UDK 539.3

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NON-AXISYMMETRIC SOLUTIONS TO TIME-FRACTIONAL HEAT CONDUCTION EQUATION IN A HALF-SPACE IN CYLINDRICAL COORDINATES

Non-axisymmetric solutions to time-fractional heat conduction equation with a source term in cylindrical coordinates are obtained for a half-space. The solutions are found using the Laplace transform with respect to time, the Hankel transform with respect to the radial coordinate, the finite Fourier transform with respect to the angular coordinate, and sine or cosine Fourier transform with respect to the bulk coordinate. Numerical results are illustrated graphically.

1. Introduction. The conventional thermoelasticity is based on the principles of the classical theory of heat conductivity, specifically on the classical Fourier law, which relates the heat flux vector to the temperature gradient. Nonclassical theories of heat conduction and generalized theories of thermoelasticity, in which the Fourier law and the standard heat conduction equation are replaced by more general equations, constantly attract the attention of the researchers [2, 5–7, 12, 13, 15–19, 21, 25, 32, 37, 39, 47, 48, 50, 51, 54]. Time-nonlocal dependence between the heat flux vector and the temperature gradient with the «long-tale» power kernel [37, 47, 48] (see also [11]) yields the time-fractional heat conduction equation:

$$\frac{\partial^\alpha T}{\partial t^\alpha} = a\,\Delta T\,. \tag{1}$$

Equation (1), called also as the diffusion-wave equation, is a mathematical model of important physical phenomena ranging from amorphous, colloid, glassy and porous materials through fractals, percolation clusters, random and disordered media to comb structures, dielectrics and semiconductors, polymers and biological systems. For an extensive bibliography on this subject see [4, 9, 26, 28–30, 55, 57] and references therein. Theory of thermoelasticity based on Eq. (1) was proposed in [37] and developed in the subsequent papers [38–41, 43, 44, 47, 48, 50, 51]. To study the thermoelasticity problems in the framework of fractional theory of thermal stresses it is necessary to solve the fractional heat conduction equation.

The fundamental solutions for the fractional diffusion-wave equation in one space-dimension were obtained by Mainardi [27]. Wyss [56] obtained the solutions to the Cauchy problem in terms of H-functions using the Mellin transform. Schneider and Wyss [53] converted the diffusion-wave equation with appropriate initial conditions into the integrodifferential equation and found the corresponding Green functions. Fujita [8] treated integrodifferential equation which interpolates the heat conduction equation and the wave equation. Hanyga [14] studied Green functions and propagator functions in one, two and three dimensions. Previously, in studies concerning time-fractional diffusion-wave equation in cylindrical coordinates only one or two spatial coordinates have been considered [20, 23, 24, 31, 33–35, 37, 38, 42, 45, 46, 49, 52]. In this paper, we study solutions to time-fractional heat conduction equation in a half-space in cylindrical coordinates in the case of three spatial coordinates r, φ, and z.

2. Dirichlet boundary condition. Consider the time-fractional heat conduction equation with a source term in cylindrical coordinates

$$\frac{\partial^\alpha T}{\partial t^\alpha} = a\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \varphi^2} + \frac{\partial^2 T}{\partial z^2}\right) + Q(r,\varphi,z,t)\,, \tag{2}$$

$$0 \le r < \infty,\quad 0 \le \varphi \le 2\pi,\quad 0 < z < \infty,\quad 0 < t < \infty,\quad 0 < \alpha \le 2\,,$$

under initial conditions

$$t = 0: \qquad T = f(r, \varphi, z), \qquad 0 < \alpha \le 2, \tag{3}$$

$$t = 0: \qquad \frac{\partial T}{\partial t} = F(r, \varphi, z), \qquad 1 < \alpha \le 2, \tag{4}$$

and the Dirichlet condition with the prescribed value of the temperature at the boundary

$$z = 0: \qquad T = g(r, \varphi, t), \qquad 0 < \alpha \le 2. \tag{5}$$

In Eq. (1) $\dfrac{\partial^\alpha T}{\partial t^\alpha}$ is the Caputo derivative of the fractional order α [1, 10, 22, 36]:

$$D_C^\alpha f(t) = \frac{d^\alpha f(t)}{dt^\alpha} = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha-1} \frac{d^n}{d\tau^n} f(\tau)\, d\tau, \qquad n-1 < \alpha < n,$$

where $\Gamma(x)$ is the gamma-function.

