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S -GRAPHS AND BRAID INDEX OF LINKS

We study relationship between reduction operations on link diagrams and S-graphs associated with them. We are motivated by the problem of computing the braid index of a link and some well known conjectures concerning the braid index of a link and the writhe of its diagrams. Possible counterexamples are discussed in terms of both S-graphs and link diagrams. We also indicate a relation of S-graphs to singular links regarded up to an appropriate equivalence relation.

1. Introduction. The present paper can be considered a continuation of the work [12]. For the reader's convenience and the sake of self-containedness, we recall here some definitions and facts from [12].

The well-known conjecture in knot theory asserts that for every closed braid diagram D which represents a link L and has the smallest number of strands $b(D)$ among all closed braids representing L, the writhe $w(D)$ of D is uniquely determined. Moreover, for any closed braid D' representing L that has $b(D)+k$ strands, the following inequalities hold:

$$w(D) - k \le w(D') \le w(D) + k .$$

We shall refer to this conjecture as ***Conjecture 1.***

By the result of Yamada [16], Conjecture 1 can be equivalently reformulated in terms of closed braids (see [7, 12]).

The first part of Conjecture 1 is known as the Jones conjecture. A stronger version of it is known as the generalized Jones conjecture (see [4, 5]):

Let $\mathcal{B}_{\mathcal{K}}$ denote the set of all closed braid diagrams representing a link $\mathcal{K}$. Let $\Phi : \mathcal{B}_{\mathcal{K}} \to \mathbb{N} \times \mathbb{Z}$ be a map with $\Phi(D) := (b(D), w(D))$ for $D \in \mathcal{B}_{\mathcal{K}}$, where $w(D)$ and $b(D)$ denote the writhe and the number of strands, respectively, of the diagram D. Then there exists a unique $w_{\mathcal{K}} \in \mathbb{Z}$ with $\Phi(\mathcal{B}_{\mathcal{K}}) = \{(b_{\mathcal{K}} + x + {}$ ${}+y, w_{\mathcal{K}} + x - y) \mid x, y \in \mathbb{N}\}$.

It is well known that the Morton − Frank − Williams inequality (MFW inequality) gives a lower bound for the braid index of a link in terms of two variable HOMFLY(PT)-polynomial (see [3, 6, 8, 10]). More precisely, let $P_L(v,z)$ be HOMFLY(PT)-polynomial of two variables v, z of a link L and let $\mathrm{span}_v P_L(v,z)$ denote the difference between the maximal and minimal degrees of variable v in the polynomial. Moreover, let $b(L)$ denote the braid index of the link L. Then we have the following:

$$\mathrm{span}_v P_L(v,z) \le 2\big(b(L) - 1\big).$$

By using MFW inequality or its cabling versions one can calculate the braid index of a knot in many cases. Unfortunately MFW inequality and any its cabling versions are not sharp for links (see [9, 10, 13, 14]). It is also known (see [14]) that if MFW inequality is sharp for a knot L or any of its cable, then L cannot serve a counterexample to the first part of Conjecture 1 that is to the Jones conjecture.

N. M. Dunfield, S. Gukov, and J. Rasmussen [2] have found Khovanov − Rozansky homology version of MFW inequality (KR−MFW inequality). K. Kawamuro [4] showed that this inequality can be used to detect the braid index of some knots for which MFW inequality fails to detect it. There are however infinity many knots for which KR−MFW inequality also fails [4].

There is another approach to the study of the generalized Jones conjecture and the related conjectures. In [7], J. Malešič and P. Traczyk studied the reduction operations of link diagrams. Such operations allow, for given a link diagram, to make of it another one with a smaller number of Seifert circles. The motivation for introducing the link operations was the problem of calculating the braid index of a link, from one side, and the conjectures mentioned above, from the other side. The characteristic feature of the reduction procedure is that the Seifert graph of a link diagram allows to keep control over the number of Seifert circles in the diagram. Such moves will be called MT reduction operations.

