# GertJan van Heijst

## TO THE MEMORY OF VYACHESLAV VLADIMIROVICH MELESHKO



This memorial paper is dedicated to Vyacheslav Vladimirovich Meleshko, professor at the Faculty of Mechanics and Mathematics of the Taras Shevchenko National University of Kiev, who passed away after a tragic accident in Kiev on 14 November 2011.

Slava (as he was known to his friends) Meleshko was born on 7 October 1951 in Dnepropetrovsk, Ukraine. After having studied at the Faculty of Mechanics and Mathematics of the Shevchenko Kiev State University he defended his PhD thesis in physics and mathematics at the Institute of Mechanics of the National Academy of Sciences, also in Kiev, in 1976. A few years later, in 1984, Slava Meleshko defended his Doctor's thesis on 'Laws of steady state wave processes in finite elastic bodies and wave guides' at the Lomonosov Moscow State University. He was a respected member of the school of mechanics founded by Prof. Andrey Ulitko and Prof. Victor Grinchenko, corresponding member and academician, respectively, of the National Academy of Sciences (NAS) of Ukraine. After finishing his PhD thesis, Slava Meleshko worked at the Institute of Mechanics of the NAS, after which in 1982 he obtained a research position at the Institute of Hydromechanics of the NAS, also in Kiev. In 1992 he became head of the newly founded Department of Vortex Motion in that institute. Since 2002 Slava Meleshko was appointed as a professor and held a chair at (and later became head of) the Department of Theoretical and Applied Mechanics of the Faculty of Mechanics and Mathematics of the Taras Shevchenko National University of Kiev.

Professor Meleshko was an internationally recognized expert in fundamental aspects of the wide field of mechanics, in particular elasticity, fluid mechanics, and acoustics. In the field of fluid dynamics, Slava Meleshko has made important contributions to the theory of vortex dynamics, mixing, and chaotic advection. His profound background in methods of mathematical physics enabled him to make crucial steps in the analysis of various

fundamental problems. It is due to his extensive knowledge of the theory of elasticity that Slava Meleshko was able to make significant contributions to the analytical description of Stokes flows, both being governed by the biharmonic equation. This approach was typical for him: by combining disciplines he was able make significant step in the analysis of problems in fluid and solid mechanics.

Professor Meleshko also had a very extensive knowledge of historical work in these areas, having built a private archive containing a large number of remarkable historical papers. Together with the late professor Hassan Aref he published a complete 'Bibliography of Vortex Dynamics' over the period 1858–1956, so starting with the publication of von Helmholtz's celebrated landmark paper on the subject (Meleshko & Aref [12], 2007). By his colleagues and friends Slava Meleshko was often referred to as a 'walking encyclopedia', not only in the area of the history of vortex dynamics, but also of fluid and solid mechanics in general.

Slava Meleshko was actively involved in scientific collaborations, with colleagues and students in Ukraine, but also with colleagues in various countries all over the world. In particular, in the latter category we should mention his scientific contacts and collaboration with the late Hassan Aref, with Luca Zannetti, and with the author of this paper. This collaboration was extended over a long period and has resulted in a number of remarkable joint papers. Working with Slava Meleshko was always a great pleasure, and warm friendship relationships developed throughout the years.

General, eternal human values formed an important basis for Slava Meleshko's thinking and inter-human behaviour, in his family, among his friends and colleagues, and in life in general. His friendly and interested attitude towards people, his humour, his laugh, his wide knowledge of literature and history, his open mind, his human warmth — this all made Slava Meleshko a very special, warm person.