For Laplace transform rule the Caputo derivative requires the knowledge of the initial values of the function $f(t)$ and its integer derivatives of the order $k = 1, 2, \ldots, n-1$:

$$L\{D_C^\alpha f(t)\} = s^\alpha L\{f(t)\} - \sum_{k=0}^{n-1} f^{(k)}(0^+) s^{\alpha-1-k}, \qquad n-1 < \alpha < n.$$

The solution to the initial-boundary-value problem (2)–(5) can be written as

$$T = \int_0^\infty \int_0^{2\pi} \int_0^\infty \rho f(\rho, \phi, \zeta) U_f(r, \varphi, z, \rho, \phi, \zeta, t)\, d\rho\, d\phi\, d\zeta +$$

$$+ \int_0^\infty \int_0^{2\pi} \int_0^\infty \rho F(\rho, \phi, \zeta) U_F(r, \varphi, z, \rho, \phi, \zeta, t)\, d\rho\, d\phi\, d\zeta +$$

$$+ \int_0^t \int_0^\infty \int_0^{2\pi} \int_0^\infty \rho Q(\rho, \phi, \zeta, \tau) U_Q(r, \varphi, z, \rho, \phi, \zeta, t-\tau)\, d\rho\, d\phi\, d\zeta\, d\tau +$$

$$+ \int_0^t \int_0^{2\pi} \int_0^\infty g(\rho, \phi, \tau) U_g(r, \varphi, z, \rho, \phi, t-\tau)\, d\rho\, d\phi\, d\tau, \tag{6}$$

where the corresponding fundamental solutions are found using the Laplace integral transform with respect to time t, the Hankel transform with respect to the radial coordinate r, the finite Fourier transform with respect to the angular coordinate φ, and the sin-Fourier transform with respect to the spatial coordinate z:

$$U_f(r, \varphi, z, \rho, \phi, \zeta, t) = \frac{2}{\pi^2} \sum_{n=0}^{\infty}{}' \int_0^\infty \int_0^\infty E_\alpha\left[-a(\xi^2 + \eta^2)\, t^\alpha\right] \times$$

$$\times \cos[n(\varphi - \phi)] J_n(r\xi) J_n(\rho\xi) \sin(z\eta) \sin(\zeta\eta) \xi\, d\xi\, d\eta,$$

$$U_F(r, \varphi, z, \rho, \phi, \zeta, t) = \frac{2t}{\pi^2} \sum_{n=0}^{\infty}{}' \int_0^\infty \int_0^\infty E_{\alpha,2}\left[-a(\xi^2 + \eta^2) t^\alpha\right] \times$$

$$\times \cos[n(\varphi - \phi)] J_n(r\xi) J_n(\rho\xi) \sin(z\eta) \sin(\zeta\eta) \xi\, d\xi\, d\eta,$$

$$U_Q(r,\varphi,z,\rho,\phi,\zeta,t)=\frac{2t^{\alpha-1}}{\pi^2}\sum_{n=0}^{\infty}{}'\int_0^{\infty}\int_0^{\infty}E_{\alpha,\alpha}\left[-a(\xi^2+\eta^2)t^{\alpha}\right]\times$$

$$\times\cos[n(\varphi-\phi)]J_n(r\xi)J_n(\rho\xi)\sin(z\eta)\sin(\zeta\eta)\xi\,d\xi\,d\eta\,,$$

$$U_g(r,\varphi,z,\rho,\phi,t)=\frac{2at^{\alpha-1}}{\pi^2}\sum_{n=0}^{\infty}{}'\int_0^{\infty}\int_0^{\infty}E_{\alpha,\alpha}\left[-a(\xi^2+\eta^2)t^{\alpha}\right]\times$$

$$\times\cos[n(\varphi-\phi)]J_n(r\xi)J_n(\rho\xi)\sin(z\eta)\xi\eta\,d\xi\,d\eta\,.$$

