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DYNAMIC STRESSES DUE TO TIME-HARMONIC ELASTIC WAVE INCIDENCE ON DOUBLY PERIODIC ARRAY OF PENNY-SHAPED CRACKS

The symmetric frequency-domain problem on the interaction effects in rectangular lattice system of coplanar penny-shaped cracks located in an infinite elastic solid is numerically investigated. The problem is reduced to a boundary integral equation for the crack-opening-displacement in a unit cell by means of 3D periodic elastodynamic Green's function. This function is adopted for the effective calculation by its representation in the form of exponentially-convergent Fourier integrals. A collocation method is used for the solution of the boundary integral equation. Numerical results for the mode-I dynamic stress intensity factor in the crack vicinities depending on the wave number and the lattice size are obtained and analysed.

Introduction. Investigation of different kinds of periodic structures such as phononic crystals, especially the elastic wave propagation in such structures, is of great importance because of their potential engineering applications [4, 8, 9]. In many cases periodic systems of cracks can act as the wave scatterers in periodic structures with generation of specific wave patterns due to their sharp edges. Previous known works, which take into account of the presence of multiple cracks in 3D elastic wave field, are related to the situations with a few defects [2, 5, 6]. The reason lies in the computational difficulties of the corresponding large-scale problems. As was shown in 2D configurations [7, 10], the models with periodically distributed cracks can simplify the analysis, especially by the boundary integral equations (BIEs) method and introducing effective Green's functions to consider properly the dynamic interactions between the infinite number of cracks.

There are usually two ways to deal with the periodic structures by the BIEs method: one is that the BIEs are formulated in the unit cell according to the wave equations and general Green's functions, then the Bloch conditions of periodicity are forced on the boundaries of the unit cell; the other is that the Bloch conditions are first substituted into the wave equations, then the BIEs are formulated based on the periodic Green's functions. The first approach has been successfully applied to compute band gaps of 2D phononic crystals [4]. In the present work, the second approach, which does not demand the BIEs formulation on the boundary of unit cell, is used for the analysis of normally incident plane longitudinal elastic waves on doubly periodic array of coplanar penny-shaped cracks.

1. Boundary integral equation formulation of the problem. Let us consider an infinite elastic solid containing a doubly periodic rectangular lattice array of coplanar penny-shaped cracks $S^{(mn)}$ $(m, n \in \mathbb{Z})$ of the same radius a in the plain $x_3 = 0$. The crack-surfaces are free of loads and their centres are located on the parallel lines to Ox_1 axis with the periodic distance d_1 and on the parallel lines to Ox_2 axis with the periodic distance d_2 (Fig. 1). The mechanical properties of the elastic solid are defined by the mass density ρ, the shear modulus G and the Poisson's ratio ν. Here, only the symmetric time-harmonic problem is considered, as a plane longitudinal elastic wave with the circular frequency ω and the known constant stress amplitude σ_{33}^{in} impinges on the cracks from the direction Ox_3.

The presence of multiple cracks in the solid leads to the superposed wave field, in which the total displacements vector $\mathbf{u}(\mathbf{x})$ can be expressed as

$$\mathbf{u}(\mathbf{x}) = \mathbf{u}^{\text{in}}(\mathbf{x}) + \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \mathbf{u}^{(mn)}, \tag{1}$$

where $\mathbf{u}^{\text{in}}$ is the displacement vector of the incident wave, $\mathbf{u}^{(mn)}(u_1^{(mn)}, u_2^{(mn)}, u_3^{(mn)})$ are unknown displacements of the scattered waves by the (mn)-th crack, respectively. The governing equations for the displacement vector $\mathbf{u}(\mathbf{x})$ are the well-known equations of motion [3]

$$k_1^{-2}\nabla(\nabla \cdot \mathbf{u}) - k_2^{-2}\nabla \times (\nabla \times \mathbf{u}) + \mathbf{u} = 0 \,. \tag{2}$$

Here, $k_j = \omega/c_j$, $j = 1,2$, are the wave numbers, $c_2 = \sqrt{G/\rho}$ and $c_1 = \sqrt{2(1-\nu)/(1-2\nu)}\, c_2$ are the transverse and longitudinal wave velocities, respectively, ∇ is a three-dimensional nabla operator. The radiation conditions at the infinity are, of course, required.

