While micromechanical formulations that include detailed micro-structural characteristics can give good response characteristics, it is often difficult to obtain exact closed form solutions especially when material nonlinearity is considered. Limited analytical and experimental studies have been done on understanding the nonlinear inelastic behavior of reinforced polymer composites by nano-inclusions under cyclic loading.

Some of the micromechanical models have been extended to predict inelastic or viscoplastic behaviors of polymer based composites [4, 7, 8, 14, 15, 20]. Weng [27] used the self-consistent method for analyzing effective creep behavior of polycrystals. It was assumed that inclusion and matrix exhibit elastic and linear viscoelastic behavior, respectively. Levesque et al. [18] proposed a linearized homogenization scheme for predicting nonlinear viscoelastic responses of nanocomposites. In this scheme, the homogenized micromechanical model of the Mori and Tanaka [21] was used. The particle was modeled as linear elastic, while the Schapery nonlinear viscoelastic model [25] was applied for the matrix phase. Li and Gao [19] investigated viscoelastic responses of carbon nanotube particles embedded in polyamide. In their investigation, the viscoelastic responses of nanocomposites were obtained via the Mori-Tanaka (MT) model while the matrix and inclusion were considered viscoelastic and elastic materials, respectively. It is important to notice that, some polymers used as constituents in composite systems exhibit combined viscoelastic-viscoplastic responses, e.g. high density polyethylene and polycarbonate and Epoxy resins. These combined responses can occur at early loading (small stress/ strain levels). According to these behavior of polymer, Aboudi [5] has developed a micromechanical model to predict the viscoelastic-viscoplastic response of multiphase materials. Also, the viscoelastic-viscoplastic model for polymer developed by Frank and Brockman [10] is implemented in the multiphase composites.

This investigation is devoted to the modeling and response characterization of the Epoxy PR-520 reinforced with nanofiber composed of CNTs as transversely isotropic nanocomposite system with unidirectionally aligned nanofibres subjected to monoharmonic kinematic loading. To predict the nonlinear inelastic behavior of the polymeric matrix, the Goldberg constitutive model is used. To simulate the response in terms of amplitudes at different temperature, the relations between the amplitudes of main field variables are established with making use of complex moduli concept (the approximate amplitude relations). A micromechanical model to predict the inelastic response of multiphase nanocomposites by taking into account the effects of the interface properties is developed.

1. Mechanical behavior of constituents. The large application of polymeric composites in engineering solutions, especially in aircraft, automotive, medical and ship-building industries, could be observed in the last decades. The desire to replace of the structures made of metal alloys by nanofibre-reinforced polymeric composites was caused mainly by their high specific strength and stiffness properties with a simultaneous significant mass reduction. These properties of nanocomposite allow for the application of such materials in the most responsible engineering constructions like bodies of vehicles, turbine blades etc.

1.1 Mechanical response of nanofiber. Nanoinclusions are usually very stiff and their mechanical response can be assumed as predominantly elastic

[17]. According to reported effective elastic properties for nanofiber composed of unidirectionally oriented carbon nanotubes (CNTs), they are considered to be transversely isotropic. Consequently, the stress-strain relations are governed by five independent elastic constants. By using the Hill's notation, symmetric fourth-order stiffness tensor for nanofiber can be represented by the equation [16]:

$$\mathbf{L}^P = \mathbf{L}^P(2k_p, \ell_p, n_p, 2m_p, 2p_p), \quad (1)$$

where \mathbf{L}^P is the elastic stiffness tensor and n_p, k_p, ℓ_p, m_p and p_p are the Hill's elastic moduli for particle, namely the uniaxial tension modulus, the plane-strain bulk modulus, the associated cross modulus, the transverse shear modulus and the axial shear modulus, respectively. It can be presented in terms of the common engineering constants as

$$\mathbf{L}^P = \mathbf{L}^P(2K_{23}, C_{12}, C_{11}, 2G_{23}, 2G_{12}),$$

when the CNTs are aligned in the direction x_1 [23]. Adopting this notation, all moduli and Poisson's ratio, ν_{12} , are given by:

$$E_{11} = n_p - \frac{\ell_p^2}{k_p}, \quad E_{22} = \frac{4m_p(k_p m_p - \ell_p^2)}{k_p n_p - \ell_p^2 + m_p n_p}, \quad G_{12} = p_p, \\ G_{23} = m_p, \quad \nu_{12} = \ell_p/2k_p, \quad K_{23} = k_p. \quad (2)$$

The relation between Hill's constants and components of elastic stiffness tensor, L_{ijkl}^P , can be rearranged as follows:

$$k_p = \frac{C_{22} + C_{23}}{2}, \quad m_p = \frac{C_{22} - C_{23}}{2}, \quad n_p = C_{11}, \quad \ell_p = C_{12}, \quad p_p = C_{66}. \quad (3)$$

Consequently, using these common engineering constants, the stress-strain relations are derived.

1.2. Effects of the interface. In this investigation, the conditions of load transfer in interface of inclusion and matrix will be modeled by a displacement jump that was proposed by Qu [24]. Though the interface has negligibly small volume concentration, the effects of interface can significantly increase the overall stiffness and properties of the nanocomposite at the high volume fraction of inclusions [9]. In this model, the interface can be introduced by using a layer of negligible thickness in which traction remains continuous and displacement becomes discontinuous. The second order compliance tensor, η_{ij} , was proposed by Qu in [24]. According to equation (4), it is obvious that when the tensor η_{ij} tends to zero (infinite stiffness), the displacement jump is zero and continuity in displacements are recovered. This tensor is chosen to be symmetric and positive definite and components of second order compliance tensor can be expressed in the form [9]:

$$\eta_{ij} = \gamma\delta_{ij} + (\beta - \gamma)n_i n_j, \quad (4)$$

where δ_{ij} is the Kronecker delta. It is important to address the physical meaning of the parameters γ and β . They are parameters that represent the compliance in the tangential and normal directions, respectively.

