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Dedicated to the memory of Magdalena von Götting

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PSEUDOCOMPACT PRIMITIVE TOPOLOGICAL INVERSE SEMIGROUPS

In the paper we study pseudocompact primitive topological inverse semigroups. We describe the structure of pseudocompact primitive topological inverse semigroups and show that the Tychonoff product of a family of pseudocompact primitive topological inverse semigroups is a pseudocompact topological space. Also we prove that the Stone – Čech compactification of a pseudocompact primitive topological inverse semigroup is a compact primitive topological inverse semigroup.

Introduction and preliminaries. Further we shall follow the terminology of [3–5, 9, 20]. The set of positive integers is denoted by $\mathbb{N}$.

A semigroup is a non-empty set with a binary associative operation. A semigroup S is called *inverse* if for any $x \in S$ there exists a unique $y \in S$ such that $x \cdot y \cdot x = x$ and $y \cdot x \cdot y = y$. Such an element y in S is called *inverse* of x and denoted by x^{-1}. The map defined on an inverse semigroup S which assigns to element x of S its inverse x^{-1} is called the *inversion*.

If S is a semigroup, then by $E(S)$ we denote the subset of idempotents of S, and by S^1 (respectively, S^0) we denote the semigroup S with the adjoined unit (respectively, zero). Also if a semigroup S has zero 0_S, then for any $A \subseteq S$ we denote $A^* = A \setminus \{0_S\}$.

If E is a semilattice, then the semilattice operation on E determines the partial order $\le$ on E:

$$e \le f \qquad \text{if and only if} \qquad ef = fe = e.$$

This order is called *natural*. An element e of a partially ordered set X is called *minimal* if $f \le e$ implies $f = e$ for $f \in X$. An idempotent e of a semigroup S without zero (with zero) is called *primitive* if e is a minimal element in $E(S)$ (in $(E(S))^*$).

Let S be a semigroup with zero and λ be a cardinal ≥ 1. On the set $B_\lambda(S) = (\lambda \times S \times \lambda) \sqcup \{0\}$ we define the semigroup operation as follows

$$(\alpha, a, \beta) \cdot (\gamma, b, \delta) = \begin{cases} (\alpha, ab, \delta), & \beta = \gamma, \\ 0, & \beta \ne \gamma, \end{cases}$$

and $(\alpha, a, \beta) \cdot 0 = 0 \cdot (\alpha, a, \beta) = 0 \cdot 0 = 0$, for all $\alpha, \beta, \gamma, \delta \in \lambda$ and $a, b \in S$. If $S = S^1$ then the semigroup $B_\lambda(S)$ is called the *Brandt* λ*-extension of the semigroup* S [12]. Obviously, $\mathcal{J} = \{0\} \cup \{(\alpha, \mathcal{O}, \beta) \mid \mathcal{O}$ is the zero of $S\}$ is an ideal of $B_\lambda(S)$. We put $B_\lambda^0(S) = B_\lambda(S)/\mathcal{J}$ and we shall call $B_\lambda^0(S)$ the *Brandt* λ^0*-extension of the semigroup* S *with zero* [13]. Further, if $A \subseteq S$ then we shall denote $A_{\alpha,\beta} = \{(\alpha, s, \beta) : s \in A\}$ if A does not contain zero, and $A_{\alpha,\beta} = \{(\alpha, s, \beta) : s \in A \setminus \{0\}\} \cup \{0\}$ if $0 \in A$, for $\alpha, \beta \in \lambda$. If $\mathcal{I}$ is a trivial semigroup (i. e., $\mathcal{I}$ contains only one element), then by $\mathcal{I}^0$ we denote the semigroup $\mathcal{I}$ with the adjoined zero. Obviously, for any $\lambda \ge 2$ the Brandt λ^0-extension of the semigroup $\mathcal{I}^0$ is isomorphic to the semigroup of $\lambda \times \lambda$-matrix units and any Brandt λ^0-extension of a semigroup with zero contains the semigroup of $\lambda\times\lambda$-matrix units. Further by B_λ we shall denote the semigroup of $\lambda\times\lambda$-matrix units and by $B^0_\lambda(1)$ the subsemigroup of $\lambda\times\lambda$-matrix units of the Brandt λ^0-extension of a monoid S with zero. A completely 0-simple inverse semigroup is called a *Brandt semigroup* [20]. By Theorem II.3.5 [20], a semigroup S is a Brandt semigroup if and only if S is isomorphic to a Brandt λ-extension $B_\lambda(G)$ of some group G.

Let $\{S_i : i\in\mathscr{I}\}$ be a disjoint family of semigroups with zero such that 0_i is zero in S_i for any $i\in\mathscr{I}$. We put $S=\{0\}\cup\{S_i^* : i\in\mathscr{I}\}$, where $0\notin\bigcup\{S_i^* : i\in\mathscr{I}\}$, and define a semigroup operation on S in the following way

$$s\cdot t=\begin{cases} st, & \text{if } \ st\in S_i^* \ \text{ for some } \ i\in\mathscr{I},\\ 0, & \text{otherwise.}\end{cases}$$

The semigroup S with such defined operation is called the *orthogonal sum* of the family of semigroups $\{S_i : i\in\mathscr{I}\}$ and in this case we shall write $S=\sum\limits_{i\in\mathscr{I}} S_i$.

A non-trivial inverse semigroup is called a *primitive inverse semigroup* if all its non-zero idempotents are primitive [20]. A semigroup S is a primitive inverse semigroup if and only if S is the orthogonal sum of a family of Brandt semigroups [20, Theorem II.4.3]. We shall call a Brandt subsemigroup T of a primitive inverse semigroup S *maximal* if every Brandt subsemigroup of S which contains T, coincides with T.

