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SYMMETRIC INVERSE TOPOLOGICAL SEMIGROUPS OF FINITE RANK $\le n$

We establish topological properties of the symmetric inverse topological semigroup of finite transformations $\mathscr{I}_\lambda^n$ *of the rank* $\le n$. *We show that the topological inverse semigroup* $\mathscr{I}_\lambda^n$ *is algebraically* h*-closed in the class of topological inverse semigroups. Also we prove that a topological semigroup* S *with countably compact square* $S \times S$ *does not contain the semigroup* $\mathscr{I}_\lambda^n$ *for infinite cardinal* λ *and show that the Bohr compactification of an infinite topological symmetric inverse semigroup of finite transformations* $\mathscr{I}_\lambda^n$ *of the rank* $\le n$ *is the trivial semigroup.*

In this paper all topological spaces will be assumed to be Hausdorff. We shall follow the terminology of [8, 9, 13, 30]. If A is a subset of a topological space X, then we denote the closure of the set A in X by $\mathrm{cl}_X(A)$. By ω we denote the first infinite cardinal.

A semigroup S is called an *inverse semigroup* if every a in S possesses an unique inverse, i.e. if there exists an unique element a^{-1} in S such that

$$aa^{-1}a = a \qquad \text{and} \qquad a^{-1}aa^{-1} = a^{-1}.$$

A map which associates to any element of an inverse semigroup its inverse is called the *inversion*.

A *topological* (*inverse*) *semigroup* is a topological space together with a continuous semigroup operation (and an inversion, respectively). Obviously, the inversion defined on a topological inverse semigroup is a homeomorphism. If S is a semigroup (an inverse semigroup) and τ is a topology on S such that (S,τ) is a topological (inverse) semigroup, then we shall call τ a *semigroup* (*inverse*) *topology* on S.

If S is a semigroup, then by $E(S)$ we denote the band (the subset of all idempotents) of S. On the set of idempotents $E(S)$ there exists a natural partial order: $e \le f$ *if and only if* $ef = fe = e$.

Let X be a set of cardinality $\lambda \ge 1$. Without loss of generality we can identify the set X with the cardinal λ. A function α mapping a subset Y of X into X is called a *partial transformation* of X. In this case the set Y is called the *domain* of α and is denoted by $\mathrm{dom}\,\alpha$. Also, the set $\{x \in X \mid y$ for some $\alpha = xy \in Y\}$ is called the *range* of α and is denoted by $\mathrm{ran}\,\alpha$. The cardinality of $\mathrm{ran}\,\alpha$ is called the *rank* of α and denoted by $\mathrm{rank}\,\alpha$. For convenience we denote by $\varnothing$ the empty transformation, that is a partial mapping with $\mathrm{dom}\,\varnothing = \mathrm{ran}\,\varnothing = \varnothing$.

Let $\mathscr{I}(X)$ denote the set of all partial one-to-one transformations of X together with the following semigroup operation:

$$x(\alpha\beta) = (x\alpha)\beta \quad \text{if} \quad x \in \mathrm{dom}\,(\alpha\beta) = \{y \in \mathrm{dom}\,\alpha \mid y\alpha \in \mathrm{dom}\,\beta\} \quad \text{for} \quad \alpha,\beta \in \mathscr{I}(X).$$

The semigroup $\mathscr{I}(X)$ is called the *symmetric inverse semigroup* over the set X (see [9]). The symmetric inverse semigroup was introduced in [35] by V. V. Wagner and it plays a major role in the theory of semigroups.

Put

$$\mathscr{I}_\lambda^\infty = \{\alpha \in \mathscr{I}(X) \mid \mathrm{rank}\,\alpha \text{ is finite }\},$$

$$\mathscr{I}_\lambda^n = \{\alpha \in \mathscr{I}(X) \mid \mathrm{rank}\,\alpha \le n\} \qquad \text{for} \qquad n = 1,2,3,\ldots .$$