Slava Meleshko was the loving husband of Dr Tatyana (Tanya) Sigizmundovna Krasnopolskaya and the father of their daughter Valeria Melechko. Quite remarkably, Tanya and Slava have been working closely together since the time they were both students in mechanics at Kiev National University. Their scientific careers have been parallel and very close during the years, and they have been involved in joint research on problems in solid and fluid mechanics. Since the early 1990s Tanya and Slava have been regular visitors, for shorter or longer periods, at the universities of Urbana-Champaign (USA), Eindhoven (NL) and also at Torino (It), to work with Hassan Aref, with the author of this paper (GJFvH), with Luca Zannetti, and with their colleagues. During those working visits Slava and Tanya often worked together as a scientist-couple, in most cases even sharing the same office. Owing to their open attitude, they both have established very warm relationships with many people, all over the world.

This paper is not intended to give a complete overview of the scientific achievements of Prof. Meleshko. Instead, I prefer to focus on a few topics that we worked on together, often jointly with colleagues in Kiev and Eindhoven or elsewhere. In this respect I will mention and briefly discuss a joint paper (Meleshko & van Heijst [14], 1994) on the early work of the Russian scientist Chaplygin, who carried out detailed analytical studies of an elliptical vortex in a shear flow and of a dipolar vortex structure with a continuous vorticity distribution. This work of Chaplygin was published in Russian in 1899 and 1903, respectively, and went unnoticed for a long time. The other paper that I want to mention briefly is a study of vortex interactions (Meleshko & van Heijst [15], 1994), which appeared as a special journal issue, edited by Hassan Aref. Finally, I will also highlight some unpublished work of Slava Meleshko and his colleagues on the behaviour of vortex structures in the vicinity of solid obstacles, with or without sharp edges.

These three topics in vortex dynamics mainly serve to illustrate the scientific background and interests of Slava Meleshko: history, fundamental issues, and elegant mathematical analysis.

# 1. Two papers: on Chaplygin and on vortex interactions

## 1.1. On Chaplygin's work

During his active search in historical works on vortices, Slava Meleshko became aware of the pioneering work that had been carried out by the Russian scientist Sergey A. Chaplygin around the turn of the 19<sup>th</sup> century. Among others, he published two remarkable papers, one on an elliptical vortex in a shear flow (Chaplygin [3], 1899) and one on a dipolar vortex structure (Chaplygin [4], 1903). Most likely because these papers were published in Russian, they went unnoticed by (western) scientists working on vortex dynamics.

For the elliptical vortex Chaplygin assumed a patch of uniform vorticity distributed over an elliptical area with semi-axes a(t) and b(t) that could in principle change in time, but such that the elliptical area is conserved:  $a(t) \cdot b(t) = a_0 b_0 = \text{const.}$  The orientation of the major axis with respect to a fixed coordinate frame was denoted by the time-dependent angle  $\phi(t)$ . The shear outside the elliptical patch was also assumed to have a uniform vorticity. For this inviscid flow problem Chaplygin derived the following solution

$$\frac{da}{dt} = -2aA\sin\phi\cos\phi\,, (1a)$$

$$\frac{d\phi}{dt} = 2A \frac{a^2 \sin^2 \phi - b^2 \cos^2 \phi}{a^2 - b^2} + \frac{2\omega ab}{(a+b)^2},$$
(1b)

with 2A being the vorticity of the shear flow and  $2\omega$  the additional vorticity of the vortex patch. For the case of zero external shear flow (A=0) this solution reveals a uniform rotation of the ellipse with a fixed shape ( $a=a_0$ ,  $b=b_0$ ), with an angular speed given by

$$\frac{d\phi}{dt} = \frac{2\omega a_0 b_0}{(a_0 + b_0)^2}.$$
 (2)

This result agrees with the classical 'Kirchhoff vortex', which was formulated in 1876. Furthermore, Chaplygin's general solution revealed more complicated time-dependent behaviour of the elliptical patch, depending on the ratio  $\omega/A$  (>0). One type of behaviour is a continuous rotation with a non-constant angular velocity and periodically changing aspect ratio  $z(t) \equiv b(t)/a(t)$ .