Green's relations $\mathscr{L}$, $\mathscr{R}$ and $\mathscr{H}$ on a semigroup S are defined by:

$$a\mathscr{L}b \quad \text{if and only if} \quad \{a\}\cup Sa=\{b\}\cup Sb,$$

$$a\mathscr{R}b \quad \text{if and only if} \quad \{a\}\cup aS=\{b\}\cup bS,$$

$$\mathscr{H}=\mathscr{L}\cap\mathscr{R},$$

for $a,b\in S$. For details about Green's relations see [5, § 2.1] or [11]. We observe that two non-zero elements (α_1,s,β_1) and (α_2,t,β_2) of a Brandt semigroup $B_\lambda(G)$, $s,t\in G$, $\alpha_1,\alpha_2,\beta_1,\beta_2\in\lambda$, are $\mathscr{H}$-equivalent if and only if $\alpha_1=\alpha_2$ and $\beta_1=\beta_2$ (see [20, p. 93]).

In this paper all topological spaces are Hausdorff. If Y is a subspace of a topological space X and $A\subseteq Y$, then by $\mathrm{cl}_Y(A)$ we denote the topological closure of A in Y.

We recall that a topological space X is said to be

- *compact* if each open cover of X has a finite subcover;
- *countably compact* if each open countable cover of X has a finite subcover;
- *pseudocompact* if each locally finite open cover of X is finite.

According to Theorem 3.10.22 of [9], a Tychonoff topological space X is pseudocompact if and only if each continuous real-valued function on X is bounded. Also, a Hausdorff topological space X is pseudocompact if and only if every locally finite family of non-empty open subsets of X is finite. Every compact space and every countably compact space are pseudocompact (see [9]).

We recall that the Stone − Čech compactification of a Tychonoff space X is a compact Hausdorff space βX containing X as a dense subspace so that each continuous map $f : X\to Y$ to a compact Hausdorff space Y extends to a continuous map $\overline{f} : \beta X\to Y$ [9].

A *topological semigroup* is a Hausdorff topological space with a continuous semigroup operation. A topological semigroup which is an inverse semigroup is called an *inverse topological semigroup*. A *topological inverse semigroup* is an inverse topological semigroup with continuous inversion. A *topological group* is a topological space with a continuous group operation and inversion. We observe that the inversion on a topological inverse semigroup is a homeomorphism (see [8, Proposition II.1]). A Hausdorff topology τ on a (inverse) semigroup S is called (*inverse*) *semigroup topology* if (S,τ) is a topological (inverse) semigroup.

Definition 1 **[12].** Let $\mathfrak{TSG}$ be some category of topological semigroups. Let λ be a cardinal ≥ 1 and $(S,\tau) \in \mathbf{Ob}\,\mathfrak{TSG}$ be a topological monoid. Let λ^0 be a topology on $B_\lambda(S)$ such that

(***a***) $(B_\lambda(S), \tau_B) \in \mathbf{Ob}\,\mathfrak{TSG}$;

(***b***) for some $\alpha \in \lambda$ the topological subspace $(S_{\alpha,\alpha}, \tau_B|_{S_{\alpha,\alpha}})$ is naturally homeomorphic to (S,τ).

Then $(B_\lambda(S), \tau_B)$ is called a *topological Brandt* λ*-extension of* (S,τ) *in* $\mathfrak{TSG}$.

Definition 2 **[13].** Let $\mathfrak{TSG}_0$ be some category of topological semigroups with zero. Let λ be a cardinal ≥ 1 and $(S,\tau) \in \mathbf{Ob}\,\mathfrak{TSG}_0$. Let τ_B be a topology on $B_\lambda^0(S)$ such that

(***a***) $(B_\lambda^0(S), \tau_B) \in \mathbf{Ob}\,\mathfrak{TSG}_0$;

(***b***) for some $\alpha \in \lambda$ the topological subspace $(S_{\alpha,\alpha}, \tau_B|_{S_{\alpha,\alpha}})$ is naturally homeomorphic to (S,τ).

Then $(B_\lambda^0(S), \tau_B)$ is called a *topological Brandt* λ^0*-extension of* (S,τ) *in* $\mathfrak{TSG}_0$.

We observe that for any topological Brandt λ-extension $B_\lambda(S)$ of a topological semigroup S in the category of topological semigroups there exist a topological monoid T with zero and a topological Brandt λ^0-extension $B_\lambda^0(T)$ of T in the category of topological semigroups with zero, such that the semigroups $B_\lambda(S)$ and $B_\lambda^0(T)$ are topologically isomorphic. Algebraic properties of Brandt λ^0-extensions of monoids with zero, non-trivial homomorphisms between them, and a category which objects are ingredients of the construction of such extensions were described in [17]. Also, in [14] and [17] a category which objects are ingredients in the constructions of finite (respectively, compact, countably compact) topological Brandt λ^0-extensions of topological monoids with zeros was described.

Gutik and Repovš [16] proved that any 0-simple countably compact topological inverse semigroup is topologically isomorphic to a topological Brandt λ-extension $B_\lambda(H)$ of a countably compact topological group H in the category of topological inverse semigroups for some finite cardinal $\lambda \ge 1$. Also, every 0-simple pseudocompact topological inverse semigroup is topologically isomorphic to a topological Brandt λ-extension $B_\lambda(H)$ of a pseudocompact topological group H in the category of topological inverse semigroups for some finite cardinal $\lambda \ge 1$ [15]. Next Gutik and Repovš showed in [16] that the Stone – Čech compactification $\beta(T)$ of a 0-simple countably compact topological inverse semigroup T is a 0-simple compact topological inverse semigroup. It was proved in [15] that the same is true in the case of 0-simple pseudocompact topological inverse semigroups.

In the paper [2] the structure of compact and countably compact primitive topological inverse semigroups was described and showed that any countably compact primitive topological inverse semigroup embeds into a compact primitive topological inverse semigroup.

In this paper we describe the structure of pseudocompact primitive topological inverse semigroups and show that the Tychonoff product of a family of pseudocompact primitive topological inverse semigroups is a pseudocompact topological space. Also we prove that the Stone – Čech compactification of a pseudocompact primitive topological inverse semigroup is a compact primitive topological inverse semigroup.