Obviously, $\mathcal{I}_\lambda^\infty$ and $\mathcal{I}_\lambda^n$, $n = 1,2,3,\dots$, are inverse semigroups, $\mathcal{I}_\lambda^\infty$ is an ideal of $\mathcal{I}(X)$, and $\mathcal{I}_\lambda^n$ is an ideal of $\mathcal{I}_\lambda^\infty$, for each $n = 1,2,3,\dots$. Further, we shall call the semigroup $\mathcal{I}_\lambda^\infty$ the *symmetric inverse semigroup of finite transformations* and $\mathcal{I}_\lambda^n$ the *symmetric inverse semigroup of finite transformations of the rank* $\le n$. The elements of semigroups $\mathcal{I}_\lambda^\infty$ and $\mathcal{I}_\lambda^n$ are called *finite one-to-one transformations* (*partial bijections*) of the set X. By

$$\begin{pmatrix} x_1 & x_2 & \dots & x_n \\ y_1 & y_2 & \dots & y_n \end{pmatrix}$$

we denote a partial one-to-one transformation which maps x_1 onto y_1, x_2 onto y_2, etc., and x_n onto y_n, and by 0 the empty transformation. Obviously, in such case we have $x_i \ne x_j$ and $y_i \ne y_j$ for $i \ne j$, $i,j = 1,2,3,\dots,n$.

Many topologists have studied topological properties of topological spaces of partial continuous maps $\mathcal{PC}(X)$ from a topological space X into a topological space Y with various topologies such as the Vietoris topology, generalized compact-open topology, graph topology, τ-topology, and others (see [1, 7, 11, 14, 22–25]). Since the set of all partial continuous self-transformations $\mathcal{PCT}(X)$ of the space X with the operation composition is a semigroup, many semigroup theorists have considered the semigroup of continuous transformations (see surveys [26] and [17]), or the semigroup of partial homeomorphisms of an arbitrary topological space (see [2–5, 16, 27, 31, 36]). A. A. Beĭda [6], S. D. Orlov [28, 29], and S. Subbiah [34] have considered semigroup and inverse semigroup topologies of semigroups of partial homeomorphisms of some classes of topological spaces. In this context the results of our paper yield some notable results about the topological behavior of the finite rank symmetric inverse semigroups sitting inside larger function space semigroups, or larger semigroups in general. For example, under reasonably general conditions, the inverse semigroup of partial finite bijections of rank ≤ 6 is a closed subsemigroup of a topological semigroup which contains $\mathcal{I}_\lambda^n$ as a subsemigroup.

Let λ be a non-empty cardinal. On the set $B_\lambda = \lambda \times \lambda \cup \{0\}$, where $0 \notin \lambda \times \lambda$, we define the semigroup operation « $\cdot$ » as follows

$$(a,b)\cdot(c,d) = \begin{cases} (a,d), & b = c, \\ 0, & b \ne c, \end{cases}$$

and $(a,b)\cdot 0 = 0\cdot(a,b) = 0\cdot 0 = 0$ for $a,b,c,d \in \lambda$. The semigroup B_λ is called the *semigroup of* $\lambda\times\lambda$*-matrix units* (see [9]). Obviously, for any cardinal $\lambda > 0$, the semigroup of $\lambda\times\lambda$-matrix units B_λ is isomorphic to $\mathcal{I}_\lambda^1$.

Definition 1 [**18, 32**]. Let $\mathfrak{S}$ be a class of topological semigroups. A topological semigroup $S \in \mathfrak{S}$ is called H*-closed in the class* $\mathfrak{S}$ if S is a closed subsemigroup of any topological semigroup $T \in \mathfrak{S}$ which contains S as a subsemigroup. If $\mathfrak{S}$ coincides with the class of all topological semigroups, then the semigroup S is called H*-closed*.

We remark that in [32] H-closed semigroups are called *maximal*.

Definition 2 [**21, 33**]. Let $\mathfrak{S}$ be a class of topological semigroups. A topological semigroup $S \in \mathfrak{S}$ is called absolutely H*-closed in the class* $\mathfrak{S}$ if any continuous homomorphic image of S into $T \in \mathfrak{S}$ is H-closed in $\mathfrak{S}$. If $\mathfrak{S}$ coincides with the class of all topological semigroups, then the semigroup S is called *absolutely* H*-closed*.

Definition 3 [**21, 33**]. Let $\mathfrak{S}$ be a class of topological semigroups. A semigroup S is called *algebraically* h*-closed in* $\mathfrak{S}$ if S with the discrete topology $\mathfrak{d}$ is absolutely H-closed in $\mathfrak{S}$ and $(S,\mathfrak{d})\in\mathfrak{S}$. If $\mathfrak{S}$ coincides with the class of all topological semigroups, then the semigroup S is called *algebraically* h*-closed*.

