

**AXIALLY SYMMETRIC SOUND RADIATION BY ELASTIC HOLLOW CYLINDER ROTATING IN THE AIR**

*Sound radiation from an elastic circular, empty inside, cylindrical tube of infinite length rotating with non-uniform angular velocity in the air is studied. The exact solutions of equations describing the aeroelastic interaction are obtained using the Fourier-transform over time. Numerical examples show that spectral structure of the sound radiation from an elastic tube is more complicated than that of a solid cylinder. In particular, the resonances of this structure are essentially dependent on the thickness of the rotating object and are subjected to the phenomena of dispersion.*

**1. Introduction.** In the techniques a good few of the objects or its parts rotate with variable angular velocity. The examples of these bodies are the rotors or shafts as the main elements in most power, electric and transport machines, as well as in many devices [6]. Various aspects of the corresponding problems attracted attention of many authors long ago [5]. In general, researchers studied the electromagnetic, thermal, mechanical and other characteristics of these objects, very rarely taking into consideration the fact that these bodies are often surrounded by an acoustical medium. At the same time, mechanical objects radiate sounds during rotation. The spectrum of sound radiation may vary greatly. On the one hand, it permits to know about the inner state of the rotated body. On the other hand, the rotors are the main source of vibrations, dangerous intensity of which depends on a whole number of factors [6]. In addition, inconstancy of the angular velocity can bring about an essential re-distribution of the strain-stress state in the elastic body and lead even to the destruction on the resonance frequencies. The correspondent information is contained in the acoustical field, too. Thus, sound radiation by the rotating objects is an up to date problem. For its understanding it is necessary make a careful study of the structure of wave field both inside sounding elastic object and outside it.

Complexity of the problem consists in that the several interdependent mechanisms take part during the process of sound generation by a rotating bodies in the real conditions. Often this process is studied without taking into account an elastic strain of radiator (see e.g. [12]). One of the sources of noise is the boundary layer, which is formed around rotating cylinder thanks to viscosity of acoustic medium. Recent experiments on the sound radiation by the turbulent boundary layer were performed [9], where generator of noise was an elastic cylindrical shell rotating in a water. Since the shell is thin-walled, the influence of inner wave processes in the thickness of body on the sound radiation is hard to investigate.

In papers [5, 10] we make an attempt to estimate sound field, which is formed exceptionally on account of the strain elastic waves in cylinder rotating with time-varying angular velocity. To simplify problem it was assumed that outer space is filled by an ideal (non-viscous) compressible liquid (gas). Corresponding numerical calculations were fulfilled for the case of elastic cylinder rotating in water.

Now we continue our investigation and give detailed analysis of sound field radiated in the air. The model object is an infinitely long elastic circular hollow cylinder rotating inconstancy about its axis of symmetry. We again concentrate main attention on the wave field excited by the cylinder in the surrounding medium. First, we investigate the spectral characteristics of the radiated sound. In the numerical examples for the case of the Armco iron – air interaction, the dependence of the sound pressure amplitude on frequency

and cylindrical tube thickness is studied. It turned out that the sound field consists of the series of resonances. In addition, the resonance dependence on the cylinder thickness is subjected to powerful dispersion. This effect is also illustrated by numerical calculations.

The non-constant angular velocity of the cylinder rotation causes the first and double sound harmonic excitations if the constant value of this velocity is modulated by the small sinusoidal amplitude over time oscillation. It is well illustrated by the numerical calculations for the intensity of the radiated acoustical wave obtained for different values of the frequency of disturbance angular velocity and tube thickness. We found two series of the amplitude resonances, one on the fundamental frequencies corresponding to the resonances of the spectrum, other on the frequencies two times smaller than the main one.

**2. Spectral characteristics.** Consider the case of non-uniform rotation of the elastic hollow cylinder of the infinite length around its axis of symmetry in the compressible ideal gas (air). The cylinder is empty inside. As a consequence of the rotation the centrifugal force arises. This force varies over time. Then in the material of the body, axially symmetric converging and diverging cylindrical elastic waves of the longitudinal and shear types are generated. Simultaneously, in the surrounding air sound waves excited by the radial vibration of the outer cylindrical surface are radiated. The intensity of these waves depends on the frequency and relative amplitude of the oscillation of the angular velocity.

