

THE TRAIN SOUND RADIATION

The problem of sound radiation from a train is considered. This object has been simulated as sets of point sources uniformly distributed in the domain of moving lengthened rectangle. The solution of the problem is obtained using integral Fourier-transforms over the space coordinates and time. The integrals are calculated with application of stationary phase method. Numerical analysis is carried out for acoustic pressure and sound intensity.

1. Formulation and analysis. The fast trains are significant sources of noise. In last years the problem of sound generation by trains and natural environment protection from the noise attracts more attention of researchers [4, 7–10, 13, 14]. In known works only the averaging noise characteristics were considered, but the detailed acoustic state and existence of different wave singularities near moving train was not taken into account.

In this paper we propose one approach for consideration of this problem using model of noise radiator in the form of moving lengthened rectangle covered by continuously distributed point sound sources.

Let us consider the sound emission from moving unit of railroad transport in train form. The train of length L and height h moves with velocity v_0 in positive direction axis Ox of Cartesian system of coordinates $Oxyz$. The train, as perturbation of noise, has the form of source, sharp-concentrated in direction of axis Oy and continuously distributed in plane Oxz in the frames of rectangle with train dimensions, lifted up above boundary interface of acoustical and solid elastic media on small height ε .

The base relations are equations of linear acoustics: the equation of motion and mass balance equation [3]

$$\begin{aligned} \rho \frac{\partial \mathbf{v}(\mathbf{x}, t)}{\partial t} &= -\nabla p(\mathbf{x}, t) + \mathbf{F}_p(\mathbf{x}, t), \\ \nabla \cdot \mathbf{v}(\mathbf{x}, t) &= -\frac{1}{\rho c^2} \frac{\partial p(\mathbf{x}, t)}{\partial t}, \end{aligned} \quad (1)$$

where p is the acoustic pressure, \mathbf{v} is the particle velocity vector, ρ is the acoustic medium density, c is the sound velocity, t is the time, ∇ is the Hamilton operator $\nabla = \nabla_{\perp} + \mathbf{i}_z \partial / \partial z$, $\nabla_{\perp} = \mathbf{i}_x \partial / \partial x + \mathbf{i}_y \partial / \partial y$. Here $\mathbf{F}_p(\mathbf{x}, t)$ is the vector of complex mass force determined by power of acoustic source:

$$\mathbf{F}_p(\mathbf{x}, t) = \int_{\varepsilon}^{h+\varepsilon} \int_{-L/2}^{L/2} \mathbf{F}(\mathbf{x}, \mathbf{x}', t) dx' dz', \quad (2)$$

where

$$\begin{aligned} \mathbf{F}(\mathbf{x}, \mathbf{x}', t) &= \mathbf{F}_0 G(\boldsymbol{\xi}, \boldsymbol{\xi}', t) \delta(z - z'), \\ G(\boldsymbol{\xi}, \boldsymbol{\xi}', t) &= \delta(x - x' - v_0 t) \delta(y) e^{-i\Omega_0 t}. \end{aligned} \quad (3)$$

Here $\mathbf{F}_0 = (F_{0x}, F_{0y}, F_{0z})$ is the vector of constant mass force with real amplitudes, $\delta(x)$ is the Dirac function, $\mathbf{x} = \boldsymbol{\xi} + \mathbf{i}_z z$, $\mathbf{x}' = \boldsymbol{\xi}' + \mathbf{i}_z z'$ and $\boldsymbol{\xi} = \mathbf{i}_x x + \mathbf{i}_y y$, $\boldsymbol{\xi}' = \mathbf{i}_x x' + \mathbf{i}_y y'$ are respectively radius-vectors in the space $Oxyz$ and in the plane Oxy , Ω_0 is the angular frequency.

Thus, having solution of problem for a case of point mass force $\mathbf{F}_p(\mathbf{x}, t) = \mathbf{F}(\mathbf{x}, \mathbf{x}', t)$, by double integration over x' and z' we find the solution of problem with mass force in the form of Eq. (2).