1.3. Constitutive equations for the polymer matrix. Nonlinear inelastic behavior of polymer matrix will be represented by Goldberg model under kinematic harmonic loading at different temperatures and wide range of amplitudes in constant frequency. Goldberg et al. [12] proposed a model for predicting the viscoplastic response of neat polymers, utilizing a set of state

variables as an indication of the resistance of polymeric chains against flow. It should also be mentioned that polymer's mechanical properties and loading/strain rate are the two main parameters that govern the nonlinear response of the polymer. According to this model, the inelastic strain components can be expressed in terms of the deviatoric stress components as follows

$$\dot{\varepsilon}_{ij}^{\text{in}} = 2D_0 \exp\left(-\frac{1}{2} \frac{Z}{\sigma_e}\right)^{2n} \left(\frac{s_{ij}}{2\sqrt{J_2}} + \alpha\delta_{ij}\right), \quad (5)$$

where $\dot{\varepsilon}_{ij}^{\text{in}}$ is the inelastic strain rate tensor which can be defined as a function of deviatoric stress and Z and α are the internal state variables; J_2 is the second invariant of the deviatoric stress tensor that can be expressed as a function of σ_{ij} ; the variable α is a state variable which controls the level of the hydrostatic stress effects; D_0 and n are material constants; D_0 represents the maximum inelastic strain rate and n controls the rate dependency of the material. The effective stress, σ_e , also is defined as a function of the mean stress, such that the summation of the normal stress components σ_{kk} is three times of the mean stress. The evolution of the internal stress state variable Z and the hydrostatic stress state variable α are defined by the equations

$$\dot{Z} = q(Z_1 - Z)\dot{e}_e^{\text{in}}, \quad \dot{\alpha} = q(\alpha_1 - \alpha)\dot{e}_e^{\text{in}}, \quad (6)$$

where q is a material constant representing the «hardening» rate, and Z_1 and α_1 are material constants representing the maximum values of Z and α , respectively. The initial values of Z and α are defined by the material constants Z_0 and α_0 . The term \dot{e}_e^{in} in equation (6) represents the effective deviatoric inelastic strain rate.

2. Procedure of complex moduli derivation for matrix (linearization approach). In this investigation, the approximate model of nonlinear inelastic behavior developed in [3] for the case of proportional kinematic harmonic loading has been used. In this case, the cyclic properties of the polymer are described in terms of complex moduli. It is important to notice that the inelastic deformation is considered to be incompressible and thermal expansion is dilatational, it may be more convenient in some applications to separate the isotropic stress-strain relations into deviatoric and dilatational components that can be shown by equations as

$$s_{ij} = 2G(e_{ij} - \varepsilon_{ij}^{\text{in}}), \quad \sigma_{kk} = 3K_V(\varepsilon_{kk} - \varepsilon^{\theta}), \quad (7)$$

where G is the shear modulus, K_V is the bulk modulus, $i, j, k = 1, 2, 3$ and repeated index implies a summation over. Due to incompressibility of plastic deformation, $\dot{\varepsilon}_{kk}^{\text{in}} = 0$, the plastic strain rate is deviatoric, $\dot{\varepsilon}_{ij}^{\text{in}} = \dot{e}_{ij}^{\text{in}}$. According to approximate model, if a body as a system is subjected to harmonic deformation or loading, then its response is also close to harmonic law

$$e_{ij}(t) = e'_{ij} \cos \omega t - e''_{ij} \sin \omega t, \quad s_{ij}(t) = s'_{ij} \cos \omega t - s''_{ij} \sin \omega t. \quad (8)$$

The complex amplitudes of the deviator of total strain, \tilde{e}_{ij} , inelastic strain, $\tilde{e}_{ij}^{\text{in}}$, and the stress deviator, \tilde{s}_{ij} , are related in the N^{th} cycle by the complex shear modulus, \tilde{G}_N , and plasticity factor, $\tilde{\lambda}_N$, as shown below

$$\tilde{s}_{ij} = 2\tilde{G}_N \tilde{e}_{ij}, \quad \tilde{e}_{ij}^{\text{in}} = \tilde{\lambda}_N \tilde{e}_{ij}. \quad (9)$$

Here

$$\begin{aligned} \tilde{e}_{ij} &= e'_{ij} + ie''_{ij}, & \tilde{s}_{ij} &= s'_{ij} + is''_{ij}, \\ \tilde{e}_{ij}^{\text{in}} &= e_{ij}^{\text{in}} + ie_{ij}^{\text{in}''}, & \tilde{\lambda}_N &= \lambda'_N + i\lambda_N'', & \tilde{G}_N &= G'_N + iG_N'', \end{aligned}$$

and N is the cycle number, $N = 1, 2, 3, \dots$; $(\cdot)'$ and $(\cdot)''$ denote the real and imaginary parts of complex quantities. The shear modulus and plasticity factor are functions of the intensity of the strain-range tensor, frequency and temperature

$$\tilde{G}_N = \tilde{G}_N(e_i, \omega, \theta), \quad \tilde{\lambda}_N = \tilde{\lambda}_N(e_i, \omega, \theta), \quad (10)$$

where the square of the intensity of strain-range tensor is calculated as $e_i^2 = e'_{ij}e'_{ij} + e''_{ij}e''_{ij}$.