The equation of the dynamical equilibrium of the elastic hollow cylinder rotating about its unmoved axis of symmetry at variable angular velocity is in the form [4, 11]

$$(\lambda + 2\mu)\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2}\right) + r\rho_s\Omega^2(t) = \rho_s\frac{\partial^2 u}{\partial t^2}, \quad b \leq r \leq a, \quad (1)$$

where  $u \equiv u(r, t)$  is the radial displacement;  $\Omega(t)$  is the time-variable angular velocity of the axial rotation of the body;  $\lambda, \mu$  are the Lamé parameters;  $\rho_s$  is the density of the elastic material;  $r$  is the radial co-ordinate with the origin on the axis of symmetry;  $t$  is the time;  $a$  and  $b$  are the outer and inner radii of the tube, respectively.

The pressure in the acoustical medium  $p \equiv p(r, t)$  is defined by the wave equation [8]

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}, \quad a \leq r < \infty, \quad (2)$$

where  $c$  is the sound velocity.

On the surfaces of the cylinder the following boundary conditions are satisfied [10]:

$$(\lambda + 2\mu)\frac{\partial u}{\partial r} + \lambda\frac{u}{r} + p = 0, \quad r = a, \quad (3)$$

$$\frac{\partial^2 u}{\partial t^2} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0, \quad r = a, \quad (4)$$

$$(\lambda + 2\mu)\frac{\partial u}{\partial r} + \lambda\frac{u}{r} = 0, \quad r = b, \quad (5)$$

where  $\rho$  is the fluid density.

To study the spectral characteristics of the radiated sound waves in the air we apply the integral Fourier transformation over time to Eqs (1)–(5), taking into account that all input and unknown functions satisfy the causality principle [8].

Then, in the Fourier-transforms space (steady-state regime) we obtain the exact solution of problem (1)–(5). In particular, for the Fourier-transform of acoustical pressure in the air we have following expression [10]:

$$p^F(r, \omega) = (\lambda + 2\mu)X_L^2(\omega)P(r, \omega), \quad a \leq r < \infty, \quad (6)$$

where

$$P(r, \omega) = \varphi_0(kr)\Delta_B \frac{1}{x_L\Delta}, \quad (7)$$

$$\Delta_B = J_2(x_L)[1 - \psi_{02}^-(x_L)] + \psi_2(x_L)[J_{02}^-(x_L) - J_{02}^-(y_L)],$$

$$\Delta = J_{02}^-(x_L) - \psi_{02}^-(x_L)J_{02}^-(y_L) + \varphi_0(x)[J_1(x_L) - \psi_1(x_L)J_{02}^-(y_L)],$$

$$X_L^2(\omega) = \frac{a^2}{c_L^2} \int_{-\infty}^{\infty} \Omega^2(t)e^{i\omega t} dt, \quad \varphi_0(kr) = -\zeta H_0^{(1)}(kr) \frac{1}{H_1^{(1)}(x)},$$

$$\zeta = \frac{\rho c}{\rho_s c_L}, \quad \psi_j(k_L r) = N_j(k_L r) \frac{1}{N_{02}^-(y_L)}, \quad j = 0, 1,$$

$$\psi_{02}^-(k_L r) = N_{02}^-(k_L r) \frac{1}{N_{02}^-(y_L)}. \quad (8)$$

Here and further index «*F*» denotes the Fourier-transform;  $J_n(z)$ ,  $n = 0, 1, 2$ , are the Bessel functions;  $N_n(z)$ ,  $n = 0, 1, 2$ , are the Neimann functions;  $H_n^{(1)}(z)$ ,  $n = 0, 1$ , are the Hankel functions of the first kind;  $\omega$  is the Fourier-transform parameter (the circular frequency for  $\omega \geq 0$ );  $k = \omega/c$  is the wave number for the acoustical medium;  $\alpha = c_T^2/c_L^2$ ,  $c_L^2 = (\lambda + 2\mu)/\rho_s$ ,  $c_T^2 = \mu/\rho_s$ , where  $c_L$  and  $c_T$  are the velocities of the longitudinal and shear waves in the cylinder, respectively;  $k_L = \omega/c_L$  is wave number for the elastic material and  $x_L = k_L a$ ,  $y_L = k_L b$ .