1. Primitive pseudocompact topological inverse semigroups.

Proposition 1. *Let S be a Hausdorff pseudocompact primitive topological inverse semigroup and S be an orthogonal sum of the family $\{B_{\lambda_i}(G_i)\}_{i\in\mathcal{I}}$ of topological Brandt semigroups with zeros, i. e. $S=\sum_{i\in\mathcal{I}} B_{\lambda_i}(G_i)$. Then the following statements hold:*

- (***i***) *every non-zero idempotent of S is an isolated point in $E(S)$ and $E(S)$ is a compact semilattice;*
- (***ii***) *every non-zero $\mathcal{H}$-class in S is a pseudocompact closed-and-open subset of S;*
- (***iii***) *every maximal subgroup in S is a pseudocompact subspace of S;*
- (***iv***) *every maximal Brandt subsemigroup of S is a pseudocompact space and has finitely many idempotents.*

P r o o f. (***i***). First part of the statement follows from Lemma 7 [2]. Then the continuity of the semigroup operation and inversion in S implies that the map $\mathfrak{e}:S\to E(S)$ defined by the formula $\mathfrak{e}(x)=x\cdot x^{-1}$ is continuous and hence by Theorem 3.10.24 [9], $E(S)$ is a pseudocompact subspace of S such that every non-zero idempotent in $E(S)$ is an isolated point. Therefore $E(S)$ is compact. Otherwise there exists an open neighbourhood $U(0)$ of the zero 0 of S in $E(S)$ such that the set $E(S)\setminus U(0)$ is infinite. But this contradicts the pseudocompactness of $E(S)$.

(***ii***). By Corollary 8 from [2] every non-zero $\mathcal{H}$-class in S is a closed-and-open subset of S and hence by Exercise 3.10.F(d) is pseudocompact.

Statement (***iii***) follows from (***ii***).

(***iv***). Let $B_{\lambda_i}(G_i)$ be a maximal Brandt subsemigroup of the semigroup S. Then statement (i) implies that $E(S)$ is compact and since every non-zero idempotent of S is an isolated point of $E(S)$ we conclude that $E(B_{\lambda_i}(G_i))$ is compact for every $i\in\mathcal{I}$. By Corollary 3.10.27 of [9] the product of a compact space and a pseudocompact space is a pseudocompact space, and hence we have that the space $E(B_{\lambda_i}(G_i))\times S$ is pseudocompact. Since S is a primitive inverse semigroup we conclude that $B_{\lambda_i}(G_i)=E(B_{\lambda_i}(G_i))\times S$. Now, the continuity of the semigroup operation in S implies that the map $\mathfrak{f}:E(B_{\lambda_i}(G_i))\times$ $\times S\to S$ defined by the formula $\mathfrak{f}(e,s)=e\cdot s$ is continuous, and since the continuous image of a pseudocompact space is pseudocompact we conclude that $B_{\lambda_i}(G_i)$ is pseudocompact. The last statement follows from Theorem 1 of [15]. ♦

Lemma 1. *Let U be an open non-empty subset of a topological group G and A be a dense subset of G. Then $A\cdot U=U\cdot A=G$.*

P r o o f. Since G is a topological group we have that there exists a non-empty open subset V of G such that $V^{-1}=U$. Let x be an arbitrary point of G. Then $x \cdot V$ is a nonempty open subset of G, because translations in every topological group are homeomorphisms. Then we have that $x \cdot V \cap A \neq \neq \varnothing$ and hence $x \in A \cdot V^{-1} = A \cdot U$. Therefore we get that $G \subseteq A \cdot U$. The converse inclusion is trivial. Hence $A \cdot U = G$. The proof of the equality $U \cdot A = G$ is similar. ♦

Lemma 2. *Let $\lambda \geq 2$ be any cardinal and U be an open non-empty subset of a topological inverse Brandt semigroup $B_\lambda(G)$ such that $U \neq \{0\}$. Then $A \cdot U \cdot A = B_\lambda(G)$ for every dense subset A of $B_\lambda(G)$.*

P r o o f. By Lemma 7 [2] we have that every non-zero idempotent of the topological inverse semigroup $B_\lambda(G)$ is an isolated point in $E(B_\lambda(G))$. The continuity of the semigroup operation and inversion in S implies that the map $\mathfrak{e} : S \to E(S)$ defined by the formula $\mathfrak{e}(x) = x \cdot x^{-1}$ is continuous and hence $G_{\alpha,\beta}$ is an open-and-closed subset of $B_\lambda(G)$ for all $\alpha, \beta \in \lambda$. Since A is a dense subset of $B_\lambda(G)$ we conclude that $A \cap G_{\alpha,\beta}$ is a dense subset in $G_{\alpha,\beta}$ for all $\alpha, \beta \in \lambda$. Also, since $\lambda \geq 2$ we have that $0 \in A \cdot U \cdot A$. This implies that it is sufficient to show that $G_{\alpha,\beta} \subseteq A \cdot U \cdot A$ for all $\alpha, \beta \in \lambda$.

Since $G_{\alpha,\beta}$ is an open subset of $B_\lambda(G)$ for all $\alpha, \beta \in \lambda$, without loss of generality we assume that $U \subseteq G_{\alpha_0,\beta_0}$ for some $\alpha_0, \beta_0 \in \lambda$, i. e., $U = V_{\alpha_0,\beta_0}$ for some open subset $V \subseteq G$. Fix arbitrary $\alpha, \beta \in \lambda$. Then there exists subsets $L, R \in G$ such that $A \cap G_{\alpha,\alpha_0} = L_{\alpha,\alpha_0}$ and $A \cap G_{\beta_0,\beta} = R_{\beta_0,\beta}$. It is obviously that L_{α,α_0} and $R_{\beta_0,\beta}$ are dense subsets of G_{α,α_0} and $G_{\beta_0,\beta}$, respectively. This implies that L and R are dense subsets of G. Then by Lemma 1 we have that

$$G_{\alpha,\beta} = (L \cdot V \cdot R)_{\alpha,\beta} = L_{\alpha,\alpha_0} \cdot V_{\alpha_0,\beta_0} \cdot R_{\beta_0,\beta} = \\ = (A \cap G_{\alpha,\alpha_0}) \cdot U \cdot (A \cap G_{\beta_0,\beta}) \subseteq A \cdot U \cdot A.$$