Here the prime near the summation symbol means that the term corresponding to $n=0$ should be multiplied by $1/2$; $J_n(r)$ is the Bessel function of the first kind of the order n, and

$$E_{\alpha,\beta}(z)=\sum_{n=0}^{\infty}\frac{z^n}{\Gamma(\alpha n+\beta)},\quad \alpha>0,\quad \beta>0,\quad z\in\mathbb{C}\,,$$

is the generalized Mittag-Leffler function in two parameters α and β [3, 10, 22, 36]. The essential role of the Mittag-Leffler functions in fractional calculus results from the following formula for the inverse Laplace transform [3, 10, 22, 36]:

$$L^{-1}\left\{\frac{s^{\alpha-\beta}}{s^{\alpha}+b}\right\}=t^{\beta-1}E_{\alpha,\beta}(-bt^{\alpha})\,.$$

In Figs 1 and 2 we present dependence of the fundamental solution $U_g(r,\varphi,z,\rho,\phi,t)$ on the radial coordinate r and the angular coordinate φ, respectively, for $\phi=0$, $z/\rho=0.4$ and $\sqrt{a}\,t^{\alpha/2}/\rho=0.5$. In Fig. 1 we have chosen $\varphi=0$; in Fig. 2 the value $r/\rho=1$ is assumed. In the Dirichlet boundary condition (5)

$$z=0:\qquad T=g_0\frac{1}{r}\delta(r-\rho)\delta(\varphi-\phi)\delta_{+}(t)$$

we have introduced a constant g_0 to obtain the nondimensional quantity

$$\bar{U}_g=\frac{\rho^4}{g_0 a\,t^{\alpha-1}}U_g$$

displayed in Figures.

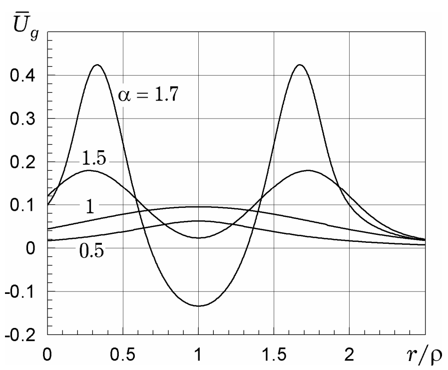

Fig. 1

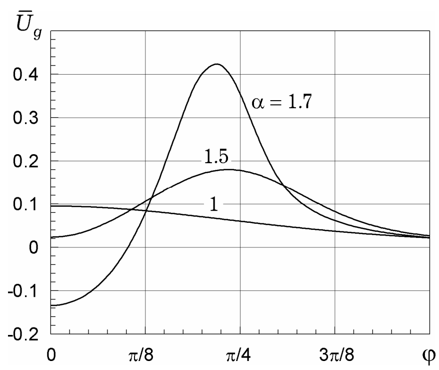

Fig. 2

3. Neumann boundary condition. Consider equations (2)–(4) under Neumann boundary condition at a surface $z = 0$. For fractional heat conduction equation, two types of Neumann boundary condition can be considered: the mathematical condition with the prescribed boundary value of the normal derivative of the temperature

$$z = 0: \qquad \frac{\partial T}{\partial z} = -G(r, \varphi, t), \qquad 0 < \alpha \le 2, \tag{7}$$

and the physical condition with the prescribed boundary value of the matter flux [5, 47, 48]

$$z = 0: \qquad D_{\mathrm{RL}}^{1-\alpha} \frac{\partial T}{\partial z} = -G(r, \varphi, t), \qquad 0 < \alpha \le 1, \tag{8}$$

$$z = 0: \qquad I^{\alpha-1} \frac{\partial T}{\partial z} = -G(r, \varphi, t), \qquad 1 < \alpha \le 2, \tag{9}$$

where $D_{\mathrm{RL}}^{1-\alpha}$ and $I^{\alpha-1}$ are the Riemann – Liouville fractional derivative and integral [1, 3, 10, 22, 36], respectively.

The integral transforms technique applied to equations (2)–(4), (7)–(9) leads to representation of solution similar to (6) with the following fundamental solutions:

$$U_f(r, \varphi, z, \rho, \phi, \zeta, t) = \frac{2}{\pi^2} {\sum_{n=0}^{\infty}}' \int_0^\infty \int_0^\infty E_\alpha\left[-a(\xi^2 + \eta^2)t^\alpha\right] \times$$

$$\times \cos[n(\varphi - \phi)] J_n(r\xi) J_n(\rho\xi) \cos(z\eta) \cos(\zeta\eta) \xi \, d\xi \, d\eta ,$$