Similarly, the spectral distribution of the particle velocity in the acoustical medium  $v^F(r, \omega) = -i(\rho\omega)^{-1}(\partial p^F/\partial r)$  can be expressed by the formula

$$v^F(r, \omega) = c_L X_L^2(\omega)V(r, \omega), \quad a \leq r < \infty, \quad (9)$$

where

$$V(r, \omega) = \varphi_1(kr)\Delta_B \frac{1}{x_L\Delta}, \quad \varphi_1(kr) = -iH_1^{(1)}(kr) \frac{1}{H_1^{(1)}(x)}. \quad (10)$$

Solutions (6) and (9) are obtained using the Sommerfeld conditions of the wave radiation at  $r \rightarrow \infty$  [8].

In other limiting case  $\omega \rightarrow 0$  was shown [10] that

$$P(r, \omega) \rightarrow 0, \quad V(r, \omega) \rightarrow 0. \quad (11)$$

In partial case of the solid cylinder when  $b \rightarrow 0$  we obtain the results of the paper [5].

**3. The time characteristics.** If the oscillation of the angular velocity  $\Omega(t)$  have form

$$\Omega(t) = \Omega_0(1 + \varepsilon_0 \sin \omega_0 t), \quad -\infty < t < \infty, \quad (12)$$

where  $\Omega_0$  is the constant angular velocity of the cylinder rotation;  $\varepsilon_0$  is the small non-dimensional parameter characterizing the amplitude of the disturbance of this velocity;  $\omega_0$  is the circular frequency, then from the Fourier-transform for  $\Omega^2(t)$  we obtain [1]

$$X_L^2(\omega) = 2\pi X_{L0}^2 \left\{ (1 + 0.5\varepsilon_0^2) \delta(\omega) - i\varepsilon_0 [\delta(\omega + \omega_0) - \delta(\omega - \omega_0)] - 0.25\varepsilon_0^2 [\delta(\omega + 2\omega_0) + \delta(\omega - 2\omega_0)] \right\}, \quad (13)$$

where  $\delta(z)$  is the Dirac function and  $X_{L0} = \Omega_0 a/c_L$ .

Applying the inverse Fourier-transform to Eqs (6), (9) and taking into account asymptotic properties (11), we obtain following formulas for the acoustical pressure and particle velocity generated by the rotating motion of the hollow cylinder at the modulated angular velocity:

$$\frac{1}{(\lambda + 2\mu)X_{L0}^2} p(r, t) = -2\varepsilon \operatorname{Im} [P(r, \omega_0) \exp(-i\omega_0 t)] - 0.5\varepsilon^2 \operatorname{Re} [P(r, 2\omega_0) \exp(-2i\omega_0 t)], \quad a \leq r < \infty, \quad (14)$$

$$\frac{1}{c_L X_{L0}^2} v(r, t) = -2\varepsilon_0 \operatorname{Im} [V(r, \omega_0) \exp(-i\omega_0 t)] - 0.5\varepsilon_0^2 \operatorname{Re} [V(r, 2\omega_0) \exp(-2i\omega_0 t)], \quad a \leq r < \infty. \quad (15)$$

For the estimation of the sound energy radiated in the acoustical medium let calculate the time average of the power over period  $T_0 = 2\pi/\omega_0$ :

$$I = \frac{1}{T_0} \int_0^{T_0} p(r, t)v(r, t) dt, \quad a \leq r < \infty. \quad (16)$$

Then, taking into account Eqs (14) and (15), we obtain

$$I = 2\varepsilon_0^2 \rho_s c_L^{-1} (\Omega_0 a)^4 \operatorname{Re} [P(r, \omega_0) V^*(r, \omega_0) + (\varepsilon_0/4)^2 P(r, 2\omega_0) V^*(r, 2\omega_0)], \quad r \geq a. \quad (17)$$

Here asterisk is a sign of complex conjugation.

**4. The numerical results.** The numerical calculations are carried out for the case of the Armco iron hollow cylinder ( $\rho_s = 7700 \text{ kg/m}^3$ ,  $c_L = 5960 \text{ m/s}$ ,  $c_T = 3240 \text{ m/s}$  [2]), immersed in the air ( $\rho = 1.293 \text{ kg/m}^3$ ,  $c = 331 \text{ m/s}$  [7]).

Fig. 1 shows the function  $|P(r, \omega)|$  (in dB) of the non-dimensional frequency  $x = ka$  (the wave outer radius of the cylinder) and the geometrical parameter  $\varepsilon = b/a$

(the relative inner radius of the cylinder) at  $r/a = 1$ . On account of the wave reflections between the cylindrical surfaces the frequency spectrum of the sound generated in the surrounding air has an explicitly expressed resonance character. Therefore, the resonance locations are sufficiently dependent on the thickness of the cylindrical objects. Namely, the sound waves on the resonance frequencies are subjected to geometrical dispersion when the tube becomes thinner. That is, the resonance locations in general are the non-monotonic function of the parameter  $\varepsilon$ .