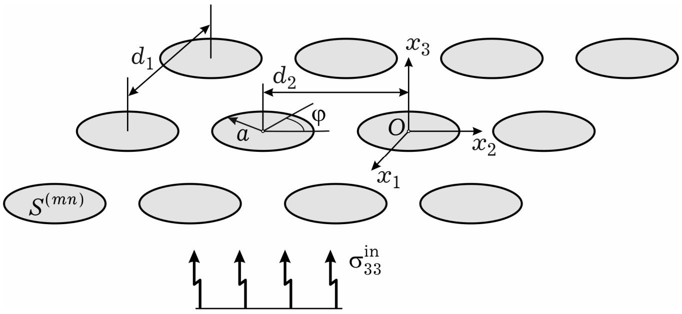


Fig. 1. Wave scattering by a doubly periodic array of penny-shaped cracks.

The boundary conditions on the crack-surfaces $S^{(mn)}$ are

$$\sigma_{33}(\mathbf{x}) = 0, \qquad \mathbf{x} \in S^{(mn)}, \qquad m,n \in \mathbb{Z} \,. \tag{3}$$

From the above definition of the displacement vector $\mathbf{u}^{(mn)}(\mathbf{x})$ it follows that the integral representations of its components $u_j^{(mn)}$, $j = 1,2,3$, $m,n \in \mathbb{Z}$, are the same as in the case of an infinite homogeneous elastic solid with a single crack [5]. Due to the symmetry of the problem they are given by

$$\begin{aligned} u_j^{(mn)}(\mathbf{x}) = {} & \frac{\partial}{\partial x_j}\left[1 + \frac{2}{k_2^2}\left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}\right)\right] \times \\ & \times \left[\iint_{S^{(mn)}} \Delta u_3^{(mn)}(\boldsymbol{\eta}) \frac{\exp(ik_1|\mathbf{x}-\boldsymbol{\eta}|)}{|\mathbf{x}-\boldsymbol{\eta}|}\, dS_\eta\right] - \\ & - 2\left[\delta_{j1}\frac{\partial}{\partial x_1} + \delta_{j2}\frac{\partial}{\partial x_2} + \frac{1}{k_2^2}\frac{\partial}{\partial x_j}\left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}\right)\right] \times \\ & \times \left[\iint_{S^{(mn)}} \Delta u_3^{(mn)}(\boldsymbol{\eta}) \frac{\exp(ik_2|\mathbf{x}-\boldsymbol{\eta}|)}{|\mathbf{x}-\boldsymbol{\eta}|}\, dS_\eta\right], \quad j = 1,2,3\,, \end{aligned} \tag{4}$$

where $\delta_{j\ell}$ is the Kronecker delta, $|\mathbf{x}-\boldsymbol{\eta}|$ is the distance between the field point and the integration point, $\Delta u_3^{(mn)}(\mathbf{x}) = [u_3^{(mn)-}(\mathbf{x}) - u_3^{(mn)+}(\mathbf{x})]/4\pi$ is the crack-opening-displacement (COD) or the displacement jump across the (mn)-th crack-surfaces in the x_3 direction.

Then in accordance to the Hooke's law, the normal stresses $\sigma_{33}^{(mn)}$ corresponding to the displacements (4) have the form

$$\sigma_{33}^{(mn)}(\mathbf{x}) = \frac{4G}{k_2^2}\sum_{j=1}^{2}\mathbf{T}_j^{\mathbf{x}}\left[\iint_{S^{(mn)}} \Delta u_3^{(mn)}(\boldsymbol{\eta})\frac{\exp(ik_j|\mathbf{x}-\boldsymbol{\eta}|)}{|\mathbf{x}-\boldsymbol{\eta}|}dS_{\boldsymbol{\eta}}\right]. \tag{5}$$