The imaginary parts of the complex moduli are determined from the condition of equality of the energies dissipated over a period and are calculated according to the formulas

$$G_N'' = \frac{\langle D' \rangle_N}{\omega e_i^2}, \quad \lambda_N'' = \frac{G_N''}{G_0}, \quad \langle (\cdot) \rangle_N = \frac{1}{T} \int_{T(N-1)}^{TN} (\cdot) dt, \quad T = \frac{2\pi}{\omega}, \quad (11)$$

where D' is the rate of dissipation of mechanical energy, G_0 is the elastic shear modulus. The real parts are found with making use of the condition that generalized cyclic diagrams $s_{aN} = s_{aN}(e_i, \omega)$ and $e_{paN} = e_{paN}(e_i, \omega)$, which relate the ranges of the stress and plastic-strain intensities in the N^{th} cycle, coincide in the frame of the complete and approximate approaches

$$\begin{aligned} G'_N(e_i, \omega) &= \sqrt{\frac{s_{aN}^2(e_i, \omega)}{4e_i^2} - G_N''^2(e_i, \omega)}, \\ \lambda'_N(e_i, \omega) &= \sqrt{\frac{e_{paN}^2(e_i, \omega)}{4e_i^2} - \lambda_N''^2(e_i, \omega)}, \end{aligned} \quad (12)$$

where G'_N and λ'_N are the sought-for real parts of shear modulus and plasticity factor.

In spite of the fact that the single-frequency approximation based on harmonic linearization has a good agreement with precise model of nonlinear behavior, it's necessary to analyze its practical accuracy for specific classes of problems.

As mentioned at the beginning of this section, the second approach (approximate approach) is based on the concept of complex moduli, which are determined by standard and modified techniques of equivalent linearization. It is important to notice that the imaginary parts of complex moduli are defined by the exact expression for rate of dissipation averaged over the period of cyclic loading while to improve the accuracy of real parts of complex moduli the modified approach is proposed as shown in equation (12). According to equation (10), the complex moduli depend on the temperature and amplitude of kinematic loading at the constant frequency. The purpose of this investigation is to study the influence of these parameters on complex moduli of polymeric nanocomposites.

3. The viscoelastic response of nanocomposite and modified Mori-Tanaka (MT) approach. By utilizing the Correspondence Principle in Viscoelasticity the constitutive relations for the inelastic behavior of the viscoelastic material

can be represented by: $\hat{\boldsymbol{\sigma}}(w) = \hat{\mathbf{L}}(w)\hat{\boldsymbol{\varepsilon}}(w)$, $\hat{\boldsymbol{\varepsilon}}(w) = \hat{\mathbf{M}}(w)\hat{\boldsymbol{\sigma}}(w)$, where $\hat{\mathbf{L}}(w)$ and $\hat{\mathbf{M}}(w)$ are the stress relaxation stiffness and creep compliance tensors, respectively. Every symbol with hat indicates the transformed function in the transformed domain, and w is the transform variable. It is important to notice that the feature of the Laplace transform is that it retains the physical dimension of the original function, which makes equations for relaxation stress and creep strain in transform domain mathematically similar to original Hooke law in time domain. As mentioned in section 3, by using the approximate amplitude relations, the complex shear moduli for matrix are derived at different temperatures for various strain amplitudes. Also, due to incompressibility of plastic deformation, the bulk modulus of polymer is considered constant and real. Then the stress relaxation stiffness tensor of polymer matrix, $\hat{\mathbf{L}}^M$, is determined as function of temperature, frequency and stress and strain amplitude.

After taking into account the effect of interface into the equivalent inclusion method, a new expression for the Eshelby's tensor will be found with slightly weakened interfaces. The new expression of Eshelby's tensor is written as:

$$\hat{\mathbf{S}}^M = \hat{\mathbf{S}} + (\mathbf{I} - \hat{\mathbf{S}})\mathbf{H}\hat{\mathbf{L}}^M(\mathbf{I} - \hat{\mathbf{S}}), \quad (13)$$

where $\hat{\mathbf{S}}$ is the original Eshelby's tensor with the components given in [23]. \mathbf{I} and $\hat{\mathbf{L}}^M$ are the fourth order identity tensor and the matrix relaxation stiffness tensor of matrix, respectively. The second term in the right hand side of equation (13) is present to introduce the interface effects. The components of tensor \mathbf{H} is presented as

$$H_{ijkl} = \gamma P_{ijkl} + (\beta - \gamma)Q_{ijkl} \quad (14)$$

expressions for tensor \mathbf{P} and \mathbf{Q} are given in [23].

Once the modified Eshelby's tensor based on equation (13) has been included into the analysis, the modified MT estimate is introduced. The new expression for the components of relaxation stiffness tensor of modified MT estimate for a two-phase aligned composite is obtained as

$$\hat{\mathbf{L}}^C = (V_0\hat{\mathbf{L}}^M + V_1\mathbf{L}^P\mathbf{A}^{\text{dil}})(V_0\mathbf{I} + V_1\mathbf{A}^{\text{dil}} + V_1\mathbf{H}\mathbf{L}^P\mathbf{A}^{\text{dil}})^{-1}, \quad (15)$$

where V_0 and V_1 are volume fraction of matrix and inclusion, respectively.

$\hat{\mathbf{L}}^C$ and \mathbf{L}^P are the relaxation stiffness tensor of composite and nanoinclusion, respectively. Also the components of dilatation tensor are recalculated as below:

$$A_{ijkl}^{\text{dil}} = [I + \hat{S}_{ijkl}^M \hat{L}_{ijkl}^M{}^{-1} (L_{ijkl}^P - \hat{L}_{ijkl}^M)]^{-1}. \quad (16)$$

It is worth mentioning that the expression for the effective inelastic properties in equation (15) depends on the nanoinclusion length in contrast to the original MT which is aspect ratio dependent.