This effect is well illustrated by Fig. 2, where curves of the

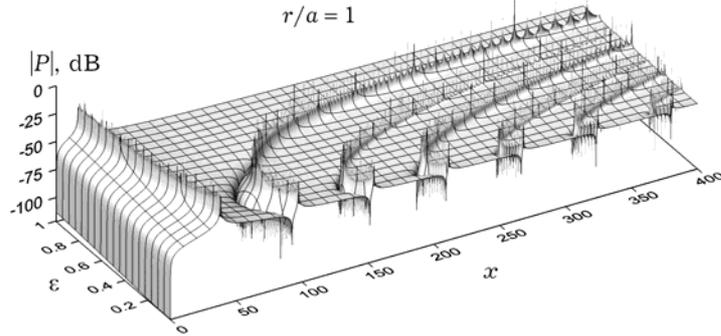


Fig. 1

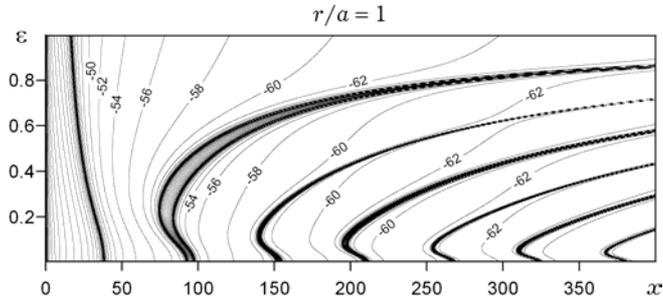


Fig. 2

identical levels of the sound spectrum amplitudes are plotted. It is shown that the first, low frequency resonance moves in the lower frequency range if the parameter  $\varepsilon$  decreases.

Additionally, Fig. 3 shows a thin structure of the spectral lines of first resonance for the discrete values of  $\varepsilon$  (the results are obtained with the non-dimensional frequency step  $\Delta x = 0.0005$ ). Dispersion is also observed for the resonances of the higher orders but only for not very great values of  $\varepsilon$ . The range of parameter  $\varepsilon$ , in which the resonance lines are moved to the lower frequencies side, narrows quickly with an increasing of the resonance order. Moreover, there are values of the cylindrical tube thickness for which a direction of the resonance curves motion is changed. That is, for continuously increasing  $\varepsilon$ , the resonances speedily move to the high frequencies side (Fig. 2). As a matter of fact, we observe radiating sound waves formed with the negative group velocity. These plots also show that the resonance amplitudes decrease rapidly with an increasing of the resonance order.