In the present paper we study the procedure of reduction of link diagrams in more systematic way. We present a series of new examples of link diagrams and Seifert graphs which can be considered as possible candidates to disprove Conjecture 2 (see below). We show that the collection of MT-operations is not complete (with respect to Conjecture 2) which leads to definition of new types of reduction operations. The connection between S-graphs and singular links is also established.

Let us recall the definition of the Seifert graph $G(D)$ of a link diagram D and S-graph (see also [7]).

The Seifert circles of D form the vertex set of $G(D)$. Furthermore, to each crossing of D that is common for two Seifert circles C_1 and C_2 there corresponds an edge in $G(D)$ which joins the two vertices associated with C_1 and C_2. Moreover, every edge in the Seifert graph $G(D)$ has a sign, according to the sign of the corresponding crossing in the link diagram D. Any Seifert graph $G(D)$ is bipartite and planar (but not planar in a canonical way) and can possess multiple edges. Note that different link diagrams may have the same Seifert graph [10]. On the other hand, given a plane bipartite (completely) signed graph G, there is a canonical procedure (up to orientation of all link components) of drawing a *special* link diagram D (see [12]) so that its Seifert graph $G(D)$ is isomorphic to the signed planar graph G. When performing the recovering procedure we keep the local rule which is indicated in Fig. 1. A link diagram is called *special* if it does not involve any separate Seifert circle [10]. Then to each special link diagram D there corresponds a plane Seifert graph $G(D)$.

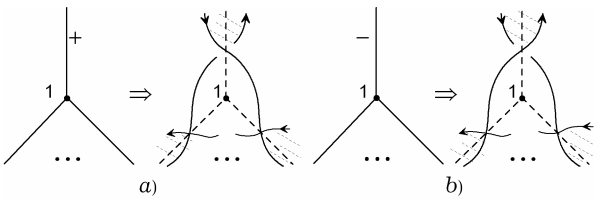


Fig. 1

In the following however, we often do not distinguish between any multiple edges of the graph. This leads to the definition of S-graph.

A set of single edges in $G(D)$ is called cyclically independent if in every (simple) cycle the number of edges from the given set is less than half. Denote by $ind_-(G(D))$ and $ind_+(G(D))$ the maximum numbers of cyclically independent (single) negative and positive edges, respectively, in $G(D)$. Murasugi and Przytycki [10] have strengthened the MFW inequalities in the following form.

For any diagram D of a link L we have

$$\mathrm{span}_v P_L(v,z) \le 2(s(D) - ind_-(G(D)) - ind_+(G(D) - 1). \qquad (1)$$

Therefore Murasugi − Przytycki inequality also gives a lower bound for the braid index of a link.

Conjecture 2. Let D be a diagram of an oriented link L. Assume that the number of Seifert circles in D is s and the writhe of D is c. Then it is possible to find another link diagram of L with the number of Seifert circles equal to $s - ind_+(G(D)) - ind_-(G(D))$ and the writhe equal to $c - ind_+(G(D)) + ind_-(G(D))$.

Conjecture 2 is true for homogeneous diagrams D of links [10]). Moreover, there is a connection between Conjectures 1 and 2. As shown in [7], any counterexample to Conjecture 2 would imply a counterexample to Conjecture 1.

Moreover, Malešič and Traczyk conjectured that a desired reduction on any given link diagram can be performed via five types of operations specified by them (see [7] and [12]). These operations were aimed to reduce any link diagram to the one with a fewer number of Seifert circles and have analogies on the level of Seifert graphs. We refer to this conjecture as ***Conjecture 3***. Note that Conjecture 3 has been disproved in [12].