The problem with $\mathbf{F}_p(\mathbf{x}, t) = \mathbf{F}(\mathbf{x}, \mathbf{x}', t)$ was solved in more general case for many moving point sound sources in moving medium [11]. This problem consists in the solving of equations for pressure in acoustic waves radiated and reflected from an elastic half-space ($z < 0$) with taking into consideration the boundary conditions on surface $z = 0$

$$\sigma_z + p_{\text{tot}} = 0, \quad \tau_{xz} = 0, \quad \tau_{yz} = 0, \quad u_{sz} = u_{\text{tot},z}, \quad (4)$$

where $p_{\text{tot}} = p_{\text{rad}} + p_{\text{ref}}$, $u_{\text{tot},z} = u_{\text{rad},z} + u_{\text{ref},z}$; σ_z , τ_{xz} , τ_{yz} and u_{sz} are respectively the components of elastic stress tensor and displacement vector, for which determination must be solved corresponding system of equations of elasticity theory [1]. As the result, we have solution of our problem for source (2) moving in motionless acoustic medium in the form:

$$\begin{aligned} p_j(\mathbf{x}, t) = & \frac{i}{8\pi^2} \left[F_{0x} \int_{\varepsilon}^{h+\varepsilon} P_j(\mathbf{x}, \mathbf{x}', t) \Big|_{x'=-L/2}^{x'=L/2} dz' - \right. \\ & - F_{0y} \frac{\partial}{\partial y} \int_{\varepsilon}^{h+\varepsilon} \int_{-L/2}^{L/2} P_j(\mathbf{x}, \mathbf{x}', t) dx' dz' + \\ & \left. + F_{0z} \int_{-L/2}^{L/2} P_j(\mathbf{x}, \mathbf{x}', t) \Big|_{z'=\varepsilon}^{z'=h+\varepsilon} dx' \right] e^{-i\Omega_0 t}, \quad j = \text{rad, ref}, \quad (5) \end{aligned}$$

where

$$\begin{aligned} P_{\text{rad}}(\mathbf{x}, \mathbf{x}', t) &= \varphi(x - x' - v_0 t, y, z - z'), \\ P_{\text{ref}}(\mathbf{x}, \mathbf{x}', t) &= R_s(x - x' - v_0 t, y, z + z') \varphi(x - x' - v_0 t, y, z + z'), \quad (6) \\ \varphi(\mathbf{x}) &= -2\pi i \frac{\exp\{iK_M[R(\mathbf{x}) + M_0 x]\}}{R(\mathbf{x})}, \end{aligned}$$

$$R(\mathbf{x}) = \sqrt{x^2 + (1 - M_0^2)(y^2 + z^2)}, \quad R_s(\mathbf{x}) = \frac{V^-(\mathbf{x})}{V^+(\mathbf{x})},$$

$$\begin{aligned} V^\pm(\mathbf{x}) &= [S_T^2 - 2S^2(\mathbf{x})]^2 + 4S^2(\mathbf{x})S_{zL}(\mathbf{x})S_{zT}(\mathbf{x}) \pm \\ & \pm N_s S_{zL}(\mathbf{x})/S_z(\mathbf{x}), \quad (7) \end{aligned}$$

$$S(\mathbf{x}) = \frac{\xi}{R(\mathbf{x})}, \quad S_{zA}(\mathbf{x}) = \sqrt{(c/c_A)^2 - S^2(\mathbf{x})}, \quad A = L, T, \quad S_z(\mathbf{x}) = \frac{z}{R(\mathbf{x})},$$

$$\xi = |\boldsymbol{\xi}|, \quad K_M = \frac{K_0}{\alpha}, \quad \alpha = 1 - M_0^2, \quad N_s = \frac{\rho}{\rho_s}.$$

Here $R_s(\mathbf{x})$ is the coefficient of sound reflection from elastic half-space, $K_0 = \Omega_0/c$ is the wave number and $M_0 = v_0/c$ is the Mach number for moving source, ρ and ρ_s are the densities of acoustical medium and solid material, c_L and c_T are velocities of longitudinal and transversal waves. Note, that formulae (6) are obtained with help of the stationary phase method [3] without taking into account small contribution in re-radiated sound field from the