In Eq. (5) $\mathbf{T}_j^{\mathbf{x}}$ are the differential operators, which are given by

$$\mathbf{T}_1^{\mathbf{x}} = -\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}+\frac{k_2^2}{2}\right)^2, \quad \mathbf{T}_2^{\mathbf{x}} = \left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}\right)\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}+k_2^2\right). \tag{6}$$

According to the Bloch-theorem and the periodicity of the problem, the CODs satisfy the following relation

$$\Delta u_3^{(mn)}(x_1+md, x_2+nd) = \Delta u_3(x_1,x_2), \qquad m,n\in\mathbb{Z}, \tag{7}$$

where $\Delta u_3 = \Delta u_3^{(00)}$ is the COD for the reference crack $S = S^{(00)}$ located in the unit cell.

Following the boundary conditions (3) and the relations (1) and (5) we obtain the BIE for the COD Δu_3 in the form

$$\sigma_{33}^{\text{in}}(\mathbf{x}) = \frac{4G}{k_2^2}\iint_S \Delta u_3(\boldsymbol{\eta})\left[\sum_{m=-b+1}^{b-1}\sum_{n=-c+1}^{c-1} R(r^{(mn)}) - L(\mathbf{x})\right]dS_{\boldsymbol{\eta}}, \quad \mathbf{x}\in S. \tag{8}$$

Here, $r^{(mn)} = \sqrt{(x_1-\eta_1-md_1)^2+(x_2-\eta_2-nd_2)^2}$, b, c are arbitrary natural numbers, the hypersingular (when $m=n=0$) kernel $R(r^{(mn)})$ describes the interaction between the reference and the neighbouring cracks located in the domains $S^{(mn)}$, $-b+1\le m\le b-1$, $-c+1\le n\le c-1$ and it is the same as for a finite number of coplanar cracks in an infinite homogeneous solid [5]

$$R(r) = \left[9-9ik_1r+(k_2^2-5k_1^2)r^2+ik_1(2k_1^2-k_2^2)r^3+\frac{1}{4}(2k_1^2-k_2^2)^2r^4\right]\times$$

$$\times\frac{\exp(ik_1r)}{r^5} - [9-9ik_2r-4k_2^2r^2+ik_2^3r^3]\frac{\exp(ik_2r)}{r^5}. \tag{9}$$

The kernel $L(x_1,x_2)$ in the BIE (8) describes the interaction of the reference crack S with the rest cracks and can be written as the following lattice sums which converge very slowly

$$L(x_1,x_2) = \sum_{m,n=-\infty}^{\infty}{}'' \sum_{j=1}^{2}\mathbf{T}_j^{\mathbf{x}}[\Phi^{mn}(x_1,x_2,x_3)]\Big|_{x_3=0}, \tag{10}$$

where the double-dash over the summation indicates that the terms with $-b+1\le m\le b-1$ and $-c+1\le n\le c-1$ are to be omitted, and the kernel Φ^{mn} is given by

$$\Phi^{mn}(x_1,x_2,x_3) = \frac{\exp\left(ik_j\sqrt{(x_1-\eta_1-md_1)^2+(x_2-\eta_2-nd_2)^2+x_3^2}\right)}{\sqrt{(x_1-\eta_1-md_1)^2+(x_2-\eta_2-nd_2)^2+x_3^2}}. \tag{11}$$

The sums (10) can be rewritten in a more convenient form by the Fourier integral expressions. For this reason the following integral [1] is used

$$\int_{-\infty}^{\infty}\exp(ixt)H_0^1\left(r\sqrt{h^2-t^2}\right)dt = 2i\frac{\exp\left(ih\sqrt{r^2+x^2}\right)}{\sqrt{r^2+x^2}}, \tag{12}$$

where $H_n^{(1)}$ is the Hankel function of the first kind and the n-th order. According to Eq. (12) we can transform the kernel term (11) to a new integral form, namely,

$$\Phi^{mn}(x_1, x_2, x_3) = \frac{1}{2i}\int_{-\infty}^{\infty} \exp(ix_3 t) \times$$

$$\times H_0^{(1)}\left(\sqrt{(x_1 - \eta_1 - md_1)^2 + (x_2 - \eta_2 - nd_2)^2}\,\sqrt{k_j^2 - t^2}\right) dt\,. \quad (13)$$