Let H be a planar bipartite graph and let E denote its edges. It is assumed that H allows multiple edges but contains no loops. Suppose we have in E the two disjoint subsets E_+ and E_- consisting of single positive and negative edges (that is marked by + and −) and the other edges in H are neutral. Assume that both E_- and E_+ are cyclically independent sets. Following [7], any graph H enhanced with the structure described above is called an S-graph. The idea was to choose in the Seifert graph $G(D)$ some maximal collections E_+ and E_- of edges that determine the numbers $ind_-(G(D))$ and $ind_+(G(D))$ and forget the signs the remaining edges. Now the strategy is to perform on the link diagram D or its Seifert graph reducing operations following only the chosen positive and negative crossings (edges).

Given a plane S-graph G, we can recover a link diagram D which it corresponds to (see [12]). The recovered diagram is not unique. First of all, we can change the orientation of all components of D. Then G is also a S-graph associated to D. The second ambiguity arises when recovering neutral edges. To each neutral edge e of the graph G we can associate n-fold, positive or negative, half-twist in D, where n can be chosen arbitrarily. However if we recover *a singular link diagram* D' from G in an appropriate way (see below), the ambiguity is only in the orientation of all the components of a diagram D. In this case, the obtained oriented singular link L' contains exactly ℓ singularities, where ℓ is the number of neutral edges of G (see Section 3).

Now we recall the definition of reduction operations on link diagrams, as in [12]. Operations ***1°–3°*** remain without any changes while the operations ***4°*** and ***5°*** are slightly generalized. Suppose we have a diagram D of a link and apply to it a reduction operation within the same link type. Let D' be the resulting link diagram. We shall say that the operation on a link diagram D is admissible if it satisfies the following condition:

$$s(D) - ind_+(G(D)) - ind_-(G(D)) \geq s(D') - ind_+(G(D')) - ind_-(G(D')).$$

Operation 1°. Cancellation of a trivial loop as shown in Fig. 2. This operation decreases the number of Seifert circles by one and changes the writhe of the diagram also by one, so it is always admissible.

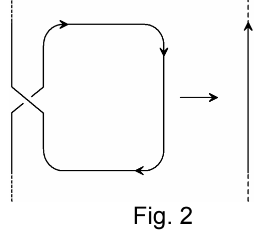


Fig. 2

Operation 2°. This operation is a modification of the Murasugi − Przytycki operation introduced and studied in [10]. Suppose we have in D a reduction site like for an Murasugi − Przytycki operation which involves two circles with a unique common crossing (see [10]). One of the circles involved in the site is the reduction circle and the second one is the engulfing circle. Moreover, it is assumed that there are no Seifert circles nested in the basic reduction circle. The situation looks like Fig. 3.

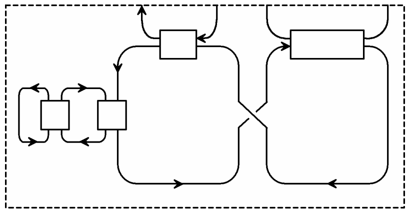


Fig. 3

The operation consists in replacing the reduction circle involved in the site with a new circle in the following way. We start in the same manner as in the Murasugi − Przytycki operation but when the first circle adjacent to the basic circle is met we circle round it and come back to the basic one. Next, we create a crossing of our new arc with the basic circle and proceed to the other end of the short arc that is being replaced by the long arc, keeping close to the reduction circle. The situation looks like as in Fig. 4.

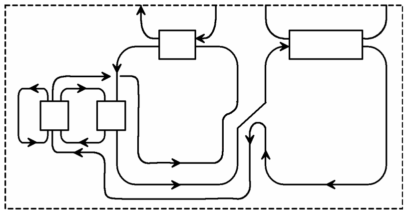


Fig. 4

This operation does not change the number of Seifert circles or the writhe, so it is of auxiliary character in the reduction process.

The operation is admissible if it decrease neither positive, nor negative indices of the Seifert graph of a link diagram. One of the three Seifert circles in the resulting link diagram is engulfed now by the basic one, so the whole configuration of the new link diagram subject to the Seifert graph is now more simple than before.

