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MODE I CRACK INITIATION IN ORTHOTROPIC VISCOELASTIC PLATE UNDER BIAXIAL LOADING

The subcritical propagation of a crack in orthotropic viscoelastic plate under time-constant biaxial external loading is investigated on the basis of generalization of Leonov – Panasyuk – Dugdale crack model for the case of orthotropic materials, which satisfy a strength condition of arbitrary form. The crack is directed along one of the anisotropy axes with external loads being applied parallel and perpendicularly to it. For finding the rheological characteristics of composite material the method of operator continued fractions is applied. The relationships for determining duration of incubational and transitional periods of crack propagation are determined on the basis of on the Volterra principle and solution of the corresponding elastic problem. The influence of the biaxiality of external loading on the safe loading and the subcritical crack growth is analyzed within the framework of the critical crack opening displacement criterion.

Introduction. Data of experimental researches [1] show stable crack propagation under low values of strength in viscoelastic materials. In connection with large mathematical complication in solving crack problems in viscoelastic materials numerical methods (such as a finite element method and boundary element method) are widely used [8, 17, 18]. For theoretical research on subcritical crack propagation in [1] the theory of subcritical crack growth in anisotropic viscoelastic medium was proposed. Efficiency of the proposed theory was demonstrated by solving the wide range of new problems on the long-term fracture for different viscoelastic materials [5, 10, 12, 13, 15]. The review of works on fracture mechanics of viscoelastic bodies is given in [11].

Much of the research on fracture of anisotropic viscoelastic cracked bodies based on the use of ordinary and modified δ_c-models of the crack [3, 19]. However, these models, as in the case of isotropic and anisotropic materials, do not allow to take into account the presence of components of the external load acting along the crack. However, as shown by experiments [4, 6], even in the case of static loading biaxiality significantly affects the ultimate state of bodies with cracks and, consequently, we can expect that this effect alone will be a prolonged exposure to stress.

To study such problems several nonclassic approaches which take into account the effect of stresses, acting along a crack, on the fracture-toughness characteristics were developed. These approaches consist in the following:

1. In studying the problems on compression of cracked bodies by forces directed along crack planes, it was proposed by Guz to take the criterion of stability loss of a material in a local near-crack tip region as a fracture criterion. The basic results in such formulation for different loading schemes and crack locations are given in [2, 7].

2. The approach based on a more complete allowance for distribution of stresses near the mode I edge crack, and not just the singular part, was proposed by Larsson and Carlsson [16]. According to this approach, it is assumed to introduce the second parameter T (in addition to K_I or J_I), which is a nonregular term in the expansion of stresses in the vicinity of a crack tip.

3. The approach based on the conception of a crack-tip process zone and on the additional necessity to satisfy the strength criterion in this zone was proposed in [14]. This approach allows to generalize the Dugdale model of crack to the case of orthotropic materials. The use of the proposed model enables one to take into account the influence of external load components acting along crack on bodies fracture.

In the present work the subcritical crack growth in orthotropic viscoelastic plate under biaxial loading is investigated based on the proposed crack model [14]. The durations of the incubation and transitional periods of subcritical crack growth are determined on the basis of Volterra principle and solving the corresponding problem of elasticity problem. The numerical results are obtained for a specific material. The influence of the degree of biaxiality of the external loading on the process of subcritical crack development is established.

Crack model. Consider a thin orthotropic plate with a crack of length $2\ell(t)$ oriented along the Ox-orthotropy axis and subjected to the action of the normal time-independent loads $\sigma_x^\infty = q$, $\sigma_y^\infty = p > 0$ applied at infinity.

Fracture of the plate material is described by the strength criterion in the general form

$$F(\sigma_1, \sigma_2, C_i) = 0\,, \tag{1}$$

where σ_1, σ_2 are the principal stresses, C_i are the material constants.

The strength condition (1) is considered as an example the Mises – Hill criterion. This criterion in the case of a plane stress state is described by

$$\frac{\sigma_x^2}{(\sigma_1^0)^2} + \frac{\sigma_y^2}{(\sigma_2^0)^2} - \frac{\sigma_x \sigma_y}{\sigma_1^0 \sigma_2^0} = 1\,, \tag{2}$$

where σ_1^0, σ_2^0 are the ultimate strengths in the x- and y-axis directions, respectively. Numerical calculations are made for $\sigma_1^0/\sigma_2^0 = 0.8$.

To investigate long-term crack propagation by means of the theory of subcritical crack growth in orthotropic viscoelastic material [1] the modified Dugdale's model [14] is used. The following assumptions are made:

1°. Process zones arising near a crack tip take the form of a narrow wedge-shaped sections on the crack continuation. In modeling, they can be replaced by slits of length d whose faces are acted upon by uniformly distributed coordinate-independent compressive stresses σ_y^0 (Fig. 1).

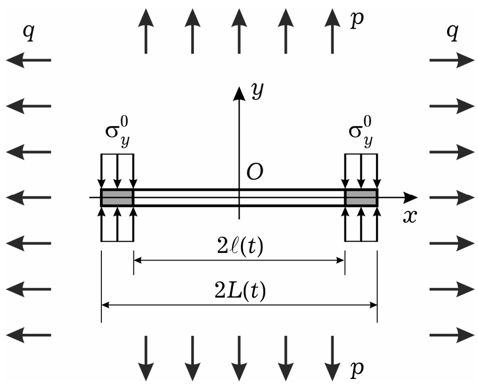


Fig. 1

2°. Stress components σ_x^0, σ_y^0 in the process zone don't depend on time and satisfy the strength criterion (1) and continuity condition on the crack front.

3°. The stress components are finite everywhere over the whole region.

