UDK 514.765+512.813.4

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COMPLETE INVARIANT KÄHLER METRICS ON COTANGENT BUNDLES OF SPHERES

For the spheres $S^n = SO(n+1)/SO(n)$, $n \ge 3$, all complete $SO(n+1)$-invariant Kähler metrics g with the canonical symplectic form as the Kähler form on the cotangent bundle T^*S^n are described. This description of the corresponding Kähler structure (J,g) (with the complex structure J) is based on the methods of symmetric Lie algebra theory. We consider also analogical complete Kähler structures (J,g) which are invariant with respect to the normalized geodesic flow on the punctured cotangent bundle $T^*S^n \setminus S^n$.

1. Introduction. Consider the sphere S^n with the standard $SO(n+1)$-invariant metric g_{S^n}. Using g_{S^n} we can identify the cotangent and tangent bundle to the sphere S^n thus obtaining the canonical symplectic structure Ω on TS^n. Therefore the metric g_{S^n} determines the geodesic flow on TS^n with the Hamiltonian function H.

In the study of $SO(n+1)$-invariant Kähler structures on TS^n, Kähler structures (J,g) with the form Ω as the Kähler form, i.e. $g(\cdot,\cdot) = \Omega(J\cdot,\cdot)$, are of particular interest because such structures arise in different problems of Riemannian and complex geometry of the (co)tangent bundles. For instance, the adapted complex structure (J_A, g_A) on TS^n [8] is the structure of this type naturally extending the Riemannian structure g_{S^n}, that is the restriction of g_A to the zero section $S^n \subset TS^n$ coincides with the homogeneous metric g_{S^n}. Moreover, (J_A, g_A) is a unique extension such that the function $\sqrt{H}$ is a solution of the homogeneous complex Monge – Ampere equation $(\partial\bar{\partial}\sqrt{H})^n = 0$ [2]. Another example of such a structure on the punctured (co)tangent bundle $TS^n \setminus S^n$ is the structure (J_S, g_S) [5, 7] which is invariant with respect of the Hamiltonian vector field of the function $\sqrt{H}$ (the normalized geodesic flow of the metric g_{S^n} on S^n).

In our paper [4] we presented a Lie algebraic method of a description of all $SO(n+1)$-invariant Kähler structures (J,g) on the domains D of the cotangent bundle T^*S^n of the n-dimensional sphere S^n with the canonical symplectic form Ω as the Kähler form. The question arises, for which structures (J,g) the metric g on D is complete. This problem is solved in the paper. Our method of the proof of the completeness is motivated by a paper of G. Patrizio and P. Wong [6], where there was proposed a proof method based on the existence of solutions of the homogeneous complex Monge – Ampere equation and associated the Monge – Ampere foliation. In our case, generally speaking, such a solution does not exist. But using the fact that the (co)tangent bundle TS^n is a cohomogeneity one manifold, that is there are $SO(n+1)$-orbits in TS^n which are hypersurfaces, we generalize the method of Patrizio and Wong and apply it for all invariant Kähler structures (J,g).

We find all Kähler structure (J, g) on the tangent bundle or the punctured tangent bundle of S^n with the complete metric g (Theorems 2 and 3). Moreover, we show that the Souriau − Rawnsley metric g_S is non-complete but there exist complete Kähler structures (J, g) on the punctured tangent bundle of S^n invariant with respect to the normalized geodesic flow of the metric g_{S^n}.

2. Invariant Kähler structures. In this subsection we shall give an exposition of the results of [4]. We will describe all invariant Kähler structures on the domains D of the cotangent bundle to sphere T^*S^n in terms of some operator-function P.