Operation 3°. Cancellation of a pair of one positive and one negative crossing in D in a situation shown in Fig. 5. This operation is always admissible and decreases the number of Seifert circles by two and preserves the writhe of the diagram D.

In the corresponding Seifert graph the two signed edges, one positive and the second negative, are contracted.

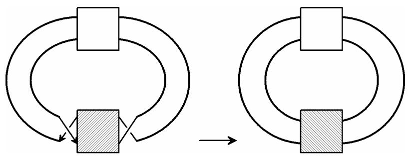
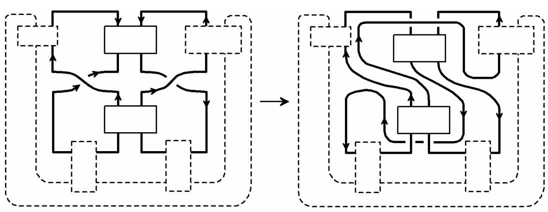


Fig. 5

Operation 4°. Assume in a link diagram D we have four Seifert circles in a circle like in Fig. 6*a*. It is also assumed that there is nothing more inside of the circle but outside the diagram can be complicated. Next, we change one short tunnel and one short bridge to obtain the long tunnel and the long bridge as indicated in Fig. 6*b*. The two single distinguished crossings are of opposite signs. As a result, one Seifert circle of the new link diagram is nested. The pair of the two single crossings in the diagram on the left is replaced with the one on the right, as indicated in Fig. 6*b*. The whole configuration of the Seifert graph in the resulting link diagram D' is simplified. The operation is admissible only if $ind_+(G(D)) = ind_+(G(D'))$ and $ind_-(G(D)) = ind_-(G(D'))$.



a) *b*)

Fig. 6

Operation 5°. This operation is similar to the previous one. A similar configuration of four Seifert circles is needed. However, the placement of the two single crossings of opposite signs is different from the above one (see Fig. 7*a*). The resulting diagram is indicated in Fig. 7*b*. In this case, the whole configuration of the Seifert graph is also simplified. This operation is admissible if it does not decrease the positive and the negative indices of the Seifert graph of the corresponding link diagrams.

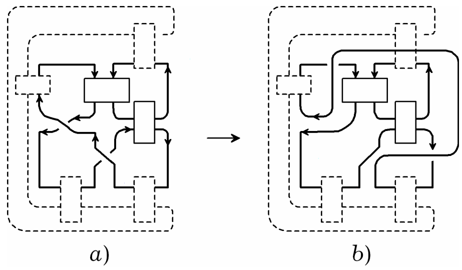


Fig. 7

There are analogies of reduction operations on Seifert graphs and S-graphs (see [7, 12]). In that case, a reduction operation on S-graph G is called admissible if the resulting signed graph G' is also S-graph. Note that the reduction on link diagrams, Seifert graphs and S-graphs are compatible (see [12]). Further properties of reduction operations on S-graphs and their relations to link diagrams can be found in [12]. Note also that many examples of link diagrams (S-graphs) from the paper [12] that disprove Conjecture 3 confirm however Conjecture 2.

In Section 2 we consider several new examples of link diagrams for which Conjecture 3 is false and provide new possible candidates to disprove Conjecture 2.

2. From S-graphs to link diagrams. Let D be a link diagram and $G(D)$ its Seifert graphs with $ind_+ = k$ and $ind_- = \ell$. Fix the sets S_+, S_- of positive and negative single edges in G which determine the numbers k and ℓ, respectively, and consider the corresponding S-graph S. Let T be the union of S_1 and S_2. As a subgraph of S, T is a forest. Moreover, let U be the set of vertices in S which are not covered by the edges from T.