This completes the proof of the lemma. ♦

Lemma 1 implies the following

Proposition 2. *Let U be an open non-empty subset of a topological inverse Brandt semigroup $B_1(G)$ such that $U \neq \{0\}$. Then for every dense subset A of $B_1(G)$ the following statements hold:*

(***i***) *$A \cdot U \cdot A = B_1(G)$ in the case when 0 is an isolated point in $B_1(G)$;*

(***ii***) *$(A \cup \{0\}) \cdot U \cdot (A \cup \{0\}) = B_1(G)$ in the case when 0 is a non-isolated point in $B_1(G)$.*

Lemma 2 and Proposition 2 imply the following

Proposition 3. *Let S be a Hausdorff primitive inverse topological semigroup such that S be an orthogonal sum of the family $\{B_{\lambda_i}(G_i)\}_{i \in \mathscr{I}}$ of topological Brandt semigroups with zeros. Let $|\mathscr{I}| > 1$ and U be an open non-empty subset of S such that $(U \cap B_{\lambda_i}(G_i)) \setminus \{0\} \neq \varnothing$ for any $i \in \mathscr{I}$. Then $A \cdot U \cdot A = S$ for every dense subset A of S.*

Remark 1. Since by Theorem II.4.3 of [20] a primitive inverse semigroup S is an orthogonal sum of a family of Brandt semigroups, i. e., S is an orthogonal sum $\sum_{i \in \mathscr{I}} B_{\lambda_i}(G_i)$ of Brandt λ_i-extensions $B_{\lambda_i}(G_i)$ of groups G_i, we have that Proposition 12 from [2] describes a base of the topology at any non-zero element of S.

Later by $\mathfrak{TISG}$ we denote the category of topological inverse semigroups, where $\mathbf{Ob}\,\mathfrak{TISG}$ are all topological inverse semigroups and $\mathbf{Mor}\,\mathfrak{TISG}$ are homomorphisms between topological inverse semigroups.

The following theorem describes the structure of primitive pseudocompact topological inverse semigroups.

Theorem 1. *Every primitive Hausdorff pseudocompact topological inverse semigroup S is topologically isomorphic to the orthogonal sum $\sum_{i\in\mathscr{I}} B_{\lambda_i}(G_i)$ of topological Brandt λ_i-extensions $B_{\lambda_i}(G_i)$ of pseudocompact topological groups G_i in the category $\mathfrak{TISG}$ for some finite cardinals $\lambda_i \geq 1$. Moreover the family*

$$\mathscr{B}(0) = \{S\backslash(B_{\lambda_{i_1}}(G_{i_1}) \cup \ldots \cup B_{\lambda_{i_n}}(G_{i_n}))^* : i_1,\ldots,i_n \in \mathscr{I},\ n \in \mathbb{N}\}, \qquad (1)$$

determines a base of the topology at zero 0 of S.

P r o o f. By Theorem II.4.3 of [20] the semigroup S is an orthogonal sum of Brandt semigroups and hence S is isomorphic to the orthogonal sum $\sum_{i\in\mathscr{I}} B_{\lambda_i}(G_i)$ of Brandt λ_i-extensions $B_{\lambda_i}(G_i)$ of groups G_i. We fix any $i_0 \in \mathscr{I}$. Since S is a topological inverse semigroup, Proposition II.2 [8] implies that $B_{\lambda_{i_0}}(G_{i_0})$ is a topological inverse semigroup. By Proposition 1, $B_{\lambda_{i_0}}(G_{i_0})$ is a pseudocompact topological Brandt λ_i-extension of pseudocompact topological group G_{i_0} in the category $\mathfrak{TISG}$ for some finite cardinal $\lambda_{i_0} \geq 1$. This completes the proof of the first assertion of the theorem.

The second statement of the theorem is trivial in the case when the set of indices $\mathscr{I}$ is finite. Therefore henceforth we assume that the set $\mathscr{I}$ is infinite.

Suppose on the contrary that $\mathscr{B}(0)$ is not a base at zero 0 of S. Then, there exists an open neighbourhood $U(0)$ of zero 0 such that $U(0)\cup \cup(B_{\lambda_{i_1}}(G_{i_1}) \cup \cdots \cup B_{\lambda_{i_n}}(G_{i_n}))^* \neq S$ for finitely many indices $i_1,\ldots,i_n \in \mathscr{I}$. Let $V(0) \subseteq U(0)$ be an open neighbourhood of 0 in S such that $V(0)\cdot V(0)\cdot V(0) \subseteq \subseteq U(0)$. Then we have that $V(0) \cup (B_{\lambda_{i_1}}(G_{i_1}) \cup \ldots \cup B_{\lambda_{i_n}}(G_{i_n}))^* \neq S$. We state that there exist a sequence of distinct points $\{x_k\}_{k\in\mathbb{N}}$ of the semigroup S and a sequence of open subsets $\{U(x_k)\}_{k\in\mathbb{N}}$ of S such that the following conditions hold:

(***i***) $x_k \in U(x_k) \subseteq B_{\lambda_{i_k}}(G_{i_k})$ for some $i_k \in \mathscr{I}$;

(***ii***) if $x_{k_1}, x_{k_2} \in B_{\lambda_{i_k}}(G_{i_k})$ for some $i_k \in \mathscr{I}$, then $k_1 = k_2$;

(***iii***) $\bigcup_{k\in\mathbb{N}} U(x_k) \subseteq S \setminus V(0)$.