Rayleigh surface wave. The integrals in Eq. (5) are calculated in similar manner with taking into consideration the finite limits of integration [3]. Then final results are in following form:

$$p_j(\mathbf{x}, t) = \frac{i}{8\pi^2} \left[F_{0x} J_{x,j}(\tilde{\boldsymbol{\xi}}, z) \Big|_{x'=-L/2}^{x'=L/2} - F_{0y} J_{xz,j}(\tilde{\boldsymbol{\xi}}, z) + F_{0z} J_{z,j}(\tilde{\boldsymbol{\xi}}, z-z') \Big|_{z'=\varepsilon}^{z'=h+\varepsilon} \right] e^{-i\Omega_0 t}, \quad j = \text{rad, ref}, \quad (8)$$

where

$$\begin{aligned} J_{x,\text{rad}}(\tilde{\boldsymbol{\xi}}, z) &= -2\pi i e^{iK_M M_0 x} \{ I_{xs,\text{rad}}(\tilde{\boldsymbol{\xi}}) [H(z-\varepsilon) - H(z-h-\varepsilon)] + I_{xe,\text{rad}}(\tilde{\boldsymbol{\xi}}, z) \}, \\ J_{xz,\text{rad}}(\tilde{\boldsymbol{\xi}}, z) &= J_{xzs,\text{rad}}(\tilde{\boldsymbol{\xi}}) [H(z-\varepsilon) - H(z-h-\varepsilon)] - J_{xze,\text{rad}}(\tilde{\boldsymbol{\xi}}, z-z') \Big|_{z'=\varepsilon}^{z'=h+\varepsilon}, \\ J_{z,\text{rad}}(\tilde{\boldsymbol{\xi}}, z) &= -2\pi i [I_{zs,\text{rad}}(y, z) H(L/2 - |x_{ss}(\tilde{\boldsymbol{\xi}}, z)|) + I_{ze,\text{rad}}(\tilde{\boldsymbol{\xi}}, z)], \\ J_{x,\text{ref}}(\tilde{\boldsymbol{\xi}}, z) &= -2\pi i e^{iK_M M_0 \tilde{x}} I_{xe,\text{ref}}(\tilde{\boldsymbol{\xi}}, z), \\ J_{xz,\text{ref}}(\tilde{\boldsymbol{\xi}}, z) &= -J_{xze,\text{ref}}(\tilde{\boldsymbol{\xi}}, z+z') \Big|_{z'=\varepsilon}^{z'=h+\varepsilon}, \\ J_{z,\text{ref}}(\tilde{\boldsymbol{\xi}}, z) &= -2\pi i [I_{zs,\text{ref}}(y, z) H(L/2 - |x_{ss}(\boldsymbol{\xi}, t)|) + I_{ze,\text{ref}}(\tilde{\boldsymbol{\xi}}, z)], \\ I_{xs,\text{rad}}(\tilde{\boldsymbol{\xi}}) &= \sqrt{\frac{2\pi i}{K_0 R(\tilde{\boldsymbol{\xi}})}} e^{iK_M R(\tilde{\boldsymbol{\xi}})}, \quad R(\boldsymbol{\xi}) = R(\mathbf{x})|_{z=0}, \\ I_{xe,\text{rad}}(\tilde{\boldsymbol{\xi}}, z) &= \frac{i}{K_0} [(z-z')^{-1} e^{iK_0 R(\tilde{\boldsymbol{\xi}}, z-z')}] \Big|_{z'=\varepsilon}^{z'=h+\varepsilon}, \quad z \neq \varepsilon, \quad z \neq h+\varepsilon, \\ J_{xzs,\text{rad}}(\tilde{\boldsymbol{\xi}}) &= J_{xzss,\text{rad}}(y) H(L/2 - |x_{ss}(\boldsymbol{\xi}, t)|) + J_{xzse,\text{rad}}(\tilde{\boldsymbol{\xi}}), \\ J_{xzss,\text{rad}}(y) &= -4i\pi^2 \left(1 + \frac{i}{2K_M} \right) e^{K_0 y}, \quad y \neq 0, \\ J_{xzse,\text{rad}}(\tilde{\boldsymbol{\xi}}) &= 2\pi i \alpha y \left\langle \sqrt{2\pi i [K_0 R^3(\tilde{\boldsymbol{\xi}})]^{-1}} \{ 1 - [2iK_M R(\tilde{\boldsymbol{\xi}})]^{-1} \} \times \right. \\ &\quad \left. \times [M_0 + \tilde{x}/R(\tilde{\boldsymbol{\xi}})]^{-1} e^{i[K_M R(\tilde{\boldsymbol{\xi}}) + M_0 \tilde{x}]} \right\rangle \Big|_{x'=-L/2}^{x'=L/2}, \\ J_{xze,\text{rad}}(\tilde{\boldsymbol{\xi}}, z) &= J_{xzse,\text{rad}}(y, z) H(L/2 - |x_{es}(\mathbf{x}, t)|) + J_{xzee,\text{rad}}(\tilde{\boldsymbol{\xi}}, z), \\ J_{xzse,\text{rad}}(y, z) &= \frac{2\pi y}{iz} \sqrt{\frac{2\pi}{iK_0 \sqrt{y^2+z^2}}} e^{iK_0 \sqrt{y^2+z^2}}, \quad z > 0, \\ J_{xzee,\text{rad}}(\tilde{\boldsymbol{\xi}}, z) &= \frac{2\pi y}{K_M z} \left\{ \frac{1}{\tilde{x}} e^{iK_M [R(\tilde{\boldsymbol{\xi}}, z) + M_0 \tilde{x}]} \right\} \Big|_{x'=-L/2}^{x'=L/2}, \\ &\quad z > 0, \quad x \neq v_0 t \pm L/2, \end{aligned} \quad (9)$$