A different type of motion according to the Chaplygin solution is an oscillation of the major axis around some mean orientation, while the eccentricity z(t) shows an oscillation with the same frequency: the ellipse performs a nutation while showing shape oscillations. For a specific choice of parameters even a steady vortex patch is possible. A third type of behavior is the shape pulsation, in which the shape of the patch changes continuously from an ellipse to a circle and vice versa.

Although Chaplygin's 1899 paper was mentioned in the review papers by Love [11, p. 123] (1901) and Auerbach [1, p. 1061] (1908), it further seems to have escaped the attention of the later fluid dynamics community. Much later, Moore & Saffman [17] (1971) and Kida [8] (1981) published work on a similar problem of an elliptical vortex patch in a strain/shear flow, apparently without being aware of the much earlier work by Chaplygin. An account of Chaplygin's analysis of this vortex problem was presented by Meleshko & van Heijst [14] (1994).

In the same paper, another remarkable work by Chaplygin was discussed, namely his analysis of the motion associated with a compact dipolar vorticity distribution in a two-dimensional unbounded inviscid flow. As one case, Chaplygin [4] (1903) considered in detail a rectilinear motion of a circular vortex (radius a) with a constant translation speed  $v_0$ . By superimposing a uniform velocity  $-v_0$  a stationary problem was obtained of a steady vortex structure placed in a potential flow with uniform velocity at infinity. By choosing a polar coordinate system  $(r,\theta)$ , with the origin in the centre of the vortex structure and  $\theta=0$  corresponding with the external flow direction, the stream function  $\psi$  of the flow inside the circle r=a has to satisfy

$$\frac{\partial^2 \Psi}{\partial r^2} + \frac{1}{r} \frac{\partial \Psi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \Psi}{\partial \theta^2} = -\omega = f(\Psi), \qquad (3)$$

where  $f(\psi)$  is an arbitrary function of  $\psi$ . Although a very similar problem was considered by Lamb [9] (1895), who suggested to adopt a linear relationship  $f(\psi) = -k^2 \psi$  (with k a constant), but without giving further details of the solution, Chaplygin also assumed a linear relationship between  $\omega$  and  $\psi$ , and arrived at the solution

$$\psi(r,\theta) = \frac{2v_0 a}{bJ_1'(b)} J_1\left(\frac{br}{a}\right) \sin \theta, \qquad r \le a, \tag{4}$$

with b=3.8317 the smallest positive root of the first-order ordinary Bessel function, i.e. of  $J_1(b)=0$ .

This solution by Chaplygin is identical to that outlined by Lamb [9] (1895) and further elaborated on by Lamb [10] (1906). Because no references were made to each other's work, it may be assumed that Chaplygin and Lamb arrived independently at the same vortex dipole solution. Although Lamb [10] (1906) devoted only a few sentences to the dipole solution, Chaplygin [4] (1903) presented a much more detailed analysis of this flow structure. For example, Chaplygin also considered a 'semi-cylindrical' vortex moving along a solid free-slip wall (see Fig. 1b). Moreover, Chaplygin generalized his solution to an asymmetric dipole structure satisfying the relationship  $\omega = -f(\psi) = n^2(\psi - \lambda)$ , with n = b/a and  $\lambda$  an arbitrary constant, see Fig. 1c.

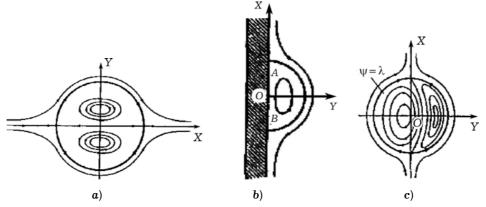


Fig. 1. Schematic drawings of: *a*) the symmetric dipole vortex; *b*) the dipole structure moving along a free-slip wall; and *c*) the asymmetric vortex dipole, according to Chaplygin [3].

Because of the pioneering work of both scientists on the analysis of this inviscid vortex structure, it was suggested to refer to this solution as the 'Chaplygin – Lamb dipole'.