Absolutely H-closed semigroups and algebraically h-closed semigroups were introduced in [33]. There they were called *absolutely maximal* and *algebraic maximal*, respectively.

In [19] are established topological properties of infinite topological semigroups of $\lambda\times\lambda$-matrix units B_λ. They showed that an infinite topological semigroup of $\lambda\times\lambda$-matrix units B_λ does not embed into a compact topological semigroup, every non-zero element of B_λ is an isolated point of B_λ, and B_λ is algebraically h-closed in the class of topological inverse semigroups.

In [20] is introduced the conception of semigroups with a tight ideal series and there they investigated their closure in semitopological semigroups, partially inverse semigroups with continuous inversion. Also they derived related results about the nonexistence of (partial) compactifications of topological semigroups with a tight ideal series. As a corollary they show that the symmetric inverse semigroup of finite transformations $\mathcal{I}_\lambda^n$ of the rank $\le n$ is algebraically closed in the class of inverse (semi)topological semigroups with continuous inversion. Since semigroups with a tight ideal series are not preserved by homomorphisms [20, Lemma 19], naturally arises the following question: *is the symmetric inverse semigroup of finite transformations* $\mathcal{I}_\lambda^n$ *of the rank* $\le n$ *is algebraically* h*-closed in the class of topological inverse semigroups?*

In this paper we shall show that for every infinite cardinal λ the finite symmetric inverse semigroup $\mathcal{I}_\lambda^n$ of the rank $\le n$ has topological properties similar to the infinite semigroup of matrix units B_λ as a topological semigroup. We show that the topological inverse semigroup $\mathcal{I}_\lambda^n$ is algebraically h-closed in the class of topological inverse semigroups. Also we prove that a topological semigroup S with countably compact square $S\times S$ does not contain the semigroup $\mathcal{I}_\lambda^n$ for infinite cardinal λ and show that the Bohr compactification of an infinite topological symmetric inverse semigroup of finite transformations $\mathcal{I}_\lambda^n$ of the rank $\le n$ is the trivial semigroup.

Theorem 1. *For any positive integer* n *the semigroup* $\mathcal{I}_\lambda^n$ *is algebraically* h*-closed in the class of topological inverse semigroups.*

P r o o f. In the case $\lambda<\omega$ the assertion of the theorem is obvious. Suppose now that $\lambda\ge\omega$. We shall prove the assertion of the theorem by induction.

Theorem 14 from [19] implies that the semigroup $\mathcal{I}_\lambda^1$ is algebraically h-closed in the class of all topological inverse semigroups. We suppose that the assertion of the theorem holds for $n=1,2,\ldots,k-1$ and we shall prove that it is true for $n=k$.

Suppose to the contrary, that there exist a topological inverse semigroup S and continuous homomorphisms h from the semigroup $\mathcal{I}_\lambda^k$ with the discrete topology into S such that $(\mathcal{I}_\lambda^k)h$ is a non-closed subsemigroup of S. Since a homomorphic image of an inverse semigroup is an inverse semigroup, Proposition II.2 of [12] implies that $\mathrm{cl}_S((\mathcal{I}_\lambda^k)h)$ is a topological inverse semigroup. Therefore, without loss of generality we can assume that $(\mathcal{I}_\lambda^k)h$ is a dense inverse subsemigroup of S.

Let $x \in S \setminus (\mathcal{I}_\lambda^k)h$ and $W(x)$ be an open neighbourhood of the point x. Since the semigroup $\mathcal{I}_\lambda^{k-1}$ is algebraically h-closed in the class of topological inverse semigroups, without loss of generality we can assume that $W(x) \cap (\mathcal{I}_\lambda^{k-1})h = \varnothing$.

Suppose that x is an idempotent of S. Then there exists an open neighbourhood $V(x) \subseteq W(x)$ such that $V(x) \cdot V(x) \subseteq W(x)$. Then since the neighbourhood $V(x)$ contains infinitely points from $(\mathcal{I}_\lambda^k)h \setminus (\mathcal{I}_\lambda^{k-1})h$ we have that $(V(x) \cdot V(x)) \cap (\mathcal{I}_\lambda^{k-1})h \neq \varnothing$. A contradiction to the assumption $W(x) \cap (\mathcal{I}_\lambda^{k-1})h = \varnothing$. There fore we have $x \cdot x \neq x$.