$$\begin{aligned}
I_{zs,\text{rad}}(y, z) &= \sqrt{\frac{2\pi}{iK_0\sqrt{y^2+z^2}}} e^{iK_0\sqrt{y^2+z^2}}, \\
I_{ze,\text{rad}}(\tilde{\xi}, z) &= \frac{-e^{iK_M[R(\tilde{\xi}, z)+M_0\tilde{x}]}}{iK_M[M_0R(\tilde{\xi}, z)+\tilde{x}]} \Bigg|_{x'=-L/2}^{x'=L/2}, \\
I_{xe,\text{ref}}(\tilde{\xi}, z) &= -\frac{i}{K_0} \left[\frac{R_s(\tilde{\xi}, z+z')}{z+z'} e^{iK_0R(\tilde{\xi}, z+z')} \right] \Bigg|_{z'=\varepsilon}^{z'=h+\varepsilon}, \\
J_{xze,\text{ref}}(\tilde{\xi}, z) &= J_{xzes,\text{ref}}(y, z)H(L/2 - |x_{es}|) + J_{xzee,\text{ref}}(\tilde{\xi}, z), \\
J_{xzes,\text{ref}}(y, z) &= R_s(\tilde{\xi}, z) \Big|_{x=x_{es}(\mathbf{x}, t)} J_{xzes,\text{rad}}(y, z), \\
J_{xzee,\text{ref}}(\tilde{\xi}, z) &= \frac{2\pi y}{K_M z} \left\{ \frac{R_s(\tilde{\xi}, z)}{\tilde{x}} e^{iK_M[R(\tilde{\xi}, z)+M_0\tilde{x}]} \right\} \Bigg|_{x'=-L/2}^{x'=L/2}, \\
& z > 0, \quad x \neq v_0 t \pm L/2, \\
I_{zs,\text{ref}}(y, z) &= R_s(\tilde{\xi}, z) \Big|_{x=x_{es}(\mathbf{x}, t)} I_{zs,\text{rad}}(y, z), \\
I_{ze,\text{ref}}(\tilde{\xi}, z) &= \frac{-R_s(\tilde{\xi}, z)e^{iK_M[R(\tilde{\xi}, z)+M_0\tilde{x}]}}{iK_M[M_0R(\tilde{\xi}, z)+\tilde{x}]} \Bigg|_{x'=-L/2}^{x'=L/2}, \\
\tilde{\xi} &= (\tilde{x} = x - x' - v_0 t, y), \quad x_{ss}(\xi, t) = x - v_0 t + M_0 y, \\
x_{es}(\mathbf{x}, t) &= x - v_0 t + M_0 \sqrt{y^2 + z^2}.
\end{aligned}$$

2. Numerical results and discussion. Analysis of total acoustical pressure is performed using relations (8) and (9). For numerical calculations it is admitted that the components of mass force vector are the same in all directions, i.e. $F_{0x} = F_{0y} = F_{0z} = F_0$. In this case $F_0 K_0 = 8\pi \cdot 10^{0.05(I_0 - 100)}$, where $I_0 = 75$ dB [2]. The train of the length $L = 360$ m and height $h = 4$ m moves on height $\varepsilon = 0.2$ m in air medium with density $\rho = 1.293$ kg/m³ and sound velocity $c = 331$ m/s [12]. Also it is assumed that ground near wheel track is covered by asphalt, which mass density is $\rho_s = 2000$ kg/m³ and velocities of elastic waves are $c_L = 3468$ m/s and $c_T = 1667$ m/s [6]. The frequency of train vibration Ω_0 is chosen from 100 to 600 Hz, taking into attention a fact that train noise is most oppressive at $\Omega_0 = 100$ Hz [5]. The numerical calculations are carried out in such manner that the moment $t = 0$ s corresponds to location of train middle in the point $x = 0$ m.