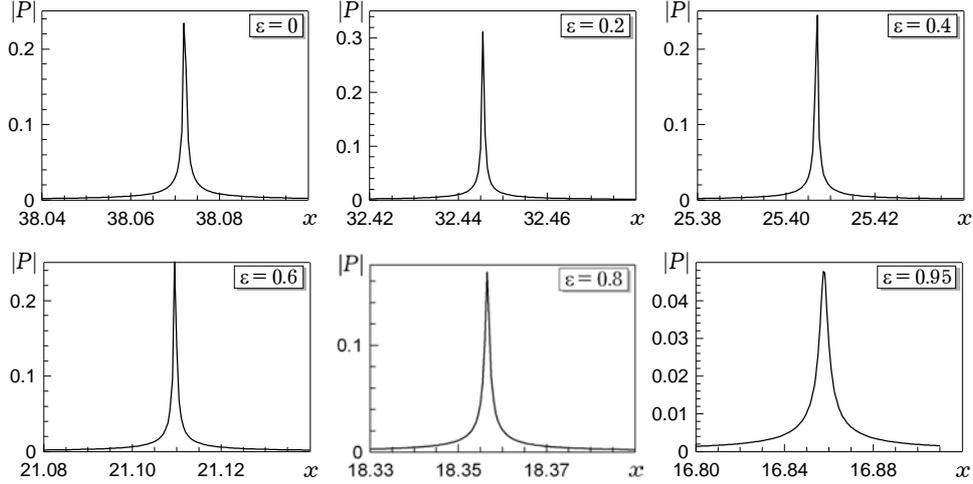


Fig. 3

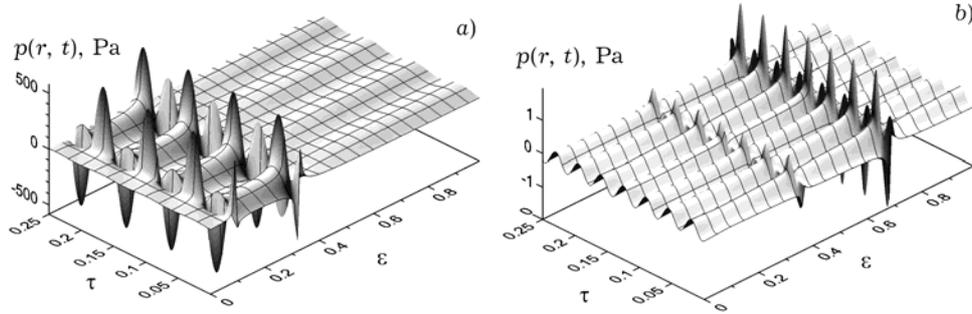


Fig. 4

In the pulse situation we can see from Eq. (14) that the acoustical waves radiate at two frequencies,  $\omega_0$  and  $2\omega_0$ . Fig. 4a demonstrates the time dependence of the sound pressure  $p(r, t)$  for different values of the thickness parameter  $\varepsilon$  and  $x_0 = 90$  ( $x_0 = k_0 a$ ,  $k_0 = \omega_0/c$ ,  $\tau = ct/a$ ). The calculations are carried out for the pressure far from the cylindrical surface,  $r/a = 10$ . The cylinder of the outer radius  $a = 0.457$  m carries out  $N_0 = 50$  revolutions per second ( $\Omega_0 = 2\pi N_0$  rad/s) with the relative amplitude of the angular velocity modulation  $\varepsilon_0 = 0.1$ . These plots illustrate the space resonances. Indeed, till

the values of  $\varepsilon$  are contained outside of resonance positions (cf. Figs 1 and 2) the acoustical signals are with the low sinusoidal amplitudes. The picture changes significantly if the parameter  $\varepsilon$  crosses the dispersive curves. Then oscillations of the signals become well noticeable although for small  $\varepsilon_0$  these amplitudes scarcely reach 400 Pa. Fig. 4b is obtained only for the second term of expression (14), that is for the component with double frequency. Here, the above mentioned effect of the space resonance is also demonstrated, but, naturally, it arises for another values of the  $\varepsilon$ . The amplitudes of these oscillations are by two order lower, because again the parameter  $\varepsilon_0$  is small. In fact, such additional signal is masked by the signal background with the frequency  $\omega_0$ , but its existence gives the evidence that the nature of the sound radiation in our case has the character of the wave field with the second harmonic.

It is well illustrated in Fig. 5a for the sound radiation intensity dependence [3]

$$N = 10 \lg(I/I_0), \quad I_0 = 10^{-12} \text{ W/m}^2, \quad (18)$$

on different  $x_0$  and  $\varepsilon$ . Here  $I_0$  is zero level of sound intensity. The value  $N$  is calculated on the basis of Eq. (17) with  $r/a = 1$ ,  $\varepsilon_0 = 0.3$ . Fig. 5a depicts the total intensity and Fig. 5b represents only that which is defined by the second component of the expression for power  $I$ .