Since $\mathcal{I}_\lambda^{k-1}$ is an inverse subsemigroup of $\mathcal{I}_\lambda^k$ Proposition II.2 [12] implies that $x^{-1} \in S \setminus (\mathcal{I}_\lambda^k)h$. Since S is a topological inverse semigroup and the semigroup $\mathcal{I}_\lambda^{k-1}$ is algebraically h-closed in the class of topological inverse semigroups, there exist open neighbourhoods $V(x)$ and $V(x^{-1})$ of the points x and x^{-1}, respectively, such that

$$V(x) \cdot V(x^{-1}) \cdot V(x) \subseteq W(x), \qquad V(x) \cap (\mathcal{I}_\lambda^{k-1})h = \varnothing,$$

$$V(x^{-1}) \cap (\mathcal{I}_\lambda^{k-1})h = \varnothing \qquad \text{and} \qquad V(x) \subseteq W(x).$$

We observe that the set $V(x) \cap (\mathcal{I}_\lambda^k)h$ is infinite, otherwise we have that $x \notin \mathrm{cl}_S((\mathcal{I}_\lambda^k)h)$. Since S is a topological inverse semigroup, the set $V(x^{-1}) \cap \cap (\mathcal{I}_\lambda^k)h$ is infinite too. Let $V = (V(x) \cap (\mathcal{I}_\lambda^k)h)h^{-1}$ and $V^* = (V(x^{-1}) \cap (\mathcal{I}_\lambda^k)h)h^{-1}$. Then the sets V and V^* are infinite, and we have $V \cap \mathcal{I}_\lambda^{k-1} = \varnothing$ and $V^* \cap \cap \mathcal{I}_\lambda^{k-1} = \varnothing$. Therefore $V \cdot V^* \cdot V \cap \mathcal{I}_\lambda^{k-1} \neq \varnothing$ and hence $((V)h \cdot (V^*)h \cdot (V)h) \cap \cap (\mathcal{I}_\lambda^{k-1})h \neq \varnothing$. But

$$((V)h \cdot (V^*)h \cdot (V)h) \subseteq V(x) \cdot V(x^{-1}) \cdot V(x) \subseteq W(x)$$

is a contradiction to the assumption $W(x) \cap (\mathcal{I}_\lambda^{k-1})h = \varnothing$. The obtained contradiction implies the assertion of the theorem. ◊

Theorem 1 implies

Corollary 1. *Let n be any positive integer and let τ be any inverse semigroup topology on $\mathcal{I}_\lambda^n$. Then $(\mathcal{I}_\lambda^n, \tau)$ is an absolutely H-closed topological inverse semigroup in the class of topological inverse semigroups.*

The following theorem generalizes Theorem 10 from [19].

Theorem 2. *A topological semigroup S with countably compact square $S \times S$ does not contain an infinite countable semigroup of matrix units.*

P r o o f. Suppose to the contrary: there exists a topological semigroup S with countably compact square $S \times S$ such that S contains an infinite countable semigroup of $\omega \times \omega$-matrix units B_ω. We numerate elements of a set X of cardinality ω by non-negative integers, i. e., $X = \{\alpha_0, \alpha_1, \alpha_2, \ldots\}$. Then we consider the sequence $\{((\alpha_0, \alpha_n), (\alpha_n, \alpha_0))_{n=1}^{\infty}\}$ in $B_\omega \times B_\omega \subset S \times S$. The countable compactness of $S \times S$ guarantees that this sequence has an accumulation point $(a, b) \in S \times S$. Since $(\alpha_0, \alpha_n) \cdot (\alpha_n, \alpha_0) = (\alpha_0, \alpha_0)$, the continuity of the semigroup operation on S guarantees that $ab = (\alpha_0, \alpha_0)$. By Lemma 4 [19], every non-zero element of the semigroup of $\omega \times \omega$-matrix units B_ω endowed with the topology induced from S is an isolated point in B_ω. So, there exists a neighbourhood $O((\alpha_0, \alpha_0)) \subseteq S$ of the point $(\alpha_0, \alpha_0) \in B_\omega$ containing no other points of the semigroup B_ω. Since $ab = (\alpha_0, \alpha_0)$, the points a, b have neighbourhoods $O(a), O(b) \subset S$ such that $O(a) \cdot O(b) \subset O((\alpha_0, \alpha_0))$. Since a is an accumulation point of the sequence (α_0, α_n), there exists a positive integer n such that $(\alpha_0, \alpha_n) \in O(a)$. Similarly there exists a positive integer $m > n$ such that $(\alpha_m, \alpha_0) \in O(b)$. Then $(\alpha_0, \alpha_n) \cdot (\alpha_m, \alpha_0) = 0 \in O(a) \cdot O(b) \bigcap B_\omega = (\alpha_0, \alpha_0)$, which is a contradiction. ◊