Let $M = G/K$ be a symmetric space with a compact connected Lie group G and a closed subgroup K. Let $\mathfrak{g}$ and $\mathfrak{k}$ be the Lie algebras of the groups G and K respectively. There is a positive definite $\mathrm{Ad}\, G$-invariant bilinear form $\langle\ ,\ \rangle$ on $\mathfrak{g}$ and a subspace $\mathfrak{m} \subset \mathfrak{g}$ such that $\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{m}$ is the $\mathrm{Ad}\, K$-invariant direct sum decomposition of $\mathfrak{g}$ and $\mathfrak{k} \perp \mathfrak{m}$. Moreover $[\mathfrak{k}, \mathfrak{m}] \subset \mathfrak{m}$ and $[\mathfrak{m}, \mathfrak{m}] \subset \mathfrak{k}$. The restriction $\langle\ ,\ \rangle \,|\, \mathfrak{m}$ determines G-invariant Riemannian metric g_{M} on $M = G/K$. The metric g_{M} identifies the cotangent bundle T^*M and the tangent bundle TM and thus we can also talk about the canonical 1-form θ on TM. The form θ and the symplectic form $\Omega \overset{\mathrm{def}}{=} d\theta$ are G-invariant with respect to the natural action of G on TM.

Consider the trivial vector bundle $G \times \mathfrak{m}$ with the two commuting Lie group actions on it: the left G-action, $l_h : (g, w) \mapsto (hg, w)$, and the right K-action $r_k : (g, w) \mapsto (gk, \mathrm{Ad}\, k^{-1}(w))$. Let $\pi : G \times \mathfrak{m} \to G \times_K \mathfrak{m}$ be the natural projection. It is well known that $G \times_K \mathfrak{m}$ and TM are isomorphic. Using the corresponding G-equivariant diffeomorphism $\varphi : G \times_K \mathfrak{m} \to TM$, $[(g, w)] \mapsto$ $\mapsto \left.\frac{d}{dt}\right|_0 g \exp(tw) K$ and the projection π we define the G-equivariant submersion $\Pi : G \times \mathfrak{m} \to TM$, $\Pi = \varphi \circ \pi$. Denote by $\tilde{\theta}$ the 1-form $\Pi^*\theta$ and by $\tilde{\Omega}$ its differential $d\tilde{\theta}$. Let ξ^ℓ be the left-invariant vector field on the Lie group G defined by a vector $\xi \in \mathfrak{g}$.

The 2-form $\tilde{\Omega}$ on the manifold $G \times \mathfrak{m}$ has the form [4]

$$\tilde{\Omega}_{(g,w)}\big((\xi_1^\ell(g), u_1), (\xi_2^\ell(g), u_2)\big) = \langle \xi_2, u_1 \rangle - \langle \xi_1, u_2 \rangle - \langle w, [\xi_1, \xi_2] \rangle, \tag{1}$$

where $g \in G$, $w \in \mathfrak{m}$, $\xi, \xi_1, \xi_2 \in \mathfrak{g}$, $u, u_1, u_2 \in \mathfrak{m} = T_w\mathfrak{m}$. The kernel $\mathcal{K} \subset$ $\subset T(G \times \mathfrak{m})$ of the 2-form $\tilde{\Omega}$ is generated by the global (left) G-invariant vector fields ζ^L, $\zeta \in \mathfrak{k}$ on $G \times \mathfrak{m}$, $\zeta^L(g, w) = (\zeta^\ell(g), [w, \zeta])$.

Let D be an open connected G-invariant subset of TM. Denote by W a unique $\mathrm{Ad}\, K$-invariant open subset of $\mathfrak{m}$ such that $\Pi^{-1}(D) = G \times W$. Let $\mathrm{Eqv}(W)$ be the set of all smooth K-equivariant mappings $A : W \to \mathrm{End}(\mathfrak{m}^{\mathbb{C}})$, $w \mapsto A_w$, i.e. for which

$$Ad_k \circ A_w \circ Ad_{k^{-1}} = A_{Ad_k w} \quad \text{on} \quad \mathfrak{m} \quad \text{for all} \quad w \in W, \quad k \in K. \tag{2}$$