In particular, for a transversely isotropic nanocomposite system containing unidirectionally aligned, identical nanoinclusions along the x_1 direction of a Cartesian coordinate system $Ox_1x_2x_3$, five independent parameters in the transformed domain based on Hill's notation can be obtained [19].

4. Numerical technique and the material properties. In the present work, for the determination of the nonlinear inelastic response of matrix (polymer), the numerical integration of Goldberg equations was adopted. To solve the implicit equation (5), one should utilize an appropriate numerical integration technique. Three step scheme of attacking the problem of complex

moduli determination was designed. At the first step, the elastic-viscoplastic response of the material to harmonic deformation was numerically calculated for different amplitudes of loading strain at various temperatures. At the second step, the stabilized cyclic stress-strain and inelastic-strain-strain diagrams were obtained for the whole set of calculated data. At the final step, the complex moduli were calculated by the averaging over the period of vibration of the results of direct integration and making use of cyclic diagrams and formulae (11) and (12). The system of nonlinear ordinary differential equations that describes the polymer response to harmonic loading in the case of pure shear consists of the one-dimensional equations of Goldberg model comprising equation (5) and evolutionary equations (6) are rewritten as follow [13]:

$$\begin{aligned}\dot{\varepsilon}_{12}^{\text{in}} &= 2D_0 \exp\left[-\frac{1}{2}\left(\frac{Z^2}{3S_{12}^2}\right)^n\right] \frac{S_{12}}{2|S_{12}|}, \\ \dot{\alpha} &= \frac{2qD_0}{\sqrt{3}}(\alpha_1 - \alpha) \exp\left[-\frac{1}{2}\left(\frac{Z^2}{3S_{12}^2}\right)^n\right] \frac{S_{12}}{|S_{12}|}, \\ \dot{Z} &= \frac{2qD_0}{\sqrt{3}}(Z_1 - Z) \exp\left[-\frac{1}{2}\left(\frac{Z^2}{3S_{12}^2}\right)^n\right] \frac{S_{12}}{|S_{12}|}.\end{aligned}\quad (17)$$

The law of strain deviator variation $e = e_0 \sin \omega t$, as well as Hooke law for shear stress $s_{12} = 2G(e_{12} - \varepsilon_{12}^{\text{in}})$, should be added to the system. It is important to notice that the known relations between the complex-value moduli, \tilde{E} , \tilde{G} and $\tilde{\nu}$, and real valued bulk modulus, K_V , exist in the form:

$$\tilde{E} = 2\tilde{G}(1 + \tilde{\nu}), \quad \tilde{\nu} = \frac{3K_V - 2\tilde{G}}{6K_V + 2\tilde{G}}.\quad (18)$$

According to the Correspondence Principle in Viscoelasticity the relaxation stiffness tensor of matrix, $\hat{\mathbf{L}}^M$, can be derived using two determined independent constants \tilde{G} and $\tilde{\nu}$. The modified MT method is applied here to obtain the effective inelastic properties of the nanocomposite. To obtain reasonable values for the parameter γ , we used the values which were obtained by Namilae and Chandra [22] with application of molecular dynamics.

The value for the parameter γ is chosen for all cases in this section 0.01 nm/GPa and β is set to zero to prevent material interpenetration. In this investigation, computations for nanocomposites reinforced by unidirectionally aligned nanofibers composed of the CNTs are presented. The values of material constants for RP-520 (Epoxy resin) that was chosen as matrix have been taken from [11] and used for calculations. The values of constants for matrix (at the different temperature) are given in Table 1. The material constants for CNTs have been taken from [26]. These values are listed below. Here $\ell_p = 10$ GPa, $k_p = 30$ GPa, $m_p = 1$ GPa, $n_p = 450$ GPa, $p_p = 1$ GPa, length and diameter of CNTs were chosen to be 6 and 1.7nm, respectively.

Table 1. The values of material constants for RP-520

Temp, °C	E_0 MPa	D_0 sec ⁻¹	Z_0 MPa	Z_1 MPa	n	q	a_0	a_1	ν
25	3250	10 ⁶	407.5	768.6	0.92	253.6	0.571	0.122	0.4
50	2980	10 ⁶	267.9	616.4	0.94	226.1	0.316	0.085	0.4
80	2520	10 ⁶	195.4	564.9	0.88	273.4	0.087	0.064	0.4

5. Numerical results and discussion. In this section, the results of investigation for examining the inelastic behavior of the transversely isotropic nanocomposite system with unidirectionally aligned nanofibres under kinematic harmonic loading are presented. Controlling parameters include temperature, amplitude of loading and volume fraction of nanofiber. The values of temperature 25, 50 and 80°C, volume fraction 3, 5 and 10 percent at the constant frequency 1Hz are considered, respectively. According to microstructural geometry of CNTs, the nanofiber aspect ratio for the transversely isotropic nanocomposites is chosen to be equal to 3.5. It is important to notice that, for the transversely isotropic nanocomposite system, five independent parameters, i.e., \hat{E}_{11} , \hat{E}_{22} , \hat{G}_{12} , $\hat{\nu}_{12}$ and \hat{K}_{23} , are used to illustrate their viscoelastic responses. The effects of volume fraction, V_f , and amplitude of harmonic loading on the complex axial Young's modulus, \hat{E}_{11} , complex transverse Young's modulus, \hat{E}_{22} , Poisson ratio, $\hat{\nu}_{12}$, and complex axial shear modulus, \hat{G}_{12} , at the constant frequency and temperature ($f = 1 \text{ Hz}$ and 25°C) are displayed in Figs. 1–3. These figures show that nanofibers volume fraction has little effect on the complex axial shear modulus, \hat{G}_{12} , and the transverse Young's moduli, \hat{E}_{22} , while its influence on the complex axial Young's modulus, \hat{E}_{11} , is profound.