Since every infinite semigroup of $\lambda \times \lambda$-matrix units B_λ contains the semigroup B_ω, Theorem 2 implies

Theorem 3. *A topological semigroup S with countably compact square $S \times S$ does not contain an infinite semigroup of matrix units.*

Theorem 2 implies

Corollary 2 [**19, Theorem 10**]. *An infinite semigroup of matrix units does not embed into a compact topological semigroup.*

A semigroup homomorphism $h : S \to T$ is called *annihilating* if $(s)h = (t)h$ for all $s, t \in S$.

A semigroup S is called *congruence-free* if it has only two congruences: identical and universal [9]. Obviously, a semigroup S is congruence-free if and only if every homomorphism h of S into an arbitrary semigroup T is an isomorphism «into» or is an annihilating homomorphism.

Theorem 1 from [16] implies that the semigroup B_λ is congruence-free for every cardinal $\lambda \geq 2$ and hence Theorem 2 implies

Theorem 4. *Every continuous homomorphism from an infinite topological semigroup of matrix units into a topological semigroup S with countably compact square $S \times S$ is annihilating.*

Theorem 4 implies

Corollary 3 [**19, Theorem 12**]. *Every continuous homomorphism from an infinite topological semigroup of matrix units into a compact topological semigroup is annihilating.*

Theorem 5. *Let $\lambda \geq \omega$ and n be a positive integer. Then every continuous homomorphism of the topological semigroup $\mathscr{I}_\lambda^n$ into a topological semigroup S with countably compact square $S \times S$ is annihilating.*

P r o o f. We shall prove the assertion of the theorem by induction. By Theorem 4 every continuous homomorphism of the topological semigroup $\mathscr{I}_\lambda^1$ into a topological semigroup S with countably compact square $S \times S$ is annihilating. We suppose that the assertion of the theorem holds for $n = 1, 2, \ldots, k-1$ and we shall prove that it is true for $n = k$.

Obviously it is sufficiently to show that the statement of the theorem holds for the discrete semigroup $\mathscr{I}_\lambda^k$. Let $h : \mathscr{I}_\lambda^k \to S$ be arbitrary homomorphism from $\mathscr{I}_\lambda^k$ with the discrete topology into a topological semigroup S with countably compact square $S \times S$. Then by Theorem 4 the restriction

$h_{\mathscr{I}_\lambda^1} : \mathscr{I}_\lambda^1 \to S$ of homomorphism h onto the subsemigroup $\mathscr{I}_\lambda^1$ of $\mathscr{I}_\lambda^k$ is an annihilating homomorphisms. Let $(\mathscr{I}_\lambda^1)h_{\mathscr{I}_\lambda^1} = (\mathscr{I}_\lambda^1)h = e$, where $e \in E(S)$. We fix any $\alpha \in \mathscr{I}_\lambda^k$ with $\operatorname{ran}\alpha = i \ge 2$. Let $\alpha = \begin{pmatrix} x_1 & x_2 & \cdots & x_i \\ y_1 & y_2 & \cdots & y_i \end{pmatrix}$, where $x_1, x_2, \ldots, x_i$, $y_1, y_2, \ldots, y_i \in X$ for some set X of cardinality λ. We fix $y_1 \in X$ and define subsemigroup T_{y_1} of $\mathscr{I}_\lambda^k$ as follows:

$$T_{y_1} = \left\{ \beta \in \mathscr{I}_\lambda^k \mid \begin{pmatrix} y_1 \\ y_1 \end{pmatrix} \cdot \beta = \beta \cdot \begin{pmatrix} y_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} y_1 \\ y_1 \end{pmatrix} \right\}.$$

Then the semigroup T_{y_1} is isomorphic to the semigroup $\mathscr{I}_\lambda^{k-1}$, the element $\begin{pmatrix} y_1 \\ y_1 \end{pmatrix}$ is zero of T_{y_1} and hence by induction assumption we have $\left(\begin{pmatrix} y_1 \\ y_1 \end{pmatrix}\right)h = (\beta)h$ for all $\beta \in T_{y_1}$.