#### 1.2. Vortex interactions

An intriguing aspect of 2D vortex dynamics is the interaction of vortex structures. It is well known that dipolar vortices may be formed as the result of two interacting vortices of opposite circulations, and that interacting vortex dipoles may display fascinating behaviour as exchange scattering, direct scattering, or even mutual trapping into a tripolar structure. Numerical simulations and laboratory experiments have elucidated many aspects of this complicated behaviour, including the stirring properties of such vortices while interacting. However, also simple point-vortex models of such interactions may yield important insight in these advection properties, as is nicely illustrated by Meleshko & van Heijst [15] (1994). In that paper, point-vortex models were used to investigate the interaction of two non-aligned vortex dipoles and also the interaction between a dipole with a monopolar vortex. These point-vortex simulations were motivated by laboratory experiments on such 2D vortex interactions in a stratified fluid, in which the flow was visualized by dye. The evolving dye patterns clearly revealed the complicated nature of the stirring induced by the interacting vortex structures. In order to study the advection properties of the interacting point vortices, Meleshko & van Heijst [15] (1994) employed the so-called contour-kinematics method, which is essentially based on the precise tracking of passive tracer points that are making up some closed contour defined at the start of the simulation. In an incompressible 2D flow, the area enclosed by any material contour is conserved, and this property provides an important tool to check the accuracy of the contour tracking during the simulation. Since generally the contours will be stretched and become convoluted, high-precision computations are required. For this purpose a special contour tracking algorithm was defined with a built-in accuracy check, see Meleshko et al. [13] (1992).

Whenever at some stage the distance between neighbouring points on the contour would become larger than some set value (due to stretching) or the angle between neighbouring segments of the contour becomes less than some prescribed value (when folding is present), additional marker points would be placed near those positions on the initial contour, and the simulation would be started all over again.

Quantitative information about the advection properties of interacting point-vortex configurations could be obtained by calculating (parts of) the areas enclosed by contours defined near or around the vortex structures in

the initial situation. For example, in the case of two colliding vortex dipoles this approach allowed to calculate how much of the area of the initial dipole 'atmosphere' would still be contained in the atmosphere of the eventual dipole structures. By comparing the contour-kinematics simulations with the dye-visualization experiments, it was demonstrated how accurate even simple point-vortex <sub>-10</sub> models are in describing the stirring properties of complicated vortex interactions: even at a detailed level the advection of the dye was captured very well by the contour-tracking method. An example is given in Fig. 2, which shows the evolution of a point-vortex dipole (initially consisting of point vortices 1 and 2) with a monopolar vortex (3). During the interaction vortex 2 paired with vortex 3 to form a new dipole (moving downwards in the picture),

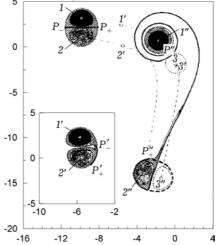


Fig. 2. Contour-kinematics simulation of the interaction of a point-vortex dipole (1, 2) with a single point vortex (3), showing 'exchange scattering' [13].

while leaving vortex 1 behind. The comparison with the dye visualization experiment (see Meleshko & van Heijst [15] (1994)) reveals a very good agreement.

Working on this paper together with Slava Meleshko was a great pleasure. His 'signature' is clearly visible, e.g. in the form of the quotations of various scientists at the beginning of each section of the paper.

# 2. Unfinished work: a vortex near a solid obstacle

An aspect of vortex dynamics that has been intriguing fluid dynamicists for many years is the behaviour of a vortex structure in close proximity of a solid obstacle. Slava Meleshko has also been fascinated by this problem, and some years ago a joint study with Shura Gourjii, Luca Zannetti and myself was started on the behaviour of a vortex dipole near a sharp-edged plate. This work is still unpublished, although a manuscript is in preparation (Meleshko et al. [16] (2013)).