Otherwise we have that $V(0)$ is a dense subset of the subspace

$$S' = S \setminus \cup(B_{\lambda_{i_1}}(G_{i_1}) \cup \ldots \cup B_{\lambda_{i_n}}(G_{i_n}))^*,$$

for some positive integer n. Since S' with induced operation from S is a primitive inverse semigroup Proposition 3 implies that $V(0)\cdot V(0)\cdot V(0) = S'$ which opposes the choice of the neighbourhood $U(0)$. The obtained contradiction implies that there exists finitely many indices $i_1,\dots,i_n,\dots,i_m \in \mathscr{I}$ where $m > n$ such that

$$U(0) \cup (B_{\lambda_{i_1}}(G_{i_1}) \cup \dots \cup B_{\lambda_{i_n}}(G_{i_n}) \cup \dots \cup B_{\lambda_{i_m}}(G_{i_n}))^* = S.$$

This completes the proof of the theorem. ♦

Proposition 4. *Let S be a primitive Hausdorff pseudocompact topological inverse semigroup which is topologically isomorphic to the orthogonal sum $\sum_{i\in\mathscr{I}} B_{\lambda_i}(G_i)$ of topological Brandt λ_i-extensions $B_{\lambda_i}(G_i)$ of topological groups G_i in the category $\mathfrak{TISG}$ for some cardinals $\lambda_i \geq 1$. Then the following conditions hold:*

(***i***) *the space S is Tychonoff if and only if for every $i \in \mathscr{I}$ the space of the topological group G_i is Tychonoff, i. e., G_i is a T_0-space;*

(***ii***) *the space S is normal if and only if for every $i \in \mathscr{I}$ the space of the topological group G_i is normal.*

P r o o f. We observe that the T_0-topological space of a topological group is Tychonoff (see Theorem 2.6.4 in [19]).

(***i***) Implication $(\Rightarrow)$ follows from Theorem 2.1.6 of [9].

$(\Leftarrow)$. Suppose that for every $i \in \mathscr{I}$ the space of the topological group G_i is Tychonoff. We fix an arbitrary element $x \in S$. First we consider the case when $x \neq 0$. Then there exists a non-zero $\mathscr{H}$-class H which contains x. By Proposition 12 from [2] there exists $i \in \mathscr{I}$ such that the topological space H is homeomorphic to the topological group G_i. Then by Proposition 1.5.8 from [9] for every open neighborhood $U(x)$ of x in H there exists a continuous map $f: H \to [0,1]$ such that $f(x) = 0$ and $f(y) = 1$ for all $y \in H \setminus U(x)$. We define the map $\tilde{f}: S \to [0,1]$ in the following way:

$$\tilde{f}(y) = \begin{cases} f(y), & y \in H, \\ 1, & y \in S \setminus H. \end{cases}$$

Since by Proposition 12 from [2] every non-zero $\mathscr{H}$-class is an open-and-closed subset of S we conclude that such defined map $\tilde{f}: S \to [0,1]$ is continuous.

Suppose that $x = 0$. We fix an arbitrary $U(0) = S \setminus (B_{\lambda_{i_1}}(G_{i_1}) \cup \dots$ $\dots \cup B_{\lambda_{i_n}}(G_{i_n}))^* \in \mathscr{B}(0)$. Then by Proposition 12 from [2], $U(0)$ is an open-and-closed subset of S. Thus we have that the map $f: S \to [0,1]$ defined by the formula

$$\tilde{f}(y) = \begin{cases} 0, & y \in U(0), \\ 1, & y \in S \setminus U(0), \end{cases}$$

is continuous, and hence by Proposition 1.5.8 from [9] the space S is Tychonoff.

Next we shall prove statement (***ii***).

$(\Rightarrow)$. Suppose that S is a normal space. By Lemma 9 of [2] we have that every $\mathscr{H}$-class of S is a closed subset of S. Then by Theorem 2.1.6 from [9] we have that every $\mathscr{H}$-class of S is a normal subspace of S and hence Definition 1 and Proposition 12 of [2] imply that for every $i \in \mathscr{I}$ the space of the topological group G_i is normal.

$(\Leftarrow)$. Suppose that for every $i \in \mathscr{I}$ the space of the topological group G_i is normal. Let F_1 and F_2 be arbitrary closed disjoint subsets of S.

At first we consider the case when $0 \notin F_1 \cup F_2$. Then there exists an open neighbourhood $U(0)$ of zero in S such that $F_1 \cup F_2 \subseteq S \setminus U(0)$, i. e., there exist finitely many $i_1, \ldots, i_n \in \mathscr{I}$ such that

$$F_1 \cup F_2 \subseteq \left(B_{\lambda_{i_1}}(G_{i_1}) \cup \ldots \cup B_{\lambda_{i_n}}(G_{i_n})\right) \setminus \{0\}.$$

By Corollary 8 of [2] every non-zero $\mathscr{H}$-class of S is open subset in S, and hence we get that the subspace $\left(B_{\lambda_{i_1}}(G_{i_1}) \cup \ldots \cup B_{\lambda_{i_n}}(G_{i_n})\right) \setminus \{0\}$ of S is a topological sum of some non-zero $\mathscr{H}$-classes of S, and hence it is an open subspace of S. Then by Theorem 2.2.7 from [9] we have that $\left(B_{\lambda_{i_1}}(G_{i_1}) \cup \ldots \cup B_{\lambda_{i_n}}(G_{i_n})\right) \setminus \{0\}$ is a normal space. Therefore, there exist disjoint open neighbourhoods $V(F_1)$ and $V(F_2)$ of F_1 and F_2 in $\left(B_{\lambda_{i_1}}(G_{i_1}) \cup \ldots \cup B_{\lambda_{i_n}}(G_{i_n})\right) \setminus \{0\}$, and hence in S, respectively.