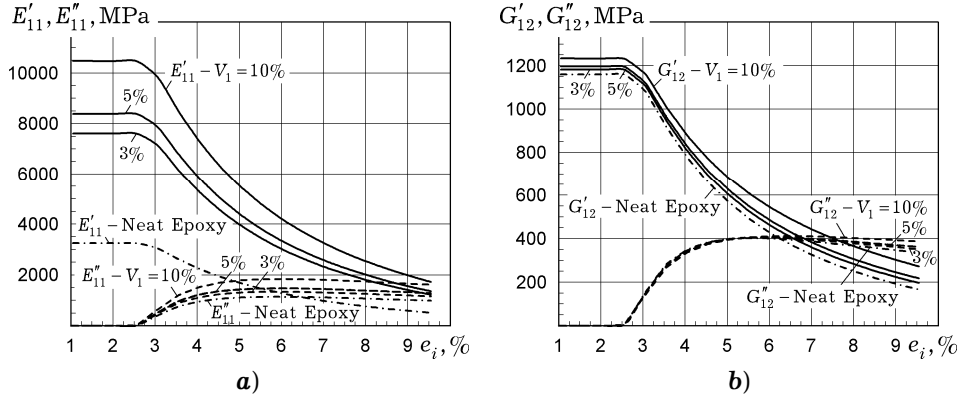


Fig. 1

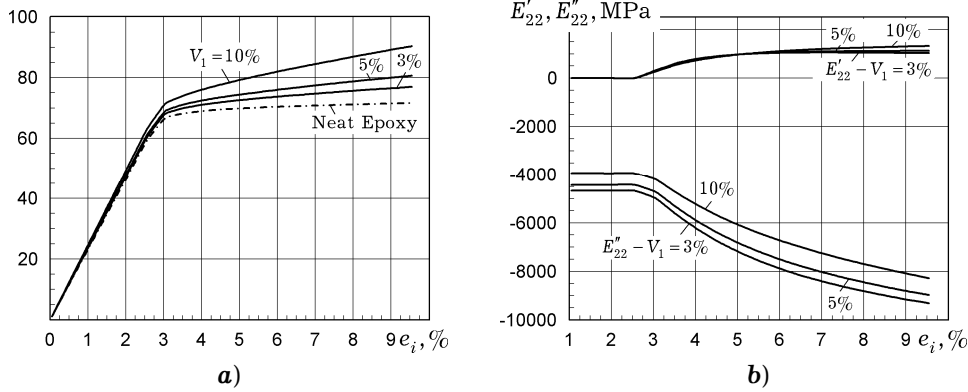


Fig. 2

In Fig. 3a and 3b, the effect of volume fraction on real and imaginary parts of Poisson ratio, $\hat{\nu}_{12}$, are presented. These results indicate that the influence of V_f for the unidirectionally aligned fibers on the Poisson ratio is small. The cyclic diagrams for neat polymer and nanocomposites with

different volume fractions at 1Hz at temperature 25°C are shown in Fig. 2a. The effect of nanofiber volume fraction is easily observable.

The influences of temperature on behavior of transversely isotropic nanocomposite system with uniaxially oriented nanofibers are clearly observed in Figs. 4–6.

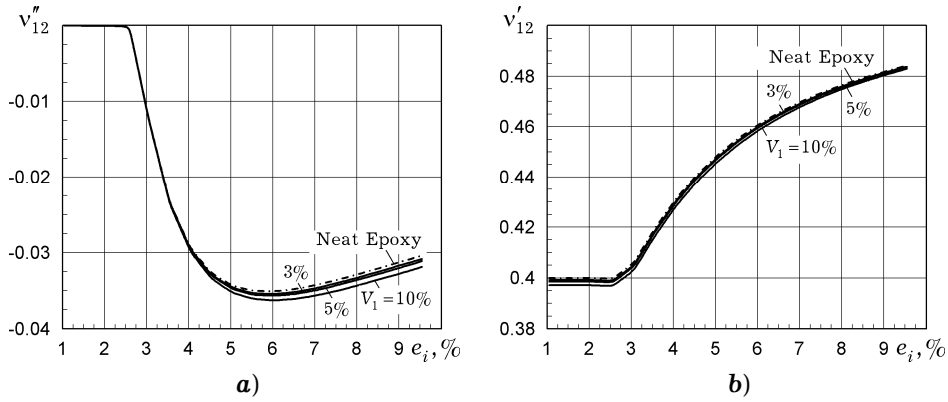


Fig. 3

According to the procedure mentioned in section 2, dependency of the complex axial shear modulus (storage modulus, G'_{12} , and loss modulus, G''_{12}) on the amplitude of strain, e_i , and temperature are shown in Fig. 4a for 5% of volume fraction of the nanofibers at 1Hz. As it is seen in this figure, the inelastic behavior occurs at lower strain amplitude with increasing of temperature. Also, the trend of storage and loss modulus behavior presented in Fig. 4a show that the values of storage modulus decrease with increase in temperature while the loss modulus increase slightly for the initial values of strain intensity at the beginning of the inelastic region (before the vicinity of 3.5% of strain intensity). This behavior of material is completely observed in cyclic diagram for different temperatures (see Fig. 5a). These results show the loss modulus increase with increasing temperature until yield point of material at the various temperatures. For higher values of strain intensity, the loss modulus decreases with increasing temperature. The peak values of the loss modulus decrease and occur earlier with increasing temperature. Within the interval of interest between 25 and 80°C, the maximum in loss modulus occurs in the vicinity of 6% of strain intensity at 25°C.