Next steps are based on substituting the expression (13) into Eq. (10), applying the Fourier integral transform to the kernel $L(x_1, x_2)$ with respect to x_1 and x_2 coordinates and performing the summation of the resulting geometrical series having $\exp[-\sqrt{\xi^2 + t^2 - k_j^2}\,]$ as the common ratio, where ξ is the integral transform parameter. Then taking the inverse transform of the resulting expressions and calculating analytically the corresponding integrals by using the cylindrical coordinate system, and after taking derivatives (6) we arrive at the following exponentially-convergent form of the kernel (10)

$$L(x_1, x_2) = \sum_{m=-b+1}^{b-1} \sum_{\ell=1}^{2}\sum_{j=1}^{2} (\delta_{2j} - \delta_{1j}) \times$$

$$\times \int_0^{\infty} \frac{\tau \exp\{-[(\delta_{2\ell} - \delta_{1\ell})(x_2 - \eta_2) + cd_2]V_j(\tau)\}}{V_j(\tau)[1 - \exp(-d_2 V_j(\tau))]} \times$$

$$\times \Omega_j(x_1 - \eta_1 + md_1, \tau)\, d\tau +$$

$$+ \sum_{n=-\infty}^{\infty} \sum_{\ell=1}^{2}\sum_{j=1}^{2} (\delta_{2j} - \delta_{1j}) \times$$

$$\times \int_0^{\infty} \frac{\tau \exp\{-[(\delta_{2\ell} - \delta_{1\ell})(x_1 - \eta_1) + bd_1]V_j(\tau)\}}{V_j(\tau)[1 - \exp(-d_1 V_j(\tau))]} \times$$

$$\times \Omega_j(x_2 - \eta_2 + nd_2, \tau)\, d\tau\,. \quad (14)$$

Here, $V_j(\tau) = \sqrt{\tau^2 - k_j^2}$ $(\operatorname{Im} V_j < 0)$, $\delta_{j\ell}$ is the Kronecker delta, $J_n(\cdot)$ is the Bessel function of the n-th order, and

$$\Omega_1(r, \tau) = J_0(\tau r)\left(\frac{k_2^2}{2} - k_1^2\right)^2 + \frac{2\tau J_1(\tau r)}{r}\left(\frac{k_2^2}{2} - k_1^2\right) + \frac{3\tau^2 J_2(\tau r)}{r^2}\,,$$

$$\Omega_2(r, \tau) = \tau^2\left(\frac{3J_2(\tau r)}{r^2} - \frac{k_2^2 J_1(\tau r)}{\tau r}\right).$$

The kernel $L(x_1, x_2)$ can be understood as a periodic Green's function of the considered problem. The parameters $b \geq 2$ and $c \geq 2$ in the kernel (14) are integers. Taking b and c to be sufficiently large, one can improve the convergence of the integrals. It should be mentioned also, that the integrand in the kernel $L(x_1, x_2)$ has singularities at the two points coinciding with the roots of the functions $V_1(\tau)$ and $V_2(\tau)$. Therefore, before the numerical procedure, a regularization of the kernels R and L in the BIE (8) is needed.