Since $\begin{pmatrix} y_1 \\ y_1 \end{pmatrix} \in \mathscr{I}_\lambda^1$, we have that $(\beta)h = (0)h$ for all $\beta \in T_{y_1}$. But $\alpha = \alpha\gamma$, where $\gamma = \begin{pmatrix} y_1 & y_2 & \cdots & y_i \\ y_1 & y_2 & \cdots & y_i \end{pmatrix} \in T_{y_1}$, and hence we have

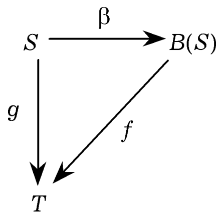
$$(\alpha)h = (\alpha\gamma)h = (\alpha)h \cdot (\gamma)h = (\alpha)h \cdot (0)h = (\alpha \cdot 0)h = (0)h = e.$$

This completes the proof of the theorem. ◊

Theorem 5 implies

Theorem 6. *Let $\lambda \ge \omega$ and n be a positive integer. Then every continuous homomorphism of the topological semigroup $\mathscr{I}_\lambda^n$ into a compact topological semigroup is annihilating.*

Recall [10] that a *Bohr compactification of a topological semigroup S* is a pair $(\beta, B(S))$ such that $B(S)$ is a compact semigroup, $\beta : S \to B(S)$ is a continuous homomorphism, and if $g : S \to T$ is a continuous homomorphism of S into a compact semigroup T, then there exists a unique continuous homomorphism $f : B(S) \to T$ such that the diagram



commutes.

Theorem 6 and Theorem 2.44 [8, Vol. 1] imply

Theorem 7. *If $\lambda \ge \omega$ and n is a positive integer, then the Bohr compactification of the topological semigroup $\mathscr{I}_\lambda^n$ is a trivial semigroup.*

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СИМЕТРИЧНІ ІНВЕРСНІ ТОПОЛОГІЧНІ НАПІВГРУПИ СКІНЧЕННОГО РАНГУ $\leq n$

Вивчаються топологічні властивості симетричних інверсних топологічних напівгруп скінченних перетворень $\mathscr{I}_\lambda^n$ *рангу* $\leq n$. *Показано, що топологічна інверсна напівгрупа* $\mathscr{I}_\lambda^n$ *є алгебраїчно* h*-замкненою в класі топологічних інверсних напівгруп. Також доведено, що топологічна напівгрупа* S *зі зліченно компактним квадратом* $S \times S$ *не містить напівгрупи* $\mathscr{I}_\lambda^n$ *для нескінченного кардинала* λ, *і показано, що компактифікація Бора нескінченної топологічної напівгрупи скінченних перетворень* $\mathscr{I}_\lambda^n$ *рангу* $\leq n$ *є тривіальною напівгрупою.*

СИММЕТРИЧНЫЕ ИНВЕРСНЫЕ ТОПОЛОГИЧЕСКИЕ ПОЛУГРУППЫ КОНЕЧНОГО РАНГА $\leq n$

Изучаются топологические свойства симметрических инверсных топологических полугрупп конечных преобразований $\mathscr{I}_\lambda^n$ *ранга* $\leq n$. *Показано, что топологическая инверсная полугруппа* $\mathscr{I}_\lambda^n$ *является алгебраически* h*-замкнутой в классе топологических инверсных полугрупп. Также доказано, что топологическая полугруппа* S *со счётно компактным квадратом* $S \times S$ *не содержит полугруппу* $\mathscr{I}_\lambda^n$ *для бесконечного кардинала* λ, *и показано, что компактификация Бора бесконечной топологической полугруппы конечных преобразований* $\mathscr{I}_\lambda^n$ *ранга* $\leq n$ *– тривиальная полугруппа.*

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