Suppose that $0 \in F_1 \cup F_2$. Without loss of generality we can assume that $0 \in F_1$. Then there exist finitely many $i_1, \ldots, i_n \in \mathscr{I}$ such that

$$F_2 \subseteq \left(B_{\lambda_{i_1}}(G_{i_1}) \cup \ldots \cup B_{\lambda_{i_n}}(G_{i_n})\right) \setminus \{0\}.$$

The assumption of the proposition implies that the set $\left(B_{\lambda_{i_1}}(G_{i_1}) \cup \right.$ $\left. \cup \ldots \cup B_{\lambda_{i_n}}(G_{i_n})\right) \setminus \{0\}$ is closed in S and hence

$$\tilde{F}_1 = F_1 \cap \left(\left(B_{\lambda_{i_1}}(G_{i_1}) \cup \ldots \cup B_{\lambda_{i_n}}(G_{i_n})\right) \setminus \{0\}\right)$$

is a closed subset of S, as well. Then the previous arguments of the proof imply that

$$\left(B_{\lambda_{i_1}}(G_{i_1}) \cup \ldots \cup B_{\lambda_{i_n}}(G_{i_n})\right) \setminus \{0\}$$

is a normal space, and hence there exist open disjoint neighbourhoods $W(\tilde{F}_1)$ and $U(F_2)$ of the closed sets $\tilde{F}_1$ and F_2 in $\left(B_{\lambda_{i_1}}(G_{i_1}) \cup \ldots \cup B_{\lambda_{i_n}}(G_{i_n})\right) \setminus \{0\}$, and hence in S, respectively. We put

$$U(F_1) = S \setminus \left(B_{\lambda_{i_1}}(G_{i_1}) \cup \ldots \cup B_{\lambda_{i_n}}(G_{i_n})\right)^* \cup W(\tilde{F}_1).$$

Then we have that $U(F_1)$ and $U(F_2)$ are open disjoint neighbourhoods of F_1 and F_2 in S, respectively. This completes the proof of statement (***ii***). ♦

Theorem 1 and Proposition 4 imply the following

Corollary 1. *Every primitive Hausdorff pseudocompact topological inverse semigroup S is a Tychonoff topological space. Moreover the topological space of S is normal if and only if every maximal subgroup of S is a normal subspace.*

By Theorem 3.10.21 from [9] every normal pseudocompact space is countably compact, and hence Corollary 1 implies the following

Corollary 2. *Every primitive Hausdorff pseudocompact topological inverse semigroup S such that every maximal subgroup of S is a normal subspace in S is countably compact.*

Proposition 5. *Every primitive pseudocompact topological inverse semigroup S is a continuous (non-homomorphic) image of the product $\tilde{E}_S \times G_S$, where $\tilde{E}_S$ is a compact semilattice and G_S is a pseudocompact topological group.*

P r o o f. By Theorem 1 the topological semigroup S is topologically isomorphic to the orthogonal sum $\sum_{i \in \mathscr{I}} B_{\lambda_i}(G_i)$ of topological Brandt λ_i-extensions $B_{\lambda_i}(G_i)$ of pseudocompact topological groups G_i in the category $\mathfrak{TIS}\mathfrak{G}$ for some finite cardinals $\lambda_i \geq 1$ and the family defined by formula (1) determines the base of the topology at zero of S.

Fix an arbitrary $i \in \mathscr{I}$. Then by Proposition 1 (***iv***) the set $E(B_{\lambda_i}(G_i))$ is finite. Suppose that $|E(B_{\lambda_i}(G_i))| = n_i + 1$ for some integer n_i. Then we have that $\lambda_i = n_i \geq 1$. On the set $E_i = (\lambda_i \times \lambda_i) \bigcup \{0\}$, where $0 \notin \lambda_i \times \lambda_i$ we define the binary operation in the following way

$$(\alpha,\beta) \cdot (\gamma,\delta) = \begin{cases} (\alpha,\beta), & \text{if } (\alpha,\beta) = (\gamma,\delta), \\ 0, & \text{otherwise,} \end{cases}$$

and $0 \cdot (\alpha,\beta) = (\alpha,\beta) \cdot 0 = 0 \cdot 0 = 0$ for all $\alpha,\beta,\gamma,\delta \in \lambda_i$. Simple verifications show that E_i with such defined operation is a semilattice and every non-zero idempotent of E_i is primitive.

By $\tilde{E}_S$ we denote the orthogonal sum $\sum_{i \in \mathscr{I}} E_i$. It is obvious that $\tilde{E}_S$ is a semilattice and every non-zero idempotent of $\tilde{E}_S$ is primitive. We determine on $\tilde{E}_S$ the topology of the Alexandroff one-point compactification τ_A: all non-zero idempotents of $\tilde{E}_S$ are isolated points in $\tilde{E}_S$ and the family

$$\mathscr{B}(0) = \{U : U \ni 0 \text{ and } \tilde{E}_S \setminus U \text{ is finite}\}$$

is the base of the topology τ_A at zero $0 \in \tilde{E}_S$. Simple verifications show that $\tilde{E}_S$ with the topology τ_A is a Hausdorff compact topological semilattice. Later we denote $(\tilde{E}_S, \tau_A)$ by $\tilde{E}_S$.

Let $G_S = \prod_{j \in \mathscr{I}} G_i$ be the direct product of pseudocompact groups G_i, $i \in \mathscr{I}$, with the Tychonoff topology. Then by Comfort – Ross Theorem (see Theorem 1.4 in [6]) we get that G_S is a pseudocompact topological group. Also by Corollary 3.10.27 from [9] we have that the product $\tilde{E}_S \times G_S$ is a pseudocompact space.

Later, for every $i \in \mathscr{I}$ by $\pi_i : G_S = \prod_{j \in \mathscr{I}} G_i \to G_i$ we denote the projection on the i-th factor.