Proposition 1. *The components of the forest T, considered as a subgraph of the plane graph S, determines in a canonical way the collection of link diagrams D with maximal number of link components so that S is an S-graph associated with $G(D)$. The number of components is equal to*

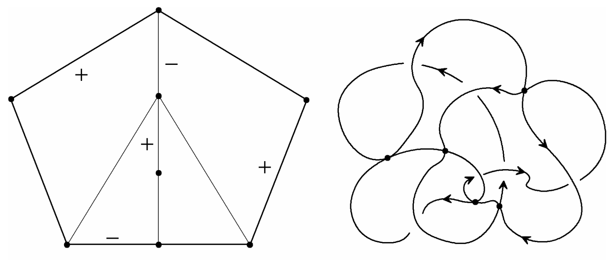
$$|S| - k - \ell .$$

P r o o f. The proof is by observation that the number of components of T summing with the number $|U|$ is just $|S| - k - \ell$. Then to each component W of T and each vertex v from U there corresponds a unique link component K going around W (around the vertex v, respectively). Moreover, since each W is a tree, K is always unknotted. ◊

Therefore, given a non-reducible S-graph G, a possible counterexample to Conjecture 2, can be found among link diagrams D that are provided by Proposition 1. In particular, we have to seek the diagrams which contain at least two components L_1 and L_2 with $b(L_1 \cup L_2) = 3$.

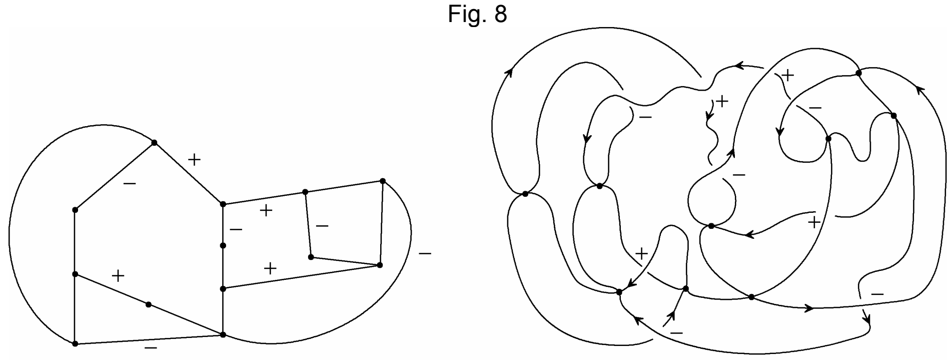
Consider now two examples of non-reducible (via MP-operations) S-graphs, G and H, pictured in Fig. 8*a* and Fig. 9*a*, respectively.

Let D be any link diagram recovering from G and W be a diagram recovering from H (see Fig. 8*b*). Here singular crossings correspond to neutral edges of S-graphs and should be replaced with multiple twists in diagrams. It is easy to verify that both D and W are not reducible via MT-operations defined for diagrams. We have $ind_-(G) = 2$, $ind_+(G) = 3$ and $|G| = 8$ and the following assertion holds.



a) *b*)

Fig. 8



a) *b*)

Fig. 9

Proposition 2. *The link diagram D is non-reducible via MT-operations. On the other hand, D is reducible in general context.*

P r o o f. The first assertion follows from the fact that the S-graph G is non-reducible via MT-operations defined for S-graphs and that the operations on S-graphs and link diagrams correspond. On the other hand, we can apply to D a sequence of moves within the same link type which results first the diagram D_1 (see Fig. 10) and finally the diagram D' (see Fig. 11).

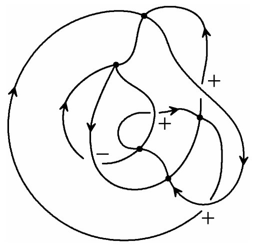


Fig. 10

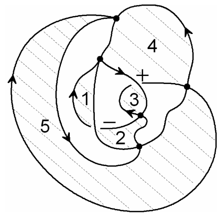


Fig. 11

Consider now the Seifert graph G' of D' indicated in Fig. 12. Here the multiple edges are replaced with a neutral edge in each inclusion, so G' is regarded as an S-graph. This S-graph is reducible via MT-operations. It follows that the new link diagram D' can be reduced via MT-operations to the other one, D_2, with $s(D_2) = 3$. Now the second assertion follows, since $|G| = 8 - 3 - 2 = 3$. ◊