When a vortex structure approaches a solid obstacle, it is likely to become affected by the vorticity generated at this object. The no-slip condition imposed by the solid wall implies the presence of a boundary layer containing oppositely-signed vorticity. Also, a sharp edge may result in flow separation and hence generate vorticity that may influence the behaviour of the vortex. In his numerical study of a vortex dipole colliding normally against an infinite flat, solid wall Orlandi [18] (1990) has demonstrated the crucial role played the boundary-layers at the wall, containing vorticity oppositely-signed to that of the primary vortex structures. After being advected away from the wall, these secondary vorticity patches pair with the primary vortex patches, resulting in the splitting of the primary dipole and the formation of two asymmetric dipoles that move away from the wall along curved paths. Depending on the initial flow conditions, these secondary vortex structures may collide and exchange partners, so that two new couples are formed: while one of these moves back towards the wall (likely to undergo a new collision), the other moves away from the solid wall. A similar behaviour was observed for the case of a vortex dipole moving towards a solid cylindrical obstacle, as reported by Verzicco et al. [20] (1995). When the cylinder diameter is comparable or larger than the dipole size, the wall curvature is not a major factor, and the dipole behaves as when colliding against a flat solid wall.

A crucial question is: what happens when the vortex dipole approaches obstacles of more complicated shapes? For example, what is the effect of a sharp edge of the obstacle on the vortex behaviour? How does the vortex dipole behave when approaching an opening in a flat plate, so when two sharp edges are present?

In the past, the problem of a vortex near a corner or a sharp edge has been addressed by quite a few authors. In an analytical approach, the problem has been formulated in terms of a potential flow problem, so completely ignoring any viscous effects. In such an approach a (potential) vortex close to the wall would travel with a constant speed parallel to the wall, as induced by its image. The potential flow near a corner between two solid walls or near the sharp edge of a plate can be conveniently mapped to the flow along a flat wall by applying a suitable conformal transformation. Not surprisingly, in such a potential-flow description the point vortex travelling along the wall towards the edge/corner would just follow the shape of the solid object, i.e. would just move around the corner or edge. Examples of such studies are presented by Karweit [7] (1975) and Sheffield (1977). It is obvious that such a modeling approach has a major drawback; a singularity exists at the corner /edge of the plate. In reality, flow separation would occur at such a point, resulting in a shear layer that rolls up into one or more secondary vortices. This problem of flow separation and edge-vortex formation has been addressed in the mixed analytical - numerical study by Meleshko et al. [16] (2013).

The problem configuration considered is shown in Fig. 3: in the x,y-plane a point-vortex dipole (with strengths  $\Gamma_1=-\Gamma_2=-\Gamma$ , separation distance b) approaches the sharp edge (corner angle  $\beta$ ) of a solid plate (x>0, y=0) at an offset distance  $x_0$ . By applying the conformal transformation  $z=\zeta^\alpha$ , with

 $\alpha = 2 - \beta/\pi$ , the problem is mapped from the complex z-plane to a point-vortex dipole normally colliding against a flat wall in the complex  $\zeta$ -plane. In order to satisfy the Kutta condition at the sharp edge (in other words: remove the singularity), an additional point vortex - termed the 'Kutta vortex' - is placed at some position close to the edge. As in the studies of Cortelezzi & Leonard [5] (1993) and Zannetti & Iollo [21] (2003), the time-dependent strength of this Kutta vortex is adjusted during the numerical simulation in order to model the rolling-up of the separated shear layer, in such a way that the Kutta condition (no singularity) at the edge is satisfied. The absolute value of the circulation of this Kutta vortex is thus a monotonically increasing function of time. At

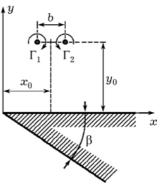
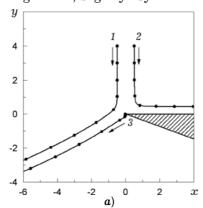


Fig. 3. The configuration of the point-vortex dipole approaching the sharp edge of a solid plate [16].

the moment when the Kutta condition would require the vortex strength to decrease, the circulation has reached a maximum; it is then frozen at that value, and the vortex is released. At the same time a new Kutta vortex is placed near the edge, again to satisfy the Kutta condition, and the whole procedure is repeated. This newly introduced Kutta vortex simulates the roll-up of the detached vortex sheet, now with oppositely-signed vorticity.