Now, for every $i \in \mathscr{I}$ we define the map $f_i : E_i \times G_S \to B_{\lambda_i}(G_i)$ by the formulae $f_i((\alpha,\beta),g) = (\alpha,\pi_i(g),\beta)$ and $f_i(0,g) = 0_i$, where 0_i is zero of the semigroup $B_{\lambda_i}(G_i)$, and put $f = \bigcup_{i \in \mathscr{I}} f_i$. It is obvious that the map $f : \tilde{E}_S \times G_S \to S$ is well defined. The definition of the topology τ_A on $\tilde{E}_S$ implies that for every $((\alpha,\beta),g) \in E_i \times G_i \subseteq \tilde{E}_S \times G_i$ the set $\{(\alpha,\beta)\} \times G_i$ is open in $\tilde{E}_S \times G_S$ and hence the map f is continuous at the point $((\alpha,\beta),g)$. Also for every $U(0) = S \setminus (B_{\lambda_{i_1}}(G_{i_1}) \cup \ldots \cup B_{\lambda_{i_n}}(G_{i_n}))^*$ the set $f^{-1}(U(0)) = (\tilde{E}_S \setminus ((\lambda_{i_1} \times \lambda_{i_1}) \cup \ldots \cup (\lambda_{i_n} \times \lambda_{i_n}))) \times G_S$ is open in $\tilde{E}_S \times G_S$, and hence the map f is continuous. ♦

The following theorem is an analogue of Comfort – Ross Theorem for primitive pseudocompact topological inverse semigroup.

Theorem 2. *Let $\{S_i : i \in \mathscr{I}\}$ be a non-empty family of primitive Hausdorff pseudocompact topological inverse semigroups. Then the direct product $\prod_{j\in\mathscr{I}} S_j$ with the Tychonoff topology is a pseudocompact topological inverse semigroup.*

P r o o f. Since the direct product of the non-empty family of topological inverse semigroups is a topological inverse semigroup, it is sufficient to show that the space $\prod_{j\in\mathscr{I}} S_j$ is pseudocompact. Let $\tilde{E}_{S_j}$, G_{S_j}, and $f_j : \tilde{E}_{S_j} \times G_{S_j} \to S_j$ be, respectively, the semilattice, the group and the map, defined in the proof of Proposition 5 for any $j \in \mathscr{I}$. Since the space $\prod_{j\in\mathscr{I}} (\tilde{E}_{S_j} \times G_{S_j})$ is homeomorphic to the product $\prod_{j\in\mathscr{I}} \tilde{E}_{S_j} \times \prod_{j\in\mathscr{I}} G_{S_j}$ we conclude that by Theorem 3.2.4, Corollary 3.10.27 from [9] and Theorem 1.4 from [6] the space $\prod_{j\in\mathscr{I}} (\tilde{E}_{S_j} \times G_{S_j})$ is pseudocompact. Now, since the map $\prod_{j\in\mathscr{I}} f_j : \prod_{j\in\mathscr{I}} (\tilde{E}_{S_j} \times G_{S_j}) \to \prod_{j\in\mathscr{I}} S_j$ is continuous we have that $\prod_{j\in\mathscr{I}} S_j$ is a pseudocompact topological space. ♦

Theorem 2 implies the following

Corollary 3. *Let $\{S_i : i \in \mathscr{I}\}$ be a non-empty family of Brandt Hausdorff pseudocompact topological inverse semigroups. Then the direct product $\prod_{j\in\mathscr{I}} S_j$ with the Tychonoff topology is a pseudocompact topological inverse semigroup.*

Remark 2. E. K. van Douwen [7] showed that Martin's Axiom implies the existence of two countably compact groups (without non-trivial convergent sequences) such that their product is not countably compact. Hart and van Mill [18] showed that Martin's Axiom for countable posets implies the existence of a countably compact group the square of which is not countably compact. Tomita in [21] showed that under $MA_{\text{countable}}$ for each positive integer k there exists a group such that its k-th power is countably compact but its $2k$-th power is not. In particular, there was proved that for each positive integer k there exists $\ell = k,\ldots,2k-1$ and a group the ℓ-th power of which is not countably compact. In [22] Tomita constructed a topological group under $MA_{\text{countable}}$ the square of which is countably compact but its cube is not. Also, Tomita in [23] showed that the existence of $2^{\mathfrak{c}}$ mutually incorparable selective ultrafilters and $2^{\mathfrak{c}} = 2^{2^{\mathfrak{c}}}$ imply that there exists a topological group G such that G^{γ} is countably compact for all cardinals $\gamma < æ$, but $G^{æ}$ is not countably compact for every cardinal $æ \leq 2^{\mathfrak{c}}$. Using these results and the construction of finite topological Brandt λ^0-extensions proposed in [17] we may show that statements similar to aforementioned hold for Hausdorff countably compact Brandt topological inverse semigroups and hence for Hausdorff countably compact primitive topological inverse semigroups.

2. The Stone – Čech compactification of a pseudocompact primitive topological inverse semigroup. Let a Tychonoff topological space X be a topological sum of subspaces A and B, i. e., $X = A \oplus B$. It is obvious that every continuous map $f : A \to K$ from A into a compact space K (respectively, $f : B \to K$ from B into a compact space K) extends to a continuous map $\hat{f} : X \to K$. This implies the following

Proposition 6. *If a Tychonoff topological space X is a topological sum of subspaces A and B, then βX is equivalent to $\beta A \oplus \beta B$.*

The following theorem describes the structure of the Stone – Čech compactification of a primitive pseudocompact topological inverse semigroup.

Theorem 3. *Let S be a primitive pseudocompact topological inverse semigroup. Then the Stone – Čech compactification of S admits a structure of primitive topological inverse semigroup with respect to which the inclusion mapping of S into βS is a topological isomorphism. Moreover, βS is topologically isomorphic to the orthogonal sum $\sum_{i \in \mathscr{I}} B_{\lambda_i}(\beta G_i)$ of topological Brandt λ_i-extensions $B_{\lambda_i}(\beta G_i)$ of compact topological groups βG_i in the category $\mathfrak{TISG}$ for some finite cardinals $\lambda_i \geq 1$.*

P r o o f. By Theorem 1, every primitive pseudocompact topological inverse semigroup S is topologically isomorphic to the orthogonal sum $\sum_{i \in \mathscr{I}} B_{\lambda_i}(G_i)$ of topological Brandt λ_i-extensions $B_{\lambda_i}(G_i)$ of pseudocompact topological groups G_i in the category $\mathfrak{TISG}$ for some finite cardinals $\lambda_i \geq 1$, such that any non-zero $\mathscr{H}$-class of S is an open-and-closed subset of S, and the family $\mathscr{B}(0)$ defined by formula (1) determines a base of the topology at zero 0 of S.