Figs. 1 show the structure of instantaneous total acoustical pressure $p(t) = \text{Re } p_{\text{tot}}$ (in Pascals) calculated in point $x = 0$ m, $y = -20$ m on height $z = 10$ m, when velocity of train motion is $v_0 = 150$ km/h, for frequencies $\Omega_0 = 100, 200, 400, 600$ Hz. It is observed that signal amplitude is modulated in time. With source frequency increasing the more and more modes of low-frequency vibrations are excited upon train length. This means that the train must be considered as peculiar acoustical resonator of linear type.

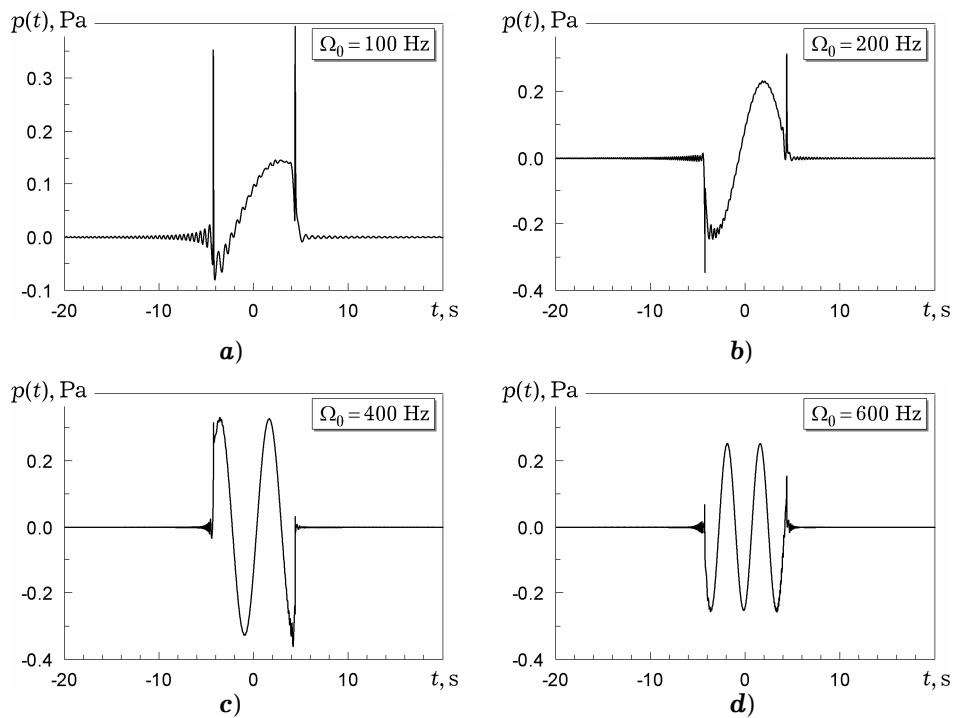


Fig. 1

Fig. 2 illustrates the time change of sound intensity

$$I(t) = 20 \lg(|p_{\text{tot}}(\mathbf{x}, t)|/p_0)$$

in dB, at $p_0 = 2 \cdot 10^{-5}$ Pa, calculated in the same place. The noise is emitted by train moving with velocity $v_0 = 150$ km/h and frequency $\Omega_0 = 100$ Hz. It is shown that sound intensity is characterized by high level, approximately 80 dB, for time of running in immediately near point of observation. In the moments of coming to and leaving of the train sound intensity gets sudden increasing connected with the forward and back head acoustical waves effect.