Examples of the calculated point-vortex trajectories are given in Figs 4 and 5. In these simulations most parameter values ( $\Gamma_1$ ,  $\Gamma_2$ , b,  $y_0$ , and  $\beta$ ) were kept fixed, while only the offset distance  $x_0$  was varied. Fig. 4 shows the trajectories for the case  $x_0 = 0.0$ . As the dipole (vortices 1 and 2) approaches the plate, it is seen to split, with the positive vortex (2) moving along the plate, away from the edge. The strength  $\Gamma_3$  of the Kutta vortex increases steadily towards an asymptotic value (see panel (b)), until this vortex is released and pairs with the negative point vortex (1) of the original dipole, now forming a new, slightly asymmetric dipole that moves away from the edge.



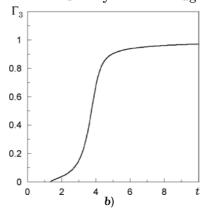
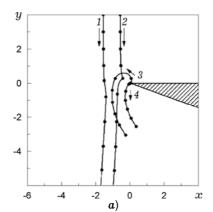


Fig. 4. Point-vortex simulation of a dipole approaching the sharp edge of a plate for  $\Gamma_1==-\Gamma_2=1.0\,,\;b=1.0\,,\;\beta=20^\circ\,,y=4.0\,,$  and offset distance  $x_0=0.0\,:\,a$ ) calculated trajectories of the original vortices (1, 2) and the Kutta vortex (3); b) evolution of the strength  $\Gamma_3$  of the Kutta vortex [16].



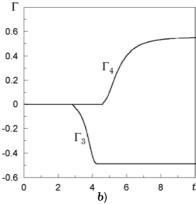


Fig. 5. As Fig. 4, but now for an offset distance  $x_0=-1.1$ . In this case two Kutta vortices are formed, whose circulations  $\Gamma_3(t)$  and  $\Gamma_4(t)$  are shown in panel (b) [16].

For a larger offset  $x_0=-1.1$  (see Fig. 5), the dipole does not split up, but instead passes along the plate edge while undergoing only a marginal change. At the plate edge, two Kutta vortices are produced: at first a vortex (3) is formed with negative circulation, which is soon followed by another secondary vortex (4) of positive circulation. Together, these vortices 3 and 4 form an asymmetric secondary dipole that slowly moves away from the edge along a curved trajectory.

Additional simulations have been carried out with the contour-tracking method, and some results for a small offset value ( $x_0 = -0.5$ ) are displayed in Fig. 6. The behaviour of the values is similar to that observed in Fig. 4, in which the dipole splits up into one vortex moving along the plate in a direction away of the edge, and one vortex pairing with the Kutta vortex produced at the edge. The contour evolution nicely reveals the advection of tracer material (indicated in grey) during the interaction process, in particular the spiraling structure of the filament around the Kutta vortex.

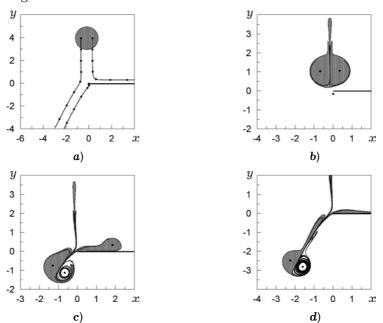


Fig. 6. Sequence of contour tracking simulations of the point-vortex dipole approaching the sharp-edged plate with parameter values as in Fig. 4 and offset distance  $x_0=-0.5$  [16].