$$U_F(r, \varphi, z, \rho, \phi, \zeta, t) = \frac{2t}{\pi^2} {\sum_{n=0}^{\infty}}' \int_0^\infty \int_0^\infty E_{\alpha,2}\left[-a(\xi^2 + \eta^2)t^\alpha\right] \times$$

$$\times \cos[n(\varphi - \phi)] J_n(r\xi) J_n(\rho\xi) \cos(z\eta) \cos(\zeta\eta) \xi \, d\xi \, d\eta ,$$

$$U_Q(r, \varphi, z, \rho, \phi, \zeta, t) = \frac{2t^{\alpha-1}}{\pi^2} {\sum_{n=0}^{\infty}}' \int_0^\infty \int_0^\infty E_{\alpha,\alpha}\left[-a(\xi^2 + \eta^2)t^\alpha\right] \times$$

$$\times \cos[n(\varphi - \phi)] J_n(r\xi) J_n(\rho\xi) \cos(z\eta) \cos(\zeta\eta) \xi \, d\xi \, d\eta ,$$

$$U_G(r, \varphi, z, \rho, \phi, t) = \frac{2at^{\alpha-1}}{\pi^2} {\sum_{n=0}^{\infty}}' \int_0^\infty \int_0^\infty E_{\alpha,\alpha}\left[-a(\xi^2 + \eta^2)t^\alpha\right] \times$$

$$\times \cos[n(\varphi - \phi)] J_n(r\xi) J_n(\rho\xi) \cos(z\eta) \xi \, d\xi \, d\eta .$$

$$U_j(r, \varphi, z, \rho, \phi, t) = \frac{2a}{\pi^2} {\sum_{n=0}^{\infty}}' \int_0^\infty \int_0^\infty E_\alpha\left[-a(\xi^2 + \eta^2)t^\alpha\right] \times$$

$$\times \cos[n(\varphi - \phi)] J_n(r\xi) J_n(\rho\xi) \cos(z\eta) \xi \, d\xi \, d\eta .$$

Figs 3 and 4 show dependence of the fundamental solution $U_G(r, \varphi, z, \rho, \phi, t)$ to the mathematical Neumann boundary-value problem on the radial coordinate r and the angular coordinate φ, respectively, for $\phi = 0$, $z = 0$, and $æ = 0.5$. In Fig. 3 the value $\varphi = 0$ has been chosen; in Fig. 4 we have assumed $r/\rho = 1$. The corresponding results to the physical Neumann boundary-value problem are depicted in Figs 5 and 6. The non-dimensional quantity are introduced as follows

$$\bar{U}_G = \frac{\rho^3}{G_0 a t^{\alpha-1}} U_G, \qquad \bar{U}_j = \frac{\rho^3}{j_0 a} U_j .$$