The effect of temperature on complex axial Young's modulus, E_{11} , is shown in Fig. 4b. As it is seen in this figure, the trends of real and imaginary parts of the complex axial Young's modulus with respect to amplitude of strain, e_0 , and temperature are completely the same as for the complex axial shear modulus, G_{12} . It is important to notice as temperature approach to the glass transition value the effects of temperature become more pronounced. The sensitivity of the complex axial shear modulus and axial Young's modulus on temperature are clearly observed in Fig. 4a and 4b.

The cyclic diagrams for transversely isotropic nanocomposite system with different temperatures at 1Hz and 5% of volume fraction of the nanofibers are shown in Fig. 5a. The influence of temperature on strength of material is easily observable. This figure show that, the strength of material decreases with increasing temperature in elastic and inelastic regions and the inelastic behavior occurs at lower strain amplitude with increase of temperature. The variations of complex transverse Young's modulus with respect to different temperature and wide range of strain amplitude of loading are presented in Fig. 5b. As it is seen the values of real and imaginary parts of transverse

Young's modulus (E'_{22} and E''_{22}) decrease with increasing temperature at the constant volume fraction and frequency. This figure shows that the sensitivity of transverse Young's modulus to temperature variations at the high values is more profound than at the region of lower temperature.

In Fig. 6a and 6b, the effect of temperature on real and imaginary parts of Poisson ratio, \hat{v}_{12} , are presented. These results indicate that the influence of temperature for the transversely isotropic nanocomposite system with unidirectionally aligned fibers on the Poisson ratio is small. Also, with increasing temperature, the values of Poisson ratio increase in the inelastic region because this behavior is governed by the nonlinear response of matrix material (polymer).