As part of this study, the behaviour of a dipolar vortex near the edge of the plate has also been investigated in laboratory experiments in a rotating fluid tank. Again, the background rotation was employed to ensure that the flow was in good approximation 2D. The dipolar vortex was generated by horizontally moving a thin-walled open cylinder through the fluid, while gradually lifting it out. Fig. 7 shows a sequence of dye-visualization snapshots taken at successive stages of the dipole collision for the case of an offset  $x_0 \approx 0$ . It can be clearly observed that the dipolar vortex splits into two halves, with the positive half pairing with the negative secondary vortex formed at the plate edge and hence moving away. The other half of the original dipole is arrested by the plate. At this no-slip wall a boundary layer is formed in which oppositely-signed (positive) vorticity is built up. This wall-produced vorticity is advected away from the wall, forming a patch that pairs with the negative primary vortex to form an asymmetric dipole that moves along a curved path.

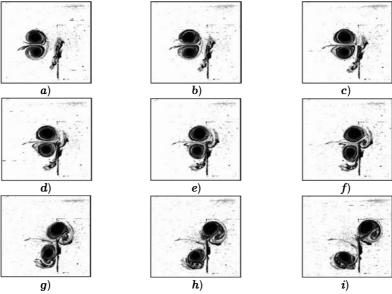


Fig. 7. Dye-visualization experiment showing the evolution of a quasi-2D vortex dipole in a rotating fluid approaching a sharp-edged plate at an offset distance  $x_0 \approx 0$  [16].

Apart from the latter aspect (which is entirely due to the no-slip character of the solid wall), the flow evolution of the vortex dipole encountering the plate edge is very similar to what is predicted by the point-vortex model with the Kutta vortex. This shows once more that – despite its simplicity – the point-vortex model captures the essential features of the flow remarkably well.

# 3. Some concluding remarks

As indicated before, the discussion of the material in the preceding sections was merely meant to give an impression of the topics Slava Meleshko was interested in, and of his approach to such problems. The selection of the three papers is biased by my personal involvement in this joint work with Slava, which in my opinion nicely illustrates his deep interest in historical aspects of science, his extensive interest in fundamental issues, and his mathematical approach. Of course, we could have made a different selection of his extensive list of publications, e.g. in the field of elasticity problems, wave phenomena, viscous and inviscid stirring and mixing, or Stokes flows. Giving a complete overview of his work in these different areas was not the main aim of this memorial paper. For those readers wishing to have a detailed overview of Slava Meleshko's scientific oeuvre, the list of publications and books at the end of this article may be valuable.

In January 2011 a workshop entitled 'Physics of Mixing' was held at the Lorentz Center in Leiden, the Netherlands. The meeting was attended by scientists working on various aspects of stirring and mixing problems, including Hassan Aref and Slava Meleshko. During the workshop a photograph was taken (by Slava's wife Tanya Krasnopolskaya) of Hassan, Slava and the author of this article, with Hassan sitting at the desk of Lorentz. This was the last time the three of us were together. On 9 September 2011, Hassan Aref died suddenly at his home, while sitting in his favourite chair. After this unexpected, sad event the initiative was taken to write a memorial paper dedicated to Hassan Aref, with Slava acting as coordinator. In particular during the last weeks before the deadline for the submission of this obituary paper, the email correspondence between the authors (Slava Meleshko, Mark Stremler, Alexey Borisov, GertJan van Heijst; see Borisov et al. [2] (2011)) was intensive and very frequent, with many email exchanges each day. On the afternoon of 14 November 2011, the last day before the deadline, Slava reminded his co-authors that the paper had to be completed and submitted the next day at noon. Returning home after his working day at the university he was killed in a tragic accident.



While writing this text I realized more and more how sorely we miss our unique colleague and warm friend Slava Meleshko.

Acknowledgement. The work referred to in section 2 will be published in Meleshko et al. [16], and contains important contributions from Shura Gourjii and Luca Zannetti. The laboratory experiments described in that paper were conducted by our former student Paul Berkvens.

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## **APPENDIX**

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