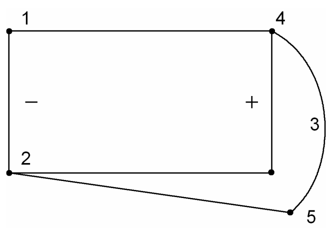


Fig. 12

In the similar way, we have the following $ind_{-}(H) = 5$, $ind_{+}(H) = 4$ and $|H| = 13$. Moreover, the following assertion holds.

Proposition 3. *The link diagram W is non-reducible via MT-operations. On the other hand, W is reducible in general context.*

P r o o f. The first assertion follows directly from the fact that the S-graph H is non-reducible via MT-operations defined for S-graphs. On the other hand, we can apply to W a sequence of moves within the same link type, so that the resulting diagram W' looks like in Fig. 13. The Seifert graph H' corresponding of W' (or rather the S-graph derived from it) is depicted in Fig. 14.

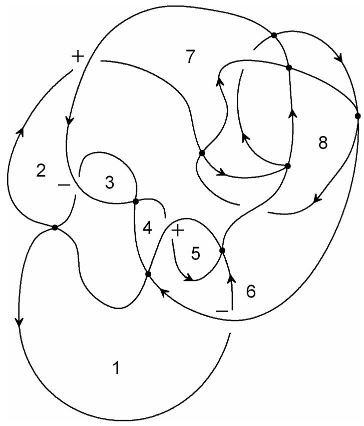


Fig. 13

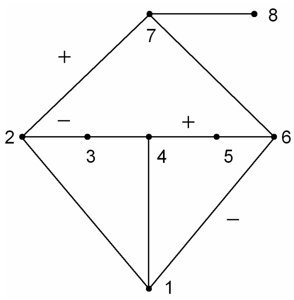


Fig. 14

It is easily seen that H' is reducible via MT-operations, so the diagram W' can be reduced via MT-operations to the other one, W_2, with $s(W_2) = 4$. Now the second assertion follows directly from the equality $13 - 5 - 4 = 4$.

Therefore, in order to reduce S-graphs G and H and the corresponding link diagrams we need new reduction operations (different from MT-operations). They look somewhat complicated and likely can not be decomposed into more simple and typical ones.

Example 1. Consider an example of S-graph R depicted in Fig. 15. We have the following: $ind_-(R) = 6$, $ind_+(R) = 5$ and $|R| = 16$. Let D be any singular link diagram recovering from R. Let $\mathcal{L}_R$ be the collection of all link diagrams D that are associated with the S-graph R. This means that R can be obtained for Seifert graph $G(D)$ by forgetting the signs of some (non-essential) edges, so that D can be recovered from R. Let $b(\mathcal{L}_R)$ be the maximum of the numbers $b(D)$, where the maximum is taken over all diagrams from $\mathcal{L}_R$. We do not know if $b(\mathcal{L}_R) = 5$. If $b(\mathcal{L}_R) > 5$, then some D from the collection gives a counterexample to Conjecture 2. Note also that each link component of the link represented by D is unlinked.

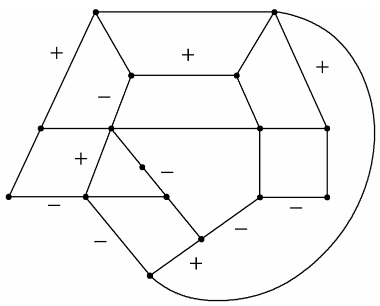


Fig. 15

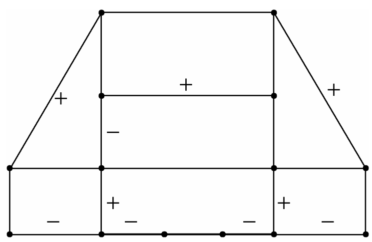


Fig. 16

In Fig. 16 we show another example of a non-reduced S-graph R_1 which also can serve a counterexample to Conjecture 2 (when passing to the corresponding link diagram).