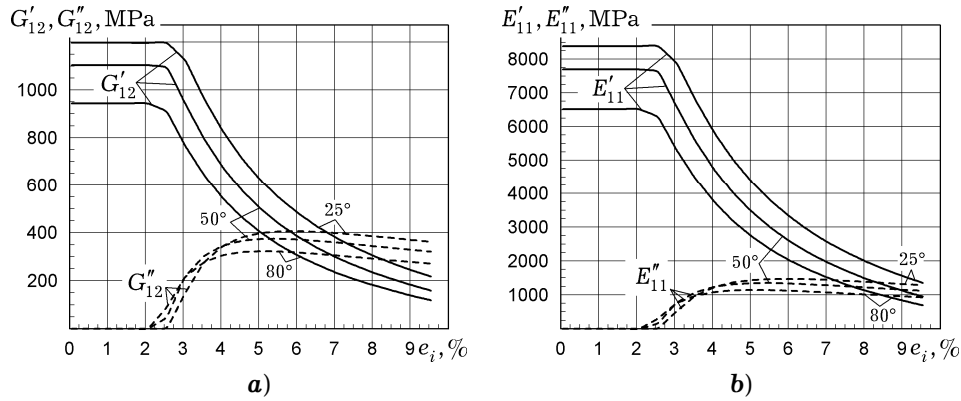


Fig. 4

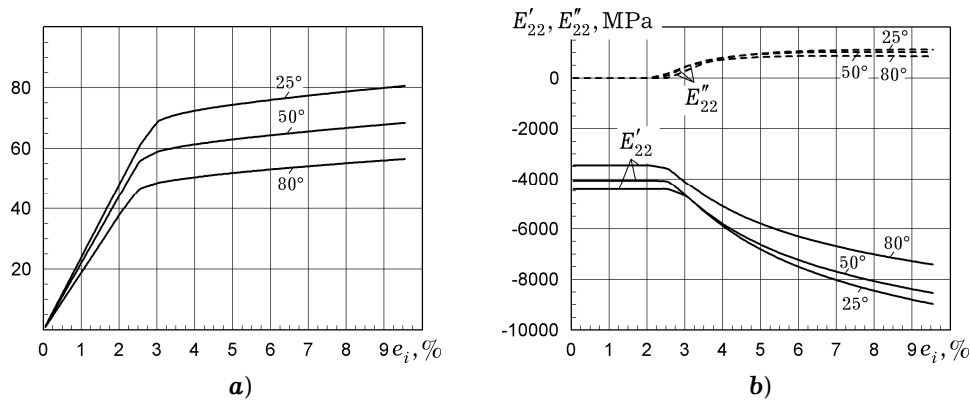


Fig. 5

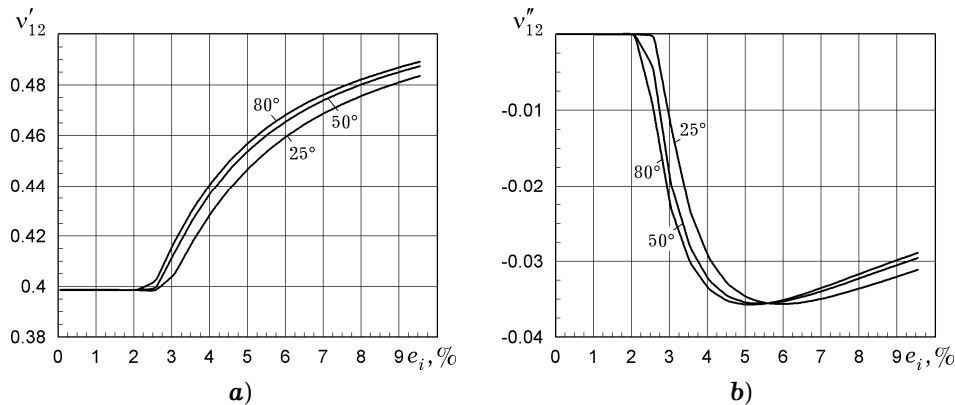


Fig. 6

Conclusions. In the paper, the effect of temperature and amplitude of excitation strain on complex modulus of nanocomposite under kinematic harmonic loading has been investigated. A micromechanics model is developed to determine effective inelastic properties of nanocomposite under monoharmonic loading by taking into account detailed micro-structural geometries and constitutive models of the constituents. By using the Correspondence Principle in viscoelasticity, the modified Mori-Tanaka method and effects of interface between inclusion and matrix is extended to the transformed domain. By applying the presently developed model, a numerical analysis for determination of the complex moduli of polymeric nanocomposite reinforced with nanofibers is conducted under the different temperature and kinematic harmonic loading. Characterization of the complex moduli dependence on temperature and amplitude of strain intensity is performed. In this investigation, composites reinforced with unidirectionally aligned nanofibers is considered. The volume fraction and temperature are considered as the controlling parameters. Results demonstrate the significant dependence of loss moduli on the temperature within the wide interval of it before the glass transition temperature. For nanocomposites containing unidirectionally aligned nanofibers, numerical results indicate that the increase of the nanofibers volume fraction enhances significantly their axial complex moduli but has little influences on their transverse, shear complex moduli. In addition, the effect of the unidirectionally aligned nanofiber orientation on the shear complex moduli is negligibly small. Furthermore, for the transversely isotropic nanocomposite system with uniaxially aligned nanofibers, both the storage and loss moduli are found to increase monotonically with the increase of the nanofiber volume fraction while decrease with increasing temperature. The procedure presented in this paper for obtaining dependencies can be useful for estimation of the mechanical and thermal degradation of polymeric composites and can be subsequently applied for the determination of fatigue, crack growth and residual life of composite structures.

1. *Киричок І. Ф., Сенченков І. К.* Осесиметричні резонансні коливання і вібророзігрів термов'язкопружної циліндричної оболонки з п'єзоелектричними сенсорами і актуаторами // *Мат. методи та фіз.-мех. поля.* – 2014. – **57**, № 3. – С. 53–61.
Kirichok I. F., Senchenkov I. K. Axisymmetric resonant vibrations and vibration heating of a thermoviscoelastic cylindrical shell with piezoelectric sensors and actuators // *J. Math. Sci.* – 2016. – **217**, No. 2. – P. 198–209.
2. *Киричок І. Ф., Сенченков І. К., Червінко О. П.* Вимушені коливання і вібророзігрів в'язкопружних балок з п'єзоелектричними сенсорами і актуаторами // *Мат. методи та фіз.-мех. поля.* – 2014. – **57**, № 2. – С. 112–124.
Kiryuchok I. F., Senchenkov I. K., Chervinko O. P. Forced vibrations and vibration heating of viscoelastic beams with piezoelectric sensors and actuators // *J. Math. Sci.* – 2016. – **215**, No. 2. – P. 143–158.
3. *Сенченков І. К., Жук Я. О., Карнаухов В. Г.* Моделирование термомеханического поведения физически нелинейных материалов при моногармоническом нагружении // *Прикл. механика.* – 2004. – **40**, № 9. – С. 3–34.
Senchenkov I. K., Zhuk Y. A., Karnaukhov V. G. Modeling the thermomechanical behavior of physically nonlinear materials under monoharmonic loading // *Int. Appl. Mech.* – 2004. – **40**, No. 9. – P. 943–969.
4. *Aboudi J.* Mechanics of composite materials: A unified micromechanical approach. – Amsterdam: Elsevier, 1991. – xi+328 p.
5. *Aboudi J.* Micromechanically established constitutive equations for multiphase materials with viscoelastic-viscoplastic phases // *Mech. Time-Depend. Mater.* – 2005. – **9**, No. 2-3. – P. 121–145.
6. *Beards C. F.* Structural vibration: Analysis and damping. – London: Arnold, 1996. – xii+276 p.
7. *Belayachi N., Benseddiq N., Naït-Abdelaziz M.* Behaviour of the heterogeneous glassy polymers: computational modelling and experimental approach // *Compos. Sci. & Technol.* – 2008. – **68**, No. 2. – P. 367–375.