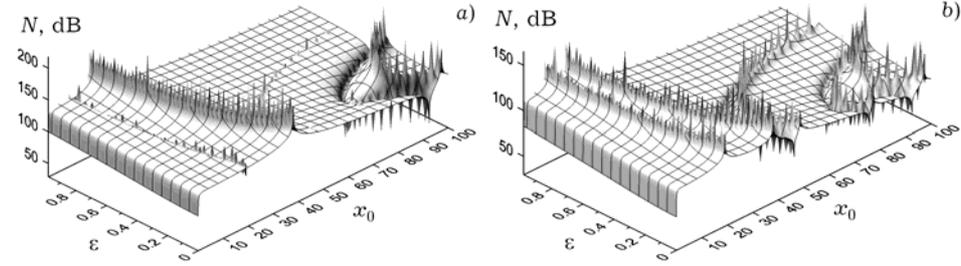


Fig. 5

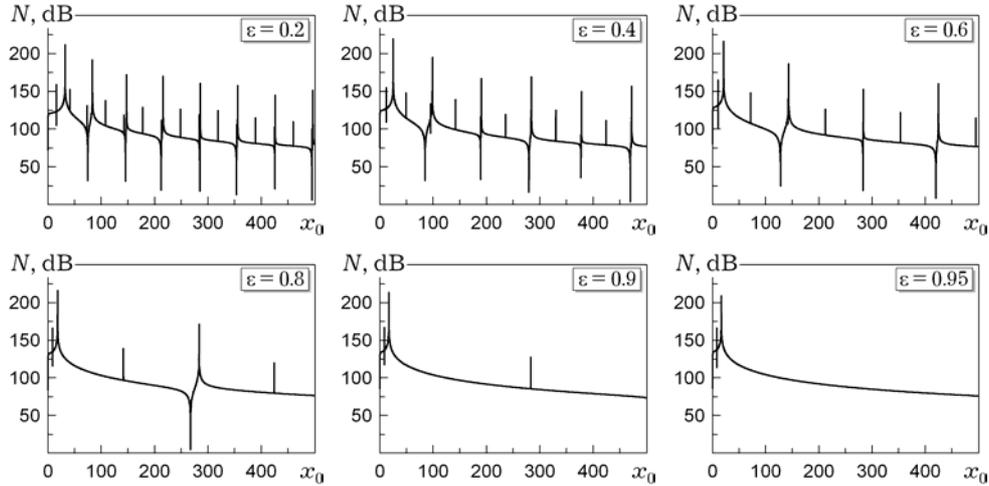


Fig. 6

These plots also discover both the resonances of the radiation amplitudes and the dispersive character of the wave formation. In more detail the structure of the radiation intensity as the function of the frequency is displayed in Fig. 6 (also in dB) for the discrete values of  $\varepsilon$  (all other parameters are as in Fig. 5). These illustrations show that the resonances are of high quality with

the fairly intensive amplitudes. The resonances of the double frequency are fairly good visible, especially for the low frequency range. For comparison, in the case of cylinder rotating in a water [10] we note that corresponding low-level spectral lines are practically masked. The plots describe very well the motion of the resonance locations with a change of the geometrical parameter  $\varepsilon$ . It is shown that the first resonance line moves, extending and decreasing, to the low frequency range. All other resonance lines diverged quickly and move to the high frequencies. It is connected with the re-reflection of the elastic waves on the boundary surfaces of the hollow cylinder.

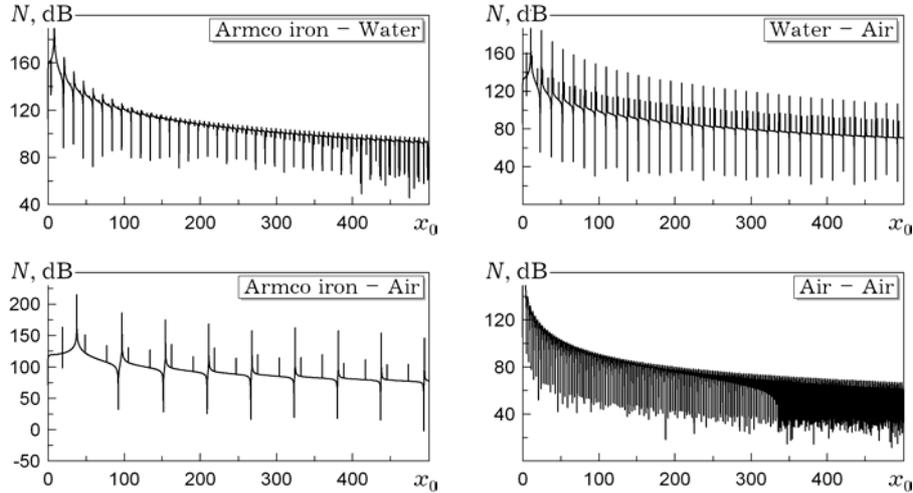


Fig. 7

In Fig. 7 the analogous curves for the intensity are shown for several cases of the solid cylinder-surrounded medium interface: Armco iron – water [10], water – air, Armco iron – air, and air – air. A last case simulates sound radiation from air vortex. The structure of the frequency dependencies of sound wave intensity radiated by the water and air hollow cylinders in air for the different geometrical parameter  $\varepsilon$  is demonstrated in Fig. 8. It is shown that sound generation by the air cylinders (in the frame of our model) is without resonances. In the other cases the resonances of the first and second harmonics form a quasi-periodical structure.