By Theorem 2, $S \times S$ is a pseudocompact topological space. Now by Theorem 1 of [10], we have that $\beta(S \times S)$ is equivalent to $\beta S \times \beta S$, and hence by Theorem 1.3 [1], S is a subsemigroup of the compact topological semigroup βS.

By Proposition 6 for every non-zero $\mathscr{H}$-class $(G_i)_{k,\ell}$, $k, \ell \in \lambda_i$, we have that $\mathrm{cl}_{\beta S}((G_i)_{k,\ell})$ is equivalent to $\beta (G_i)_{k,\ell}$, and hence it is equivalent to βG_i. Therefore we get that $\sum_{i \in \mathscr{I}} B_{\lambda_i}(G_i) \subseteq \beta S$. Suppose that $\sum_{i \in \mathscr{I}} B_{\lambda_i}(G_i) \neq \beta S$. We fix an arbitrary $x \in \beta S \setminus \sum_{i \in \mathscr{I}} B_{\lambda_i}(G_i)$. Then Hausdorffness of βS implies that there exist open neighbourhoods $V(x)$ and $V(0)$ of the points x and 0 in βS, respectively, and there exist finitely many $i_1, \ldots, i_n \in \mathscr{I}$ such that $V(0) \cap \beta S \supseteq S \setminus (B_{\lambda_{i_1}}(G_{i_1}) \cup \ldots \cup B_{\lambda_{i_n}}(G_{i_n}))^*$. Then we have that

$$V(x) \cap S \subseteq (B_{\lambda_{i_1}}(G_{i_1}) \cup \ldots \cup B_{\lambda_{i_n}}(G_{i_n}))^* \subseteq$$

$$\subseteq (B_{\lambda_{i_1}}(\beta G_{i_1}) \cup \ldots \cup B_{\lambda_{i_n}}(\beta G_{i_n}))^*,$$

and since by Theorem 1, λ_i is finite for every $i \in \mathscr{I}$, we get a contradiction with the initial assumption.

This completes the proof of the theorem. ♦

Theorem 3 implies the following

Corollary 4. *Let S be a primitive countably compact topological inverse semigroup. Then the Stone – Čech compactification of S admits a structure of primitive topological inverse semigroup with respect to which the inclusion mapping of S into βS is a topological isomorphism.*

Remark 3. Theorem 3 and Corollary 4 give the positive answer to the Question 1, which we posed in [2].

We define the series of categories as follows:

(*i*) Let $\mathbf{Ob}(\mathscr{B}^*(\mathscr{CCTG}))$ be all Hausdorff 0-simple countably compact topological inverse semigroups;

let $\mathbf{Ob}(\mathscr{B}^*(\mathscr{PCTG}))$ be all Hausdorff pseudocompact topological inverse Brandt semigroups;

let $\mathbf{Ob}(\mathscr{PPCTG})$ be all primitive Hausdorff pseudocompact topological inverse semigroups;

let $\mathbf{Ob}(\mathscr{PCCTG})$ be all primitive Hausdorff pseudocompact topological inverse semigroups.

(*ii*) Let $\mathbf{Mor}(\mathscr{B}^*(\mathscr{CCTG})))$, $\mathbf{Mor}(\mathscr{B}^*(\mathscr{PCTG}))$, $\mathbf{Mor}(\mathscr{PPCTG})$, and $\mathbf{Mor}(\mathscr{PCCTG})$ be continuous homomorphisms of corresponding topological inverse semigroups.

Comfort and Ross [6] proved that the Stone – Čech compactification of a pseudocompact topological group is a topological group. Therefore the functor of the Stone – Čech compactification β from the category of pseudocompact topological groups back into itself determines a monad. Similar result Gutik and Repovš proved in [17] for the category of all Hausdorff 0-simple countably compact topological inverse semigroups $\mathscr{B}^*(\mathscr{CCTG})$. In the our case by Theorem 3 and Corollary 4 we get the same

Corollary 5. *The functor $\beta : \mathscr{B}^*(\mathscr{CCTG}) \to \mathscr{B}^*(\mathscr{CCTG})$ of the Stone – Čech compactification (respectively, $\beta : \mathscr{B}^*(\mathscr{PCTG}) \to \mathscr{B}^*(\mathscr{PCTG})$, $\beta : \mathscr{PPCTG} \to \to \mathscr{PPCTG}$, and $\beta : \mathscr{PCCTG} \to \mathscr{PCCTG}$) determines a monad.*

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ПСЕВДОКОМПАКТНІ ПРИМІТИВНІ ТОПОЛОГІЧНІ ІНВЕРСНІ НАПІВГРУПИ

Вивчаються гаусдорфові псевдокомпактні примітивні топологічні інверсні напівгрупи. Описано структуру таких напівгруп і показано, що тихоновський добуток сім'ї псевдокомпактних примітивних топологічних інверсних напівгруп є псевдокомпактним топологічним простором. Також доведено, що компактифікація Стоуна – Чеха псевдокомпактної примітивної топологічної інверсної напівгрупи є компактною примітивною топологічною інверсною напівгрупою.

ПСЕВДОКОМПАКТНЫЕ ПРИМИТИВНЫЕ ТОПОЛОГИЧЕСКИЕ ИНВЕРСНЫЕ ПОЛУГРУППЫ

Изучаются хаусдорфовы псевдокомпактные примитивные топологические инверсные полугруппы. Описана структура таких полугрупп и показано, что тихоновское произведение семьи псевдокомпактных примитивных топологических инверсных полугрупп является псевдокомпактным топологическим пространством. Также доказано, что компактификация Стоуна – Чеха псевдокомпактной примитивной топологической инверсной полугруппы является компактной примитивной топологической инверсной полугруппой.

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