In this case, we have $ind_-(R_1) = 5$, $ind_+(R_1) = 5$ and $|R_1| = 14$. Let E_1 be a link diagram, recovered from the S-graph R_1. We do not know if $b(\mathcal{L}_{R_1}) = 4$. If $b(\mathcal{L}_{R_1}) > 4$, there is a diagram $E_1 \in \mathcal{L}_{R_1}$ which gives a counter-example to Conjecture 2.

3. Connection with singular links. We noted in Section 2 that the same S-graph may correspond to different link diagrams. To make this correspondence one-to-one (for connected link diagrams and up to change the orientation of all components of link diagrams), we can associate with each S-graph G a *special* singular link diagram D in the following way. We proceed in the same way as in the case of recovering the ordinary link diagrams. The only difference is that to each neutral edge of G we assign a flat singularity of D. In our case, on singular links or diagrams it is convenient to choose appropriate equivalence relations. Instead of vertex rigid isotopy which usually is considered on singular links, we regard more weak equivalence relation which allows a half-twist in a flat singularity, as depicted in Fig. 17. In the remaining part, it is allowed to use the same five Reidemeister moves as for spatial graphs (see, for example, [11]).

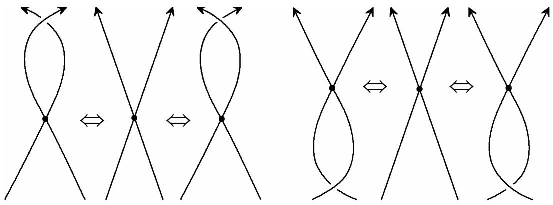


Fig. 17

Proposition 4. *Up to orientation of components, the above correspondence between special connected singular link diagrams and plane connected S-graphs is one-to-one.*

P r o o f. For a fixed orientation of one component of a recovering link diagram, the ambiguity can occur in recovering any neutral edge of the S-graph, which in the standard situation case is replaced with a multiple positive or negative half-k-twist in a diagram. As for the case of singular links, it corresponds to a standard flat singularity with orientation of strands induced from the fixed orientation of one component, since the S-graph and the corresponding link diagrams are connected. ◊

It is not difficult to verify that an analogue of Alexander theorem holds for singular links in classical setting (see, for example, [1]) and so for singular links in our setting. Moreover, the well known Vogel algorithm [15] can be also adopted for singular links in our setting. Here we have only to use classical Reidemeister moves (Reidemeister moves for classical link diagrams without singularities). It follows the braid index of a singular link L, $b(L)$, defined as the smallest number of strands to represent L as a closure of some singular braid, is also well defined. Under this agreement, we can reformulate our problem as follows.

Problem 1. Suppose we have an S-graph G and let D be the corresponding singular link diagram, with the link type L. Is it true that for any such graph G and diagram D we have $|G| - ind_+(G) - ind_-(G) \geq b(L)$?

The positive answer to this question proves also Conjecture 2. On the other hand, if for some non-reducible via MT-operations S-graph G and a diagram $D \in \mathcal{L}_G$ we have $|G| - ind_+(G) - ind_-(G) < b(L)$, this probably gives a counterexample to Conjecture 2. The latter justifies introducing singular link diagrams and the weak isotopy as an equivalence relation on singular links in our setting.

Let S be an S-graph, pictured in Fig. 18*a* and T be a singular link diagram recovered from S (see Fig. 18*b*). We have $ind_-(S) = 2$, $ind_+(S) = 2$ and $|S| = 7$. We can directly show that T is non-reducible via MT-operations, but the singular link diagram T is reducible in general context to some singular link diagram T' with $s(T') = 3$. This means that $b(T) = 3$.