Denote by $\mathrm{Alm}(W)$ the set of all maps $P \in \mathrm{Eqv}(W)$ such that the operator $P_w : \mathfrak{m}^{\mathbb{C}} \to \mathfrak{m}^{\mathbb{C}}$ and its real part $R_w = \mathrm{Re}\, P_w : \mathfrak{m} \to \mathfrak{m}$ are nondegenerate for each $w \in W$. Such a K-equivariant mapping $P \in \mathrm{Alm}(W)$ determines a complex (left) G-invariant subbundle $\mathcal{F}(P) \subset T^{\mathbb{C}}(G \times W)$ generated by (left) G-invariant vector fields ξ^L, $\xi \in \mathfrak{m}$, and ζ^L, $\zeta \in \mathfrak{k}$ on $G \times W$, where

$$\xi^L(g,w) = (\xi^{\ell}(g),\, iP_w(\xi)), \qquad \zeta^L(g,w) = (\zeta^{\ell}(g),\, [w,\zeta]).$$

The subbundle $\mathcal{F}(P)$ is (right) K-invariant by (2) and because the vector fields ζ^L, $\zeta \in \mathfrak{k}$, span the (right) K-invariant subbundle (kernel) $\mathcal{K}$. Therefore $F(P) \overset{\mathrm{def}}{=} \Pi_*(\mathcal{F}(P))$ is a well-defined (smooth) complex subbundle of $T^{\mathbb{C}}D$ ($\mathcal{K}^{\mathbb{C}} \subset \mathcal{F}(P)$). Moreover, since $\mathcal{F}(P) + \overline{\mathcal{F}(P)} = T^{\mathbb{C}}(G \times W)$ and $\mathcal{F}(P) \cap \overline{\mathcal{F}(P)} = \mathcal{K}$ we have $F(P) + \overline{F(P)} = T^{\mathbb{C}}D$ and $F(P) \cap \overline{F(P)} = 0$. In other words, the mapping P determines an almost-complex structure J_P on $D \subset TM$ with $F(P)$ as the subbundle of its $(0,1)$-vectors: $\Gamma F(P) = \{X + iJ_P X, X \in \Gamma(TD)\}$.

Let g_P denote a $(2,0)$-tensor on TD, where $g_P(Y_1, Y_2) \overset{\mathrm{def}}{=} \Omega(J_P Y_1, Y_2)$ for vector fields Y_1, Y_2 on TD. The almost-complex structure J_P is an almost Kähler structure with the Kähler form Ω if $(\Omega(F(P), F(P)) = 0$ and g_P is a Riemannian metric on TD. Such a structure is a Kähler structure if the subbundle $F(P) \subset T^{\mathbb{C}}(TD)$ is involutive, i.e. is closed under the Lie bracket $[F(P), F(P)] \subset F(P)$. Since $g_P(\cdot,\cdot) = \Omega(J_P \cdot, \cdot)$ this structure will be denoted by the pair (J_P, g_P) or, (J_P, Ω).

Let G be the Lie group $SO(n+1)$ and K be its subgroup isomorphic to $SO(n)$. Let $\langle\ ,\ \rangle$ be the normalized trace form $-\frac{1}{2}\mathrm{Tr}$ on $\mathfrak{g} = so(n+1)$ associated with the standard representation of $so(n+1)$. Put $r(w) = |w| = \sqrt{\langle w, w\rangle}$. Denote by O some open connected subset of $[0, +\infty)$ such that $O = r^{-1}(W)$.

Now we can describe all $SO(n+1)$-invariant Kähler structures on $D \subset TM$ with the Kähler form Ω. To this end consider the operator-function $P : W \to \mathrm{End}(\mathfrak{m}^{\mathbb{C}})$:

$$P_w(\xi) = \psi(|w|)\xi + \frac{\lambda(|w|) - \psi(|w|)}{r^2}\langle w, \xi\rangle w, \qquad \xi \in \mathfrak{m}, \qquad w \in W, \quad (3)$$

where $\psi,\ \lambda : O \to \mathbb{C}$ are smooth functions. Remark that the one-dimensional complex vector space $\langle w\rangle$ and its orthogonal complement $\langle w\rangle^{\perp}$ in $\mathfrak{m}^{\mathbb{C}}$ are the eigensubspaces of P_w with eigenvalues $\lambda(|w|)$ and $\psi(|w|)$ respectively.