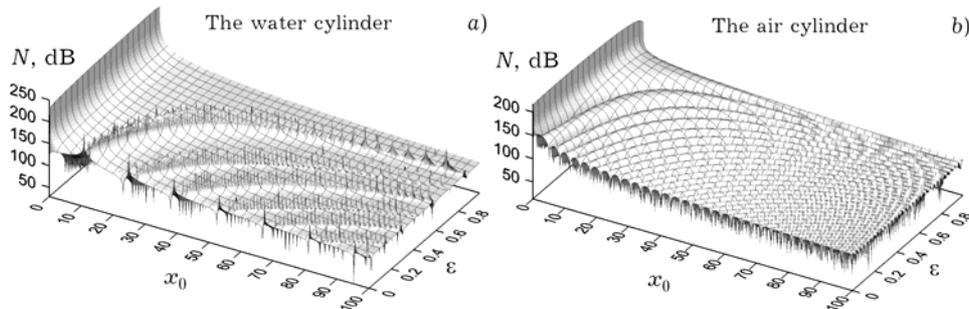


Fig. 8

Finally, in Fig. 9 the distributions of the sound wave intensity of the Armco iron cylinder in the air near the cylindrical surface,  $1 \leq r/a \leq 2$ , are shown when the elastic tube thickness is a continuous variable. The pictures are obtained for  $x_0 = 5, 10, 25, 50, 75, 100, 200$  and  $400$ . These plots are interesting because in this case both the primary resonances (with  $x_0$ ) and the secondary resonances (with  $2x_0$ ) are expressed explicitly. In other words, it is

demonstrated that continuous variation of the parameter  $\varepsilon$  causes to the consequence of the visible sound intensity splash for the arbitrary frequencies of the radiation.