3. Efficient numerical solution of the BIE. To determine explicitly the singularities in the kernel R of BIE (8), this equation is identically transformed to

$$\iint_S \frac{\Delta u_3(\boldsymbol{\eta})}{|\mathbf{x}-\boldsymbol{\eta}|^3}\,dS_\eta + qk_2^2\iint_S \frac{\Delta u_3(\boldsymbol{\eta})}{|\mathbf{x}-\boldsymbol{\eta}|}\,dS_\eta + \iint_S \Delta u_3(\boldsymbol{\eta})\Bigg[\frac{4(1-\nu)}{k_2^2}\times$$

$$\times R(|\mathbf{x}-\boldsymbol{\eta}|) - \frac{1}{|\mathbf{x}-\boldsymbol{\eta}|^3} - \frac{qk_2^2}{|\mathbf{x}-\boldsymbol{\eta}|}\Bigg]dS_\eta - \frac{4(1-\nu)}{k_2^2}\times$$

$$\times \iint_S \Delta u_3(\boldsymbol{\eta})\Bigg[\sum_{m=-b+1}^{b-1}\sum_{n=-c+1}^{c-1}(1-\delta_{0m})(1-\delta_{0n})\times$$

$$\times R\left(\sqrt{(x_1-\eta_1+md_1)^2+(x_2-\eta_2+nd_2)^2}\right) + L(\mathbf{x})\Bigg]dS_\eta =$$

$$= \frac{1-\nu}{G}\sigma_{33}^{\text{in}}, \qquad \mathbf{x}\in S, \tag{15}$$

Here, $q = \dfrac{7-12\nu+8\nu^2}{8(1-\nu)}$, $|\mathbf{x}-\boldsymbol{\eta}| = r^{(00)}$, the hypersingular and weakly singular kernels are contained in the first and second integrals, respectively, and they have the same form as in the static case. As a consequence, to describe the appropriate local behavior of the solution at the crack-front correctly, the following representation for the COD is used

$$\Delta u_3(\mathbf{x}) = \sqrt{a^2-x_1^2-x_2^2}\,\alpha(\mathbf{x}), \qquad \mathbf{x}\in S, \tag{16}$$

where $\alpha(\mathbf{x})$ is a new unknown function of sufficient smoothness. It is important that the multiplicative square-root separation in the COD allows us to compute the SIF accurately without using any special boundary elements at the crack-front. With considering the ansatz (16) the singular integrals of the BIE (15) are recast into the following forms, where the integrals on the right-hand side are regular

$$\iint_S \frac{\sqrt{a^2-\eta_1^2-\eta_2^2}}{|\mathbf{x}-\boldsymbol{\eta}|^3}\alpha(\boldsymbol{\eta})\,dS_\eta = -\pi^2\alpha(\mathbf{x}) - \frac{1}{2}\pi^2 x_1\frac{\partial\alpha(\mathbf{x})}{\partial x_1} - \frac{1}{2}\pi^2 x_2\frac{\partial\alpha(\mathbf{x})}{\partial x_2} +$$

$$+\frac{\pi^2}{32}[4a^2-x_1^2-3x_2^2]\frac{\partial^2\alpha(\mathbf{x})}{\partial x_1^2} + \frac{\pi^2}{8}x_1x_2\frac{\partial^2\alpha(\mathbf{x})}{\partial x_1\partial x_2} +$$

$$+\frac{\pi^2}{32}[4a^2-3x_1^2-x_2^2]\frac{\partial^2\alpha(\mathbf{x})}{\partial x_2^2} +$$

$$+\iint_S \frac{\sqrt{a^2-\eta_1^2-\eta_2^2}}{|\mathbf{x}-\boldsymbol{\eta}|^3}\Bigg[\alpha(\boldsymbol{\eta})-\alpha(\mathbf{x})-(\eta_1-x_1)\frac{\partial\alpha(\mathbf{x})}{\partial x_1} -$$

$$-(\eta_2-x_2)\frac{\partial\alpha(\mathbf{x})}{\partial x_2} - \frac{1}{2}(\eta_1-x_1)^2\frac{\partial^2\alpha(\mathbf{x})}{\partial x_1^2} -$$

$$-(\eta_1-x_1)(\eta_2-x_2)\frac{\partial^2\alpha(\mathbf{x})}{\partial x_1\partial x_2} - \frac{1}{2}(\eta_2-x_2)^2\frac{\partial^2\alpha(\mathbf{x})}{\partial x_2^2}\Bigg]dS_\eta, \tag{17}$$