Theorem 1 [**4**]. *Let $G = SO(n+1)$ and let K be its subgroup isomorphic to $SO(n)$. Let (J, Ω) be a G-invariant Kähler structure on the domain $D \subset T(G/K)$. If $n \geq 3$ then $J = J_P$, where the smooth mapping $P : W \to \mathrm{End}(m^{\mathbb{C}})$ has form* (3). *If $n \geq 2$ then the smooth mapping P* (3) *defines a Kähler structure on D if and only if real parts of the functions $\lambda(|w|), \psi(|w|) : W \to \mathbb{C}$ are positive and $\psi(r)$ is given by formula*

$$\psi(r) = r\frac{\cosh(\alpha(r))}{\sinh(\alpha(r))}, \qquad \text{where} \qquad \frac{d\alpha(r)}{dt} = \frac{1}{\lambda(r)}, \qquad \text{or} \qquad \psi(r) = r. \quad (4)$$

3. Complete Kähler metrics g_P on TS^n and T^0S^n. We continue with the previous notation but throughout this subsection it is assumed that $G = SO(n+1)$ and $K = SO(n)$.

To find all complete metrics g_P on $D \subset TS^n$ note that the tangent space TS^n is a cohomogeneity one manifold, i.e. the Lie group $SO(n+1)$ acts on this manifold with a codimension one orbit. The zero section $S^n \subset TS^n$ is a singular orbit. With the exception of this one case, all $SO(n+1)$-orbits in TS^n are hypersurfaces (principal orbits). The geometry of cohomogeneity one manifolds is now well understood. But we will use only one fundamental fact on the structure of these manifold [1]:

> the unit smooth vector field U on TS^n which is g_P-orthogonal to each $SO(n+1)$-orbit on TS^n is a geodesic vector field, i.e. its integral curves are geodesics of the metric g_P.

We describe the vector field U in terms of some Hamiltonian vector field. To this end consider the Hamiltonian function $H : TS^n \to \mathbb{R}$ associated with the invariant metric g_{S^n} on the sphere S^n (induced by the form $\langle\, ,\, \rangle$ on $\mathfrak{m}$). Each G-orbit in TS^n (principal or singular) is a level surface of H. We claim that the vector field $J_P X_H$ on $D \subset TS^n$, where X_H is the Hamiltonian vector field of H, is g_P-orthogonal to each G-orbit in D. Indeed, since the function H is G-invariant $dH(Y) = 0$ for each vector field Y tangent to G-orbits in D. Then by definition of a Hamiltonian vector field we have

$$0 = dH(Y) = \Omega(-X_H, Y) = g_P(J_P X_H, Y).$$

Since the operator $J_P(x)$ is orthogonal with respect to the form $g_P(x)$, we have $U = J_P X_H / \| X_H \|$, where

$$\| J_P X_H \|^2 = \| X_H \|^2 \overset{\text{def}}{=} g_P(X_H, X_H) = \Omega(J_P X_H, X_H).$$

Moreover,

$$dH(U) = \Omega(-X_H, U) = g_P(J_P X_H, J_P X_H) / \| X_H \| = \| X_H \|, \tag{5}$$

and for any vector field Z on D

$$dH(Z) = \Omega(-X_H, Z) = g_P(J_P X_H, Z) \le \| X_H \| \cdot \| Z \|.$$

The function $\| X_H \|$ on D is G-invariant so that $\| X_H \| = p(H)$ (is a function of H) for some smooth positive function p.

Using the vector field $J_P X_H$ and the method of [6] we shall calculate the distance between level sets $\{H = a\}$ and $\{H = b\}$ in D with respect to the metric induced by g_P. Let $\gamma = \gamma(t)$, $t \in [0, T]$, be the integral curve of the vector field $U = J_P X_H / \| X_H \|$ with the initial point $x_a \in \{H = a\}$. There exists a function h on some subset of $\mathbb{R}$ such that the function $h(H(\gamma(t)))$ is linear in t. It is easy to verify that

$$h(s) = \int_a^s \frac{1}{p(\tau)}\, d\tau, \tag{6}$$

because by (5)

$$\frac{d}{dt}h(H(\gamma(t))) = h'(H(\gamma(t)))dH(\gamma'(t)) = h'(H(\gamma(t)))\| X_H(\gamma(t))\| = 1\,.$$