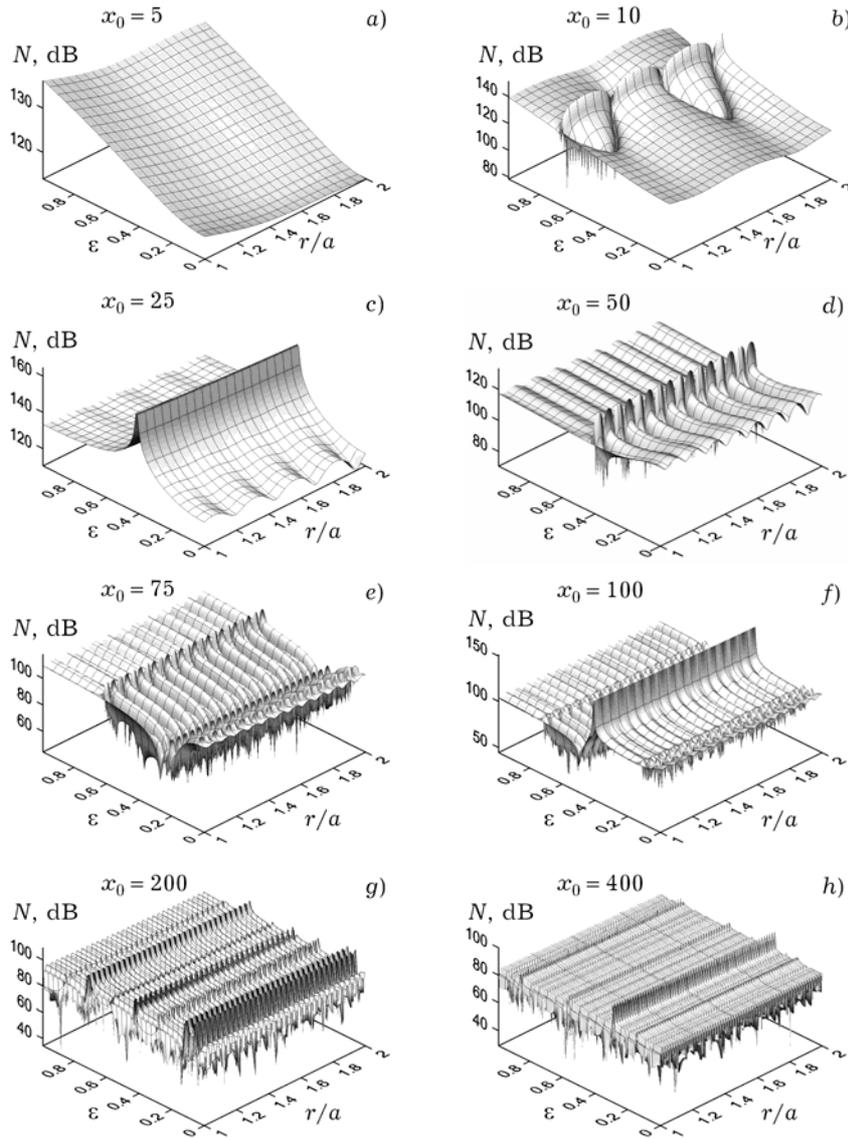


Fig. 9

**4. Conclusions.** The rotation of the hollow circular elastic cylinder with an inconstant angular velocity is a cause of the sound wave radiation in surrounded acoustical medium. More precisely, a source of the wave propagation in the cylinder and air is variable over time centrifugal force excited by the rotating elastic body. A result of the time modulation of this rotating motion is the complicated spectral structure of the generated sound waves with clearly expressed resonance character. The resonance properties are also reflected on the stationary excited sound signals. As a result of the fact that centrifugal force is proportional to the second power of the angular velocity, the generated sound contains the first and second oscillation harmonics.

The investigation of the thin structure of frequency characteristics is a necessary precondition for the sound radiation control and diagnostic of the work of rotating cylindrical elements of the machines. From other hand, the

rotating elastic cylinder can be considered as low powerful all-directional transducer of the acoustic signals in air on the basic and double frequencies.

The analysis of the numerical calculations shows that the major peculiarities of the sound wave structure are following:

- The amplitudes of the radiated acoustical pressure and wave intensity in the air posses the sequence of the resonances caused by the superposition of the outgoing and ingoing cylindrical waves in the elastic material of the rotated elastic tube.

- The resonances locations are connected with the phase velocities  $v_j^{\text{ph}} = cx/x_j^{\text{res}}(x)$ ,  $j = 1, 2, \dots$ , of the resonance wave propagation. The phase velocities as well as the group velocities  $v_j^{\text{gr}} = dv_j^{\text{ph}}/dx$ ,  $j = 1, 2, \dots$ , of these waves are subjected to the dispersion phenomenon caused by the varying cylinder thickness parameter.

- The resonance lines are of good quality and high intensity.

- The resonances corresponding to the solid cylinder case [5] are of the constructive type. For the hollow cylinder the resonances are divided into the constructive and destructive classes. This effect is visible very well for the thin elastic cylindrical shells.

- The first resonance is particular in the sense that its location in contrast to the positions of all rest the resonances, is low mobile with a change of the cylindrical wall thickness.

- The series of the low-level resonances presented in the sound wave intensity slightly masked on the background of the high-amplitude resonances are good discovered by the cylindrical tube thickness changing at the fixed frequencies of the angular velocities oscillation. However, this masking for the cylinder in the air is not so strong as for the cylinder in the water [10].

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## **ОСЕСИМЕТРИЧНЕ ВИПРОМІНЮВАННЯ ЗВУКУ ПРУЖНИМ ПОРОЖНИСТИМ ЦИЛІНДРОМ, ЩО ОБЕРТАЄТЬСЯ В ПОВІТРІ**

*Вивчається випромінювання звуку пружною круговою, пустою всередині циліндричною трубою безмежної довжини, що обертається з несталю кутвою швидкістю в повітрі. Точні розв'язки рівнянь, що описують аеропружну взаємодію середовищ, одержані з використанням інтегрального перетворення Фур'є за часом. Числові приклади показують, що спектральна структура звукового випромінювання від пружної труби, набагато складніша, ніж у випадку суцільного пружного циліндра. Зокрема, резонанси цієї структури суттєво залежать від товщини обертального об'єкту і підлягають явищу дисперсії.*

## **ОСЕСИМЕТРИЧНОЕ ИЗЛУЧЕНИЕ ЗВУКА УПРУГИМ ПОЛЫМ ЦИЛИНДРОМ, ВРАЩАЮЩИМСЯ В ВОЗДУХЕ**

*Изучается излучение звука упругой круговой, пустой изнутри цилиндрической трубой бесконечной длины, вращающейся с переменной угловой скоростью в воздухе. Точные решения уравнений, описывающих аэроупругое взаимодействие сред, получены с использованием интегрального преобразования Фурье по времени. Числовые примеры показывают, что спектральная структура звукового излучения от упругой трубы, намного сложнее, чем в случае сплошного упругого цилиндра. В частности, резонансы этой структуры существенно зависят от толщины вращающегося объекта и подвергаются явлению дисперсии.*

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