8. *Eshelby J. D.* The determination of the elastic field of an ellipsoidal inclusion, and related problem // Proc. Roy. Soc. Lond. **A**. – 1957. – **241**, No. 1226. – P. 376–396.
9. *Esteva M., Spanos P. D.* Effective elastic properties of nanotube reinforced composites with slightly weakened interfaces // J. Mech. Mater. Struct.. – 2009. – **4**, No. 5. – P. 887–900.
10. *Frank G. F., Brockman R. A.* A viscoelastic-viscoplastic constitutive model for glassy polymers // Int. J. Solids Struct. – 2001. – **38**, No. 30-31. – P. 5149–5164.
11. *Gilat A., Goldberg R. K., Roberts G. D.* Incorporation of the effects of temperature and unloading into the strain rate dependent analysis of polymer matrix materials utilizing a state variable approach // J. Earth & Space. – 2006. – **4**. – P. 1–8.
12. *Goldberg R. K.* Computational simulation of the high strain rate tensile response of polymer matrix composites. – NASA/TM-2002-**211489**. – P. 1–16.
13. *Hashemi M., Zhuk Y. A.* Characterization of complex moduli for plastic material under harmonic loading // Современные проблемы естественных наук: Тез. докл. междунар. конф. «Тараповские чтения – 2016» (Харьков, 1–15 марта 2016 г.) – Харьков, 2015. – С. 76.
14. *Hashin Z., Shtrikman S.* On some variational principles is anisotropic, and nonhomogeneous elasticity // J. Mech. Phys. Solids. – 1962. – **10**, No. 4. – P. 335–342.
15. *Hill R.* A self-consistent mechanics of composite materials // J. Mech. Phys. Solids. – 1965. – **13**, No. 4. – P. 213–222.
16. *Hill R.* Theory of mechanical properties of fibre-strengthened materials: I. Elastic behaviour // J. Mech. Phys. Solids. – 1964. – **12**, No. 4. – P. 199–212.
17. *Khan Z. H., Husain M.* Carbon nanotube and its possible applications // Indian J. Eng. & Mater. Sci. – 2005. – **12**. – P. 529–551.
18. *Lévesque M., Gilchrist M. D., Bouleau N., Derrien K., Baptiste D.* Numerical inversion of the Laplace-Carson transform applied to homogenization of randomly reinforced linear viscoelastic media // Comput. Mech. – 2007. – **40**, No. 4. – P. 771–789. – Doi 10.1007/s00466-006-0138-6.
19. *Li K., Gao X.-L., Roy A. K.* Micromechanical modeling of viscoelastic properties of carbon nanotube-reinforced polymer composites // Mech. Adv. Mater. Struct. – 2006. – **13**, No. 4. – P. 317–328.
20. *McLaughlin R. A.* A study of the differential scheme for composite materials // Int. J. Eng. Sci. – 1977. – **15**, No. 4. – P. 237–244.
21. *Mori T., Tanaka K.* Average stress in matrix and average elastic energy of materials with misfitting inclusions // Acta Metallurgica. – 1973. – **21**, No. 5. – P. 571–574. – [https://doi.org/10.1016/0001-6160\(73\)90064-3](https://doi.org/10.1016/0001-6160(73)90064-3).
22. *Namilae S., Chandra N.* Multiscale model to study the effect of interfaces in carbon nanotube-based composites // J. Eng. Mater. Technol. – 2005. – **127**, No. 2. – P. 222–232.
23. *Qiu Y. P., Weng G. J.* On the application of Mori-Tanaka's theory involving transversely isotropic spheroidal inclusions // Int. J. Eng. Sci. – 1990. – **28**, No. 11. – P. 1121–1137.
24. *Qu J.* The effect of slightly weakened interfaces on the overall elastic properties of composite materials // Mech. Mater. – 1993. – **14**, No. 4. – P. 269–281.
25. *Schapery R. A.* On the characterization of nonlinear viscoelastic materials // Polym. Eng. & Sci. – 1969. – **9**, No. 4. – P. 295–310.
26. *Shi D.-L., Feng X.-Q., Huang Y. Y., Hwang K.-C., Gao H.* The effect of nanotube waviness and agglomeration on the elastic property of carbon nanotube-reinforced composites // Trans. ASME. J. Eng. Mater. Technol. – 2004. – **126**, No. 3. – P. 250–257.
27. *Weng G. J.* A self-consistent relation for the time-dependent creep of polycrystals // Int. J. Plasticity. – 1993. – **9**, No. 2. – P. 181–198.

ВПЛИВ ТЕМПЕРАТУРИ НА ЦИКЛІЧНІ ВЛАСТИВОСТІ ТРАНСВЕРСАЛЬНО-ІЗОТРОПНОЇ НАНОКОМПОЗИТНОЇ СИСТЕМИ ПРИ ГАРМОНІЧНОМУ НАВАНТАЖЕННІ

Розвинуто термомеханічну модель для визначення ефективних непружних властивостей нанокompозита при моногармонічному навантаженні, яка враховує вплив геометричних параметрів мікроструктури та властивостей складових. За допомогою модифікованого методу Мори–Танака й принципу в'язкопружної відповідності в моделі враховано наявність приповерхневої області взаємодії на межі контакту включення і матриці. Розвинуту методикку застосовано до числового визначення комплексних модулів полімерного нанокompозита, зміцненого нановолокнами з вуглецевих нанотрубок, який знаходиться в ізотермічних умовах при заданій температурі. Проведено аналіз залежності комплексних модулів від температури та інтенсивності деформації для композита, зміцненого однонаправленими волокнами. Результати демонструють значну залежність модулів накопичення і втрат від температури в широкій області її зміни. Модулі накопичення і втрат зростають монотонно при збільшенні об'ємного вмісту волокон і зменшуються при зростанні температури. Показано, що жорсткість нанокompозита зменшується у пружній та непружній областях поведінки при збільшенні температури.

ВЛИЯНИЕ ТЕМПЕРАТУРЫ НА ЦИКЛИЧЕСКИЕ СВОЙСТВА ТРАНСВЕРСАЛЬНО-ИЗОТРОПНОЙ НАНОКОМПОЗИТНОЙ СИСТЕМЫ ПРИ ГАРМОНИЧЕСКОМ НАГРУЖЕНИИ

Развита термомеханическая модель для определения эффективных неупругих свойств нанокompозита при моногармоническом нагружении, учитывающая влияние геометрических параметров микроструктуры и свойств составляющих. При помощи модифицированного метода Мори–Танака и принципа вязкоупругого соответствия, в модели учтено наличие приповерхностной области взаимодействия на границе контакта включения и матрицы. Развита методика применяется для численного определения комплексных модулей полимерного нанокompозита упрочненного нановолокнами из углеродных нанотрубок, находящегося в изотермических условиях при заданной температуре. Проведен анализ зависимости комплексных модулей от температуры и интенсивности деформации для композита, упрочненного однонаправленными волокнами. Результаты демонстрируют существенную зависимость модулей накопления и потерь от температуры в широкой области ее изменения. Модули накопления и потерь возрастают монотонно при увеличении объемного содержания волокон и уменьшаются при возрастании температуры. Показано, что жесткость нанокompозита уменьшается в упругой и неупругой областях поведения при возрастании температуры.

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Received
03.07.16