Suppose that $x_b \in \{H = b\}$ and $x_b = \gamma(t_b)$. But the curve γ is a geodesic. Therefore the length of the curve γ, $t \in [0, t_b]$, from x_a to x_b is

$$l_\gamma = \int_0^{t_b} \| \gamma'(t) \| dt = t_b = h(H(x_b)) - h(H(x_a)) = h(b) - h(a)\,.$$

For any other curve $\gamma_0(t)$, with $\| \gamma_0' \| = 1$, starting from the point x_a and ended at a point $x_b^0 \in \{H = b\}$ we have

$$\frac{d}{dt}h(H(\gamma_0(t))) = h'(H(\gamma_0(t)))\,dH(\gamma_0'(t)) \le$$

$$\le h'(H(\gamma_0(t)))\| \gamma_0' \| \cdot \| X_H(\gamma_0(t)) \| = 1\,.$$

Thus $h(H(\gamma_0(t)) - h(H(x_a)) \le t$ and the length of the curve γ_0 from x_a to x_b^0 is larger than the length of the curve γ:

$$l_{\gamma_0} = \int_0^{t_b^0} \| \gamma_0'(t) \| dt = t_b^0 \ge h(H(\gamma_0(t_b^0)) - h(H(x_a)) =$$

$$= h(H(x_b^0)) - h(H(x_a)) = h(b) - h(a) = l_\gamma\,.$$

Therefore the distance between the level surfaces $\{H = a\}$ and $\{H = b\}$, $a, b \in \mathbb{R}^+$ is $|h(b) - h(a)|$, where the function p is given by (6). Since these level surfaces are compact sets, the metric g_P on D is complete if and only if the metric on the orbit space D/G induced by the function h is complete too.

Let us calculate the function h for the Kähler metric g_P defined by the operator-function P (3). To this end we consider on $G \times \mathfrak{m}$ the function $\tilde{H}$ and the vector field $\tilde{X}_H$ putting

$$\tilde{H}(g, w) = \langle w, w \rangle \qquad \text{and} \qquad \tilde{X}_H(g, w) = 2(w^\ell(g), 0)\,. \tag{7}$$

It is immediate that $\tilde{H} = \Pi^* H$. Moreover, $\Pi_* \tilde{X}_H = X_H$ (see [3], section 2.4). Using the fact that the complex structure J_P is defined uniquely by the complex subbundle $\mathcal{F}(P) \subset T^{\mathbb{C}}(G \times W)$, we obtain that $J_P X_H = \Pi_*(J_P \tilde{X}_H)$, where

$$J_P \tilde{X}_H(g,\, w) = 2((-R_w^{-1} S_w w)^\ell(g),\, (R_w + S_w R_w^{-1} S_w)w) =$$

$$= 2\left(-\frac{\nu(\| w \|)}{\mu(\| w \|)}w,\, \frac{\mu^2(\| w \|) + \nu^2(\| w \|)}{\mu(\| w \|)}w\right).$$

Here the operator R_w (resp. S_w) is the real (resp. imaginary) part of P_w; similarly, $\lambda = \mu + i\nu$. By (1) and (7)

$$\Pi^* \| X_H \|^2 = \Pi^*(\Omega(J_P X_H, X_H)) = \tilde{\Omega}(J_P \tilde{X}_H, \tilde{X}_H) =$$

$$= 4\,\frac{\mu^2(|w|) + \nu^2(|w|)}{\mu(|w|)}\langle\, w, w \,\rangle.$$

Thus

$$p^2(H) = 4\frac{|\lambda(\sqrt{H})|^2}{\operatorname{Re}\lambda(\sqrt{H})}H. \tag{8}$$

Suppose that the domain D is the whole space TS^n. Then the operator-function P (3) is smooth on the set $W = \mathfrak{m}$ if and only if the complex-valued functions ψ and $\frac{\lambda - \psi}{r^2}$ on the real line $\mathbb{R}$ are even smooth functions. Thus λ is an even function on $\mathbb{R}$ and the function ψ is uniquely defined by λ:

$$\psi(r) = r\frac{\cosh(\alpha(r))}{\sinh(\alpha(r))}, \tag{9}$$

where

$$\alpha(r) = \int\frac{dr}{\lambda(r)}, \qquad \alpha(0) = 0.$$

We proved (compare (6) and (8))