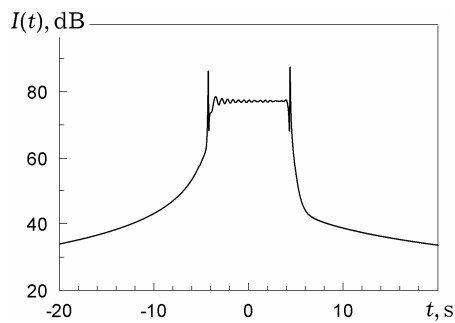


Fig. 2

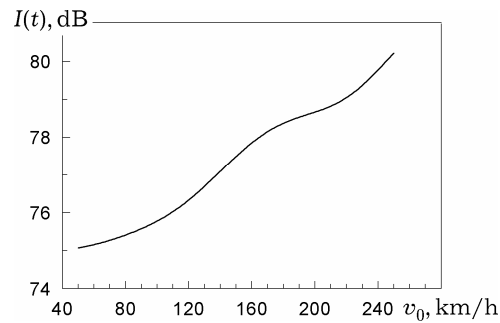


Fig. 3

The intensity of noise generated from train depends essentially on velocity of motion. From Fig. 3 may be noted that with increasing of train velocity the noise level increases, but non-monotonic only with some variations. The calculations refer to the sound intensity in time moment $t = 0$ s, i.e. when train is located symmetrically with respect to point of observation, and for velocities changing from 50 to 250 km/h.

The space distribution of sound intensity in the form of isolines on the spaciousness $x \times y = 400 \times 400 \text{ m}^2$ ($z = 10 \text{ m}$) near railroad at four time moments is displayed on Figs. 4. Here the frequency of vibrations is $\Omega_0 = 100 \text{ Hz}$ and the velocity of train motion is $v_0 = 150 \text{ km/h}$. The numerical results show that train motion is characterized by the two fronts of sound intensity, forward and back. Between these fronts noise is uniform with high level nearly 85 dB. Before the train and, in particular, outside the acoustic field is subjected to the considerable influence of interference (changes of wave phases), but have low, nearly 55 dB level.