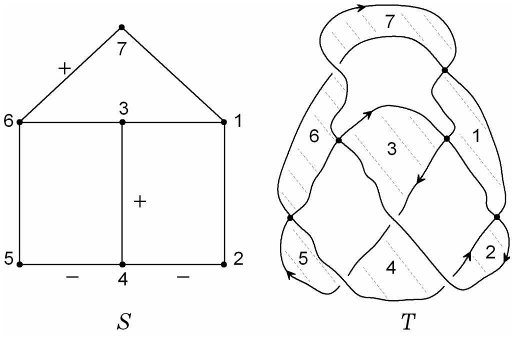


Fig. 18

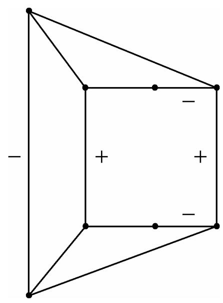


Fig. 19

Consider now the S-graph R_1, depicted in Fig. 19.

Here we have $ind_-(R_1) = 3$, $ind_+(R_1) = 2$ and $|R_1| = 8$. Let D_1 be any link diagram, recovered from the S-graph R_1. The graph R_1 is non-reducible via MT-operations, but $b(\mathcal{L}_{R_1}) = 3$. Moreover, if D_1 is a singular link diagram recovering from the S-graph R_1, then $b(D_1) = 3$. This means that any diagram $D \in \mathcal{L}_{R_1}$ cannot serve a counterexample to Conjecture 2.

The similar situation is with the S-graph W from the Fig. 7 of the paper [12]. Let D be any singular link diagram recovering from W (see Fig. 11*a* in the paper [12]). It is not difficult to show that $b(D) = 3$, so it cannot be a counterexample to Conjecture 2.

Note also that the proofs of Propositions 2 and 3 can be run also in terms of reduction operations (moves) on singular links diagrams and braid index of singular links which they represent. In the similar manner we can treat Example 1 and the other examples of non-reducible S-graphs considered in Section 2. The reduction operations we use to show that the above link diagrams, which are not reducible via MT-operations, are reducible in general sense, often are not admissible on intermediate steps of reduction. This shows that in general we have to include in the reduction process some operations which not necessarily keep over the numbers $ind_+(G)$ and $ind_-(G)$.

We are also interested in answering the following questions.

Question 1. What is the difference between the classical braid index of a singular link and the braid index of this link in our setting?

Question 2. Let T be any S-graph, $\mathcal{L}_T$ the corresponding collection of (classical) link diagrams recovered from T and W be a singular link diagram recovering from T. Is it true that $b(\mathcal{L}_T) = b(W)$?

Further analysis of Conjecture 2, S-graphs and connection with singular knots will be considered in a forthcoming paper.

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S -ГРАФИ ТА БРЕЙД-ІНДЕКС СПЛЕТЕНЬ

Вивчається співвідношення між редукційними операціями на діаграмах сплетень і асоційованими з ними S -графами в контексті обчислення брейд-індексу сплетення, а також в контексті відомих гіпотез про зв'язок між брейд-індексом сплетення і скрутом його діаграм. Наведено деякі потенційні контрприклади до цих гіпотез як на рівні графів, так і на рівні діаграм сплетень. Вказано також на зв'язок між S -графами та сингулярними сплетеннями, які розглядаються з точністю до відповідного відношення еквівалентності.

S -ГРАФЫ И БРЕЙД-ИНДЕКС ЗАЦЕПЛЕНИЙ

Изучается соотношение между редукционными операциями на диаграммах зацеплений и ассоциированными с ними S -графами в контексте вычисления брейд-индекса зацепления, а также в контексте известных гипотез о связи между брейд-индексом зацепления и кручением его диаграмм. Приведены некоторые потенциальные контрпримеры для этих гипотез как на уровне графов, так и на уровне диаграмм зацеплений. Указано также на связь между S -графами и сингулярными зацеплениями, которые рассматриваются с точностью к соответствующему отношению эквивалентности.

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