Theorem 2. *The Kähler metric g_P on the tangent bundle TS^n defined by the operator-function $P : \mathfrak{m} \to \operatorname{End}(\mathfrak{m}^{\mathbb{C}})$ (3) is complete if and only if the function λ is an even smooth complex-valued function on $\mathbb{R}$ with the positive real part, the function ψ is given by (9) and the function*

$$h(s) = \int_0^{\sqrt{s}}\frac{\sqrt{\operatorname{Re}\lambda(r)}}{|\lambda(r)|}dr$$

is unbounded as $s \to \infty$.

Suppose now that the domain D is a punctured tangent bundle $T^0S^n = TS^n \setminus S^n$. Then the operator-function P (3) is smooth on the set $W = \mathfrak{m}^0 = \mathfrak{m} \setminus \{0\}$ if and only if the complex-valued function λ on the set $(0, +\infty) \subset \mathbb{R}$ is smooth. Using similar arguments as above, we obtain

Theorem 3. *The Kähler metric g_P on the punctured tangent bundle T^0S^n defined by the operator-function $P : \mathfrak{m} \to \operatorname{End}(\mathfrak{m}^{\mathbb{C}})$ (3) is complete if and only if*

(***a***) *the function λ is a smooth function on $(0, +\infty) \subset \mathbb{R}$ with the positive real part;*

(***b***) *the function ψ is given by* (4) *and has positive real part*;

(***c***) *the function*

$$h(s) = \int_1^{\sqrt{s}}\frac{\sqrt{\operatorname{Re}\lambda(r)}\,dr}{|\lambda(r)|}$$

is unbounded as $s \to \infty$ and $s \to 0$.

Remark. The Kähler metric g_P on the punctured tangent bundle T^0S^n is invariant with respect to the Hamiltonian vector field $X_{\sqrt{H}}$ of the function $\sqrt{H}$ (the normalized geodesic flow of the metric g_{S^n} on S^n) if and only if $\psi(r) = r$ (see [3, theorem 12] and [4, subsection 4.3]). Thus the metric g_P with $\psi(r) = r$ and $\lambda(r) = 1$, constructed by Rawnsley [5] is noncomplete. But by Theorem 3 there exist complete metrics g_P on T^0S^n which are invariant with respect to the normalized geodesic flow of the metric g_{S^n}.

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ПОВНІ ІНВАРІАНТНІ КЕЛЕРОВІ МЕТРИКИ НА КОДОТИЧНИХ РОЗШАРУВАННЯХ СФЕР

Для сфер $S^n = SO(n+1)/SO(n)$, $n \ge 3$, *описані всі повні* $SO(n+1)$*-інваріантні келерові метрики* g *з канонічною симплектичною формою як келеровою формою на кодотичних розшаруваннях* T^*S^n. *Цей опис відповідних келерових структур* (J,g) *(з комплексною структурою* J*) базується на методах теорії симетричних алгебр Лі. Розглянуто також аналогічні повні келерові структури* (J,g), *інваріантні відносно нормалізованого геодезичного потоку на проколотому кодотичному розшаруванні* $T^*S^n \setminus S^n$.

ПОЛНЫЕ ИНВАРИАНТНЫЕ КЭЛЕРОВЫ МЕТРИКИ НА КОКАСАТЕЛЬНЫХ РАССЛОЕНИЯХ СФЕР

Для сфер $S^n = SO(n+1)/SO(n)$, $n \ge 3$, *описаны все полные* $SO(n+1)$*-инвариантные кэлеровы метрики* g *с канонической симплектической формой в качестве кэлеровой формы на кокасательных расслоениях* T^*S^n. *Это описание соответствующих кэлеровых структур* (J,g) *(с комплексной структурой* J*) базируется на методах теории симметрических алгебр Ли. Рассмотрены также аналогичные полные кэлеровы структуры* (J,g), *инвариантные относительно нормализованного геодезического потока на проколотом кокасательном расслоении* $T^*S^n \setminus S^n$.

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Received
20.06.07