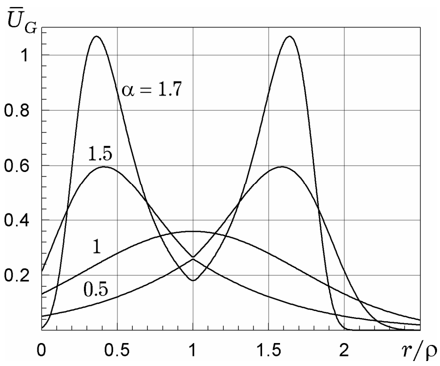

Fig. 3

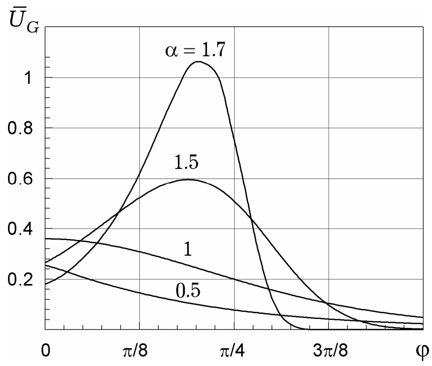

Fig. 4

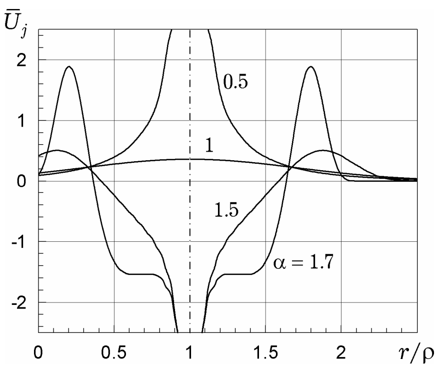

Fig. 5

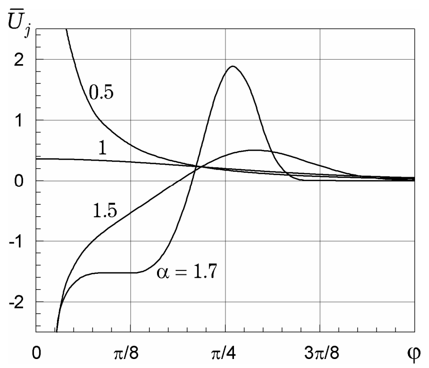

Fig. 6

4. Conclusions. The solutions to the Cauchy, source, Dirichlet and Neumann problems for time-fractional heat conduction equation have been found in a half-space in cylindrical coordinates. The considered equation in the case $0 < \alpha < 1$ interpolates the Helmholtz and heat conduction equation. The obtained solutions satisfy the appropriate initial and boundary conditions and reduce to the solutions of classical heat conduction equation in the limit $\alpha \to 1$. In the case $1 < \alpha < 2$ the time-fractional heat conduction equation interpolates the standard heat conduction equation and the classical wave equation. In the limit $\alpha \to 2$ the received solutions reduce to the solutions of wave equation. The solutions to the fractional heat conduction equation in the superdiffusion regime ($1 < \alpha < 2$) feature propagating humps, underlining the proximity to the standard wave equation in contrast to the shape of curves describing the subdiffusion regime ($0 < \alpha < 1$). It should be noted that theory of thermoelasticity based on time-fractional heat conduction equation proposed in [37] interpolates the classical theory of thermal stresses and that without energy dissipation put forward by Green and Naghdi [12].

In the axisymmetric case the corresponding results were investigated in [49]. The dependence of the fundamental solutions on the bulk coordinate z is similar both in axisymmetric and non-axisymmetric cases. For this reason and reason of space in this paper we have restricted ourselves to presenting only numerical results illustrating the dependence of solutions on the radial coordinate r and the angular coordinate φ. Curves depicted in Figures show unusual behavior of solutions in comparison with the corresponding solutions to the classical heat conduction equation.

Two types of the Neumann boundary condition have been considered. In the case of the classical heat conduction equation ($\alpha = 1$) these two types of condition are identical, but for fractional heat conduction equation, as is evident from Figures (compare Fig. 3 and Fig. 5 as well as Fig. 4 and Fig. 6), they are essentially different. Singularity of the fundamental solution U_j at a point of application of the delta pulse should be accentuated.

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НЕОСЕСИМЕТРИЧНІ РОЗВ'ЯЗКИ РІВНЯННЯ ТЕПЛОПРОВІДНОСТІ З ДРОБОВОЮ ПОХІДНОЮ ЗА ЧАСОМ У ПІВПРОСТОРІ В ЦИЛІНДРИЧНИХ КООРДИНАТАХ

Для півпростору отримано неосесиметричні розв'язки рівняння теплопровідності з дробовою похідною за часом. Для знаходження розв'язків використано перетворення Лапласа за часом t*, перетворення Ганкеля за радіальною координатою* r*, скінченне перетворення Фур'є за кутовою координатою* φ *і* $\sin$ *- або* $\cos$ *-перетворення Фур'є за просторовою координатою* z*. Числові результаті проілюстровано графіками.*

НЕОСЕСИММЕТРИЧНЫЕ РЕШЕНИЯ УРАВНЕНИЯ ТЕПЛОПРОВОДНОСТИ С ДРОБНОЙ ПРОИЗВОДНОЙ ПО ВРЕМЕНИ В ПОЛУПРОСТРАНСТВЕ В ЦИЛИНДРИЧЕСКИХ КООРДИНАТАХ

Для полупространства получены неосесимметричные решения уравнения теплопроводности с дробной производной по времени. Для нахождения решений используется преобразование Лапласа по времени t*, преобразование Ханкеля по радиальной координате* r*, конечное преобразование Фурье по угловой координате* φ *и* $\sin$ *- или* $\cos$ *-преобразование Фурье по пространственной координате* z*. Численные результаты иллюстрируются графиками.*

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