$$\iint_S \frac{\sqrt{a^2-\eta_1^2-\eta_2^2}}{|\mathbf{x}-\boldsymbol{\eta}|}\alpha(\boldsymbol{\eta})\,dS_\eta = \frac{\pi^2}{4}[2a^2-x_1^2-x_2^2]\alpha(\mathbf{x}) +$$

$$+\iint_S \frac{\sqrt{a^2-\eta_1^2-\eta_2^2}}{|\mathbf{x}-\boldsymbol{\eta}|}[\alpha(\boldsymbol{\eta})-\alpha(\mathbf{x})]\,dS_\eta .$$

Concerning the improper integrals in the kernel L of the BIE (8), they are regularized by the subtraction technique with the analytical evaluation of the special regularizing integrals and taking into account the following behaviour of the contained expressions in the vicinities of peculiar points $\tau = k_j$:

$$\frac{1}{1-\exp(-dV_j(\tau))} \sim \frac{1}{dV_j(\tau)}. \tag{18}$$

By the implementation of the above mentioned regularization techniques to the BIE (15) its regular analogue is obtained, which is suitable for the numerical solution. To this end, a collocation method is used to obtain a well-conditioned system of linear algebraic equations.

After the numerical solution of the BIE (8) or (15), the dynamic stress intensity factor (SIF) in the crack-front vicinity can be computed. In our case only the mode-I SIF K_{I} is present and can be easily determined from the relation

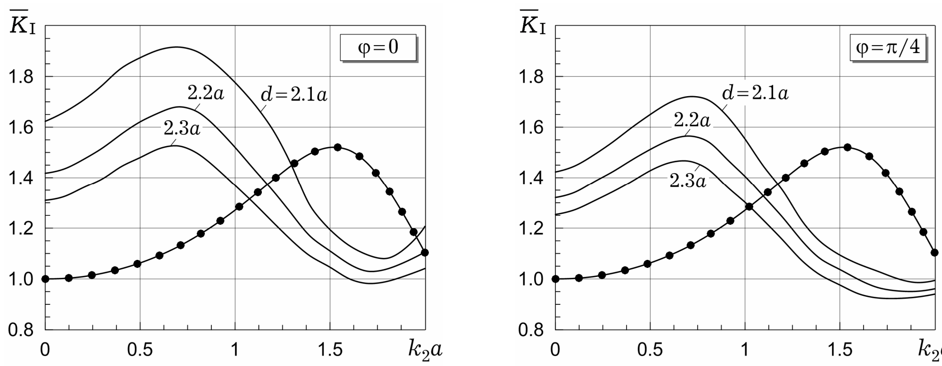
$$K_{\mathrm{I}}(\varphi) = -\frac{2G\pi\sqrt{\pi a}}{(1-\nu)}\,\alpha(\mathbf{x})\bigg|_{\substack{x_1=-a\sin\varphi\\ x_2=a\cos\varphi}}, \qquad \mathbf{x}\in S, \tag{19}$$

where φ is the angular coordinate of the crack-front point (see Fig. 1).

4. Numerical results. Numerical computations have been carried out for a doubly periodic array of penny-shaped cracks with a square lattice and different lengths of periodicity or distances between the cracks $d_1 = d_2 = d$. The Poisson's ratio of the elastic solid has been taken as $\nu = 0.3$. The static SIF $K_{\mathrm{I}}^{\mathrm{st}} = 2\sigma_{33}^{\mathrm{in}}\sqrt{a/\pi}$ has been chosen as a normalization factor for the dynamic mode-I SIF, i. e. $\bar{K}_{\mathrm{I}} = |K_{\mathrm{I}}|/K_{\mathrm{I}}^{\mathrm{st}}$.

In the calculation the crack-surface has been discretized into 217 constant boundary elements with $\Delta r = 0.1a$ in the radial direction and $\Delta\varphi = \pi/12$ in the polar coordinate direction. The convergence controlling parameters in the kernel $L(x_1, x_2)$ have been selected as $b = c = 3$, and the truncation parameters for the sum (14) as $p = 25$ and for the integral in Eq. (14) as $\tau_{\max} = 16c_2/a$.