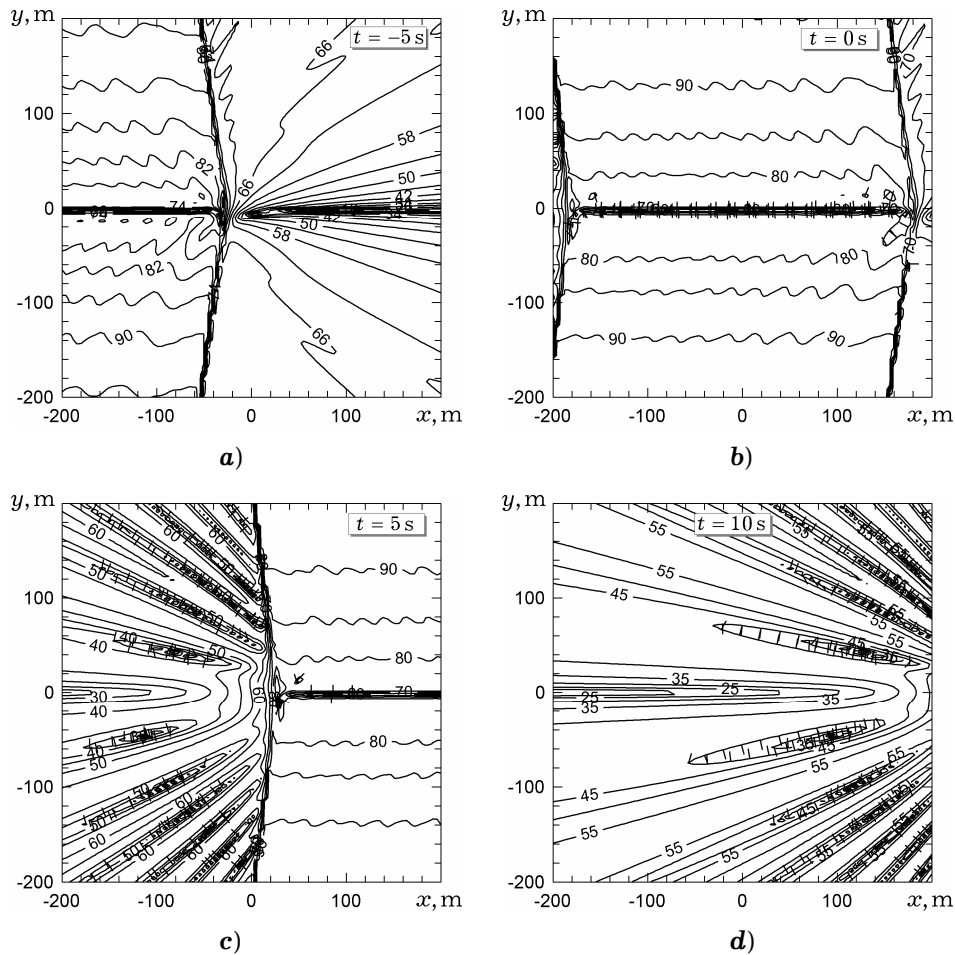


Fig. 4

Fig. 5 displays the space distribution of sound intensity $I_{av}(x, y)$ time-averaged per $T = 20 \text{ s}$:

$$I_{av}(\mathbf{x}) = 10 \lg \left\{ \frac{1}{T} \int_0^T \left[\frac{p_{tot}(\mathbf{x}, t)}{p_0} \right]^2 dt \right\}$$

on the same plane $x \times y = 400 \times 400 \text{ m}^2$, but for the height $z = 5 \text{ m}$. The calculations show the high level of averaging sound intensity, distribution of which is characterized by T -similar lines of noise intensity concentration. Also it is shown that in the limits of calculations high level locates near sides of this spaciousness.

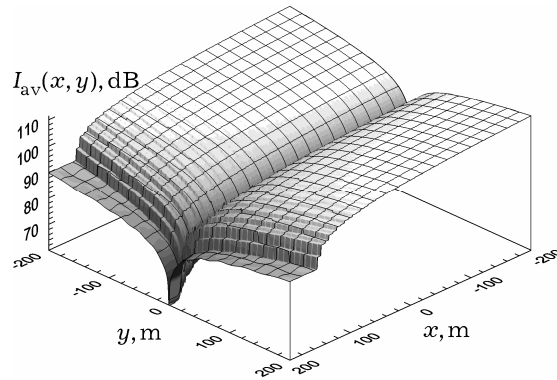


Fig. 5

3. Conclusions. The mathematical modeling of railroad noise shows that structure of acoustical signal radiated from moving train is very complex and non-homogeneous. The signals observed at some distance from wheel-gauge are presented in the form of acoustical pressure impulses with finite duration and determined by velocity of train, its length and intensity of force vibrations. It is noted that this signal is amplitude modulate, and absolute value of amplitude varies in the limits from 0.2 to 0.4 Pa. As it turned out, the train plays a part of linear resonator of low-frequency acoustic waves.

It have been confirmed the empirical noted tendency of sound intensity increasing with increasing of velocity, but non-linearly, as nonmonotonic function of train velocity.

The spaciousness of noise from fast moving train is divided upon the three sub-spaces, i.e. in front, in limits of moving object and after its, that is characterized by appropriate sound intensity level. The interface boundaries are oblique fronts of acoustical pressure jumps. In the middle sub-spaciousness, which dimensions reach several hundreds meters from wheel-gauge, instantaneous noise intensity is most significant with value more than 85 dB.

From numerical analysis it arises that acoustic intensity averaged per some time interval and calculated for train moving with velocity 150 km/h and frequency 100 Hz, at distance 200 m from railroad achieves more than 100 dB.

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ВИПРОМІНЮВАННЯ ЗВУКУ ПОТЯГОМ

Розглядається задача про випромінювання звуку потягом. Об'єкт моделюється системою точкових джерел звуку, неперервно розподілених в області рухомого видовженого прямокутника. Розв'язок задачі одержано з використанням інтегрального перетворення Фур'є за просторовими координатами і часом. Інтеграл обчислюється з застосуванням методу стаціонарної фази. Числовий аналіз проведено для акустичного тиску та інтенсивності звуку.

ИЗЛУЧЕНИЕ ЗВУКА ПОЕЗДОМ

Рассматривается задача излучения звука поездом. Объект моделируется системой точечных источников звука, непрерывно распределенных в области движущегося вытянутого прямоугольника. Решение задачи получено с использованием интегрального преобразования Фурье по пространственным координатам и времени. Интегралы вычисляются с применением метода стационарной фазы. Численный анализ выполнен для акустического давления и интенсивности звука.

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Received
26.08.09