Fig. 2 illustrates the variations of the normalized mode-I dynamic SIF with the dimensionless wave number at the two representative crack-front points with the angular coordinates $\varphi = 0$ (Fig. 2*a*) and $\varphi = \pi/4$ (Fig. 2*b*), which describe the location along the sides and diagonal of the lattice, respectively. The curves correspond to the following lengths of periodicity: $d = 2.1a$, $2.2a$, and $d = 2.3a$; and the marked curve corresponds to the SIF for a single crack subjected to the same wave loading.



a) ***b***)

Fig. 2. Normalized mode-I dynamic SIF versus the wave number.

An essential contrast in the behaviour of the $\bar{K}_\mathrm{I}$-factor versus the wave number for a single crack and multiple cracks can be found in Fig. 2. Periodic cracks lead to fast changing $\bar{K}_\mathrm{I}$-factor in comparison with the more smooth dependence of the $\bar{K}_\mathrm{I}$-factor for a single crack. This fact confirms the resonant interaction in the periodic system of closely located cracks on the wave field. At low frequencies, the dynamic SIF for interacting multiple cracks is larger than that for a single crack, and after reaching a peak value a rapid decrease and a reduction of the $\bar{K}_\mathrm{I}$-factor compared to that for a single crack are observed. These effects are more pronounced as the crack distance becomes smaller in the directions of the periodicity. The wave numbers, at which the peaks of the $\bar{K}_\mathrm{I}$-factor are achieved, are much smaller in the case of periodic cracks.

The present numerical method provides an efficient way for the analysis of cracked solids with more complicated periodic microstructures, including doubly periodic cracks with non-circular shapes and non-orthogonal lattices, multiple layers of doubly periodic cracks, partially disordered periodic systems of cracks, etc. Special attention should be paid in future to the wave reflection and transmission phenomena to analyze band gaps and resonances of elastic waves in such structures.

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ДИНАМІЧНІ НАПРУЖЕННЯ ЗА ПАДІННЯ ПРУЖНОЇ ГАРМОНІЧНОЇ ХВИЛІ НА ДВОПЕРІОДИЧНИЙ МАСИВ ДИСКОВИХ ТРІЩИН

У частотній області числово розв'язано симетричну задачу про динамічну взаємодію у прямокутно-гратковій системі компланарних дискових тріщин, розміщених у безмежному пружному тілі. Задачу зведено до граничного інтегрального рівняння відносно функції динамічного розкриття тріщини в унітарній комірці за допомогою тривимірної періодичної функції Ґріна. Цю функцію адаптовано до ефективного розрахунку через її подання у формі експоненціально збіжних інтегралів Фур'є. Для розв'язання граничного інтегрального рівняння використано числову схему методу колокацій. Для різних розмірів комірки встановлено залежності коефіцієнта інтенсивності динамічних напружень відриву в околі тріщин від хвильового числа.

ДИНАМИЧЕСКИЕ НАПРЯЖЕНИЯ ПРИ ПАДЕНИИ УПРУГОЙ ГАРМОНИЧЕСКОЙ ВОЛНЫ НА ДВУХПЕРИОДИЧЕСКИЙ МАССИВ ДИСКОВЫХ ТРЕЩИН

В частотной области численно решена симметричная задача о динамическом взаимодействии в прямоугольно-решеточной системе компланарных дисковых трещин, расположенных в бесконечном упругом теле. Задача сведена к граничному интегральному уравнению относительно функции динамического раскрытия трещины в унитарной ячейке с помощью трехмерной периодической функции Грина. Эта функция адаптирована к эффективным расчетам через представление ее в форме экспоненциально сходящихся интегралов Фурье. Для решения граничного интегрального уравнения использована численная схема метода коллокаций. Для разных размеров ячейки установлены зависимости коэффициента интенсивности динамических напряжений отрыва в окрестности трещин от волнового числа.

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