The stress components σ_x^0, σ_y^0 are determined by the system of equations [14]

$$\sigma_x^0 = \beta(\sigma_y^0 - p) + q\,,\; F(\sigma_x^0, \sigma_y^0, C_i) = 0\,, \tag{3}$$

where $\beta = \sqrt{E_1^0/E_2^0}$, E_1^0, E_2^0 are the elastic moduli along the orhotropy axes.

The length of a process zone $d(t)$ is determined as [14]

$$\frac{\ell(t)}{L(t)} = \frac{\ell(t)}{\ell(t) + d(t)} = \cos\frac{\pi p}{2\sigma_y^0}. \tag{4}$$

Viscoelastic crack opening displacement. The solution of viscoelastic problem is obtained as a result of applying the Volterra principle to the known analytical solution [1]. The viscoelastic displacement of crack faces on the interval $[\ell(t), L(t)]$ is expressed in the form

$$\delta(x,t) = T^* \cdot \delta_0(x,t), \tag{5}$$

where $\delta_0(x,t)$ and T^* are determined from corresponding elastic solution as [14]

$$\delta_0(x,t) =$$

$$= -2\frac{\sigma_y^0}{\pi}\left\{x \ln\frac{\left(L^2(t) - x\ell(t) + \sqrt{(L^2(t)-x^2)(L^2(t)-\ell^2(t))}\right)(\ell(t)+x)}{\left(L^2(t) + x\ell(t) + \sqrt{(L^2(t)-x^2)(L^2(t)-\ell^2(t))}\right)(\ell(t)-x)} - \right.$$

$$\left. - 2\ell(t)\ln\frac{\sqrt{L^2(t)-x^2} + \sqrt{L^2(t)-\ell^2(t)}}{\sqrt{x^2-\ell^2(t)}}\right\}, \tag{6}$$

$$T^* = \frac{1}{\sqrt{E_1^* E_2^*}}\sqrt{2\left(\sqrt{\frac{E_1^*}{E_2^*}} - \nu_{21}^*\right) + \frac{E_1^*}{G_{12}^*}}. \tag{7}$$

In (7) E_1^*, E_2^*, G_{12}^*, ν_{12}^* are Volterra's integral operators of the form

$$\frac{1}{E_1^*} = \frac{1}{E_1^0}[1 + \lambda_1 R^*(\beta_1)], \qquad \frac{1}{E_2^*} = \frac{1}{E_2^0}[1 + \lambda_2 R^*(\beta_2)],$$

$$\frac{1}{G_{12}^*} = \frac{1}{G_{12}^0}[1 + \lambda_G R^*(\beta_G)], \qquad \frac{1}{\nu_{21}^*} = \frac{1}{\nu_{21}^0}[1 + \lambda_\nu R^*(\beta_\nu)], \tag{8}$$

E_1^0, E_2^0, G_{12}^0, ν_{21}^0 are instantaneous elastic moduli of the material; λ_1, λ_2, λ_G, λ_ν, β_1, β_2, β_G, β_ν are rheological parameters of the material; $R^*(\beta)$ are resolvent operators of the form

$$R^*(\beta) \cdot f(t) = \int_0^t R(t-\tau, \beta) f(t)\, d\tau. \tag{9}$$

Then, the function $\delta_0(x,t)$ in the crack tip $|x| = \ell(t)$, $y = 0$ with allowance for (6) can be written down as follows

$$\delta_0(\ell,t) = \frac{4\sigma_y^0 \ell(t)}{\pi}\ln\sec\frac{\pi p}{2\sigma_y^0}. \tag{10}$$

Reducing the number of operators. For investigation of crack growth the function of integral operators (7) must be presented in form (9). For this purpose the method of integral continued fractions is applied. As is known for resolvent operators

$$(1 + \lambda R^*(\beta))^{-1} = 1 - \lambda R^*(\beta - \lambda). \tag{11}$$

From the theory of continued fractions [9]

$$\sqrt{1+\lambda R^*(\beta)} = 1 + 2\mathop{K}_{m=1}^{\infty} \frac{0.25\lambda R^*(\beta)}{1} \approx$$

$$\approx 1 + 2\mathop{K}_{m=1}^{M} \frac{0.25\lambda R^*(\beta)}{1} \overset{M=2}{=} 1 + \frac{0.5\lambda R^*(\beta)}{1 + 0.25\lambda R^*(\beta)}. \qquad (12)$$

By using (11) and (12) integral operator (7) can be written as

$$T^* = T_0 \left[1 + \sum_{j=1}^{7} \mu_j R^*(\gamma_j) \right], \qquad (13)$$

where

$$T_0 = \frac{1}{\sqrt{E_1^0 E_2^0}} \sqrt{2\left(\sqrt{\frac{E_1^0}{E_2^0}} - \nu_{21}^0\right) + \frac{E_1^0}{G_{12}^0}},$$

$$\gamma_i = \beta_i - 0.25\lambda_i, \quad i = 1,2, \quad \gamma_i = \beta_{i-2}^0, \quad i = 3,\dots,7,$$

$$\mu_1 = 0.5\lambda_1 \left(1 + \frac{0.5\lambda_2}{\gamma_1 - \gamma_2}\right)\left(1 + 0.5\sum_{j=1}^{5} \frac{\lambda_j^0}{\gamma_1 - \beta_j^0}\right),$$

$$\mu_2 = 0.5\lambda_2 \left(1 + \frac{0.5\lambda_1}{\gamma_2 - \gamma_1}\right)\left(1 + 0.5\sum_{j=1}^{5} \frac{\lambda_j^0}{\gamma_2 - \beta_j^0}\right),$$

$$\mu_i = 0.5\lambda_{j-2}^0 \left(1 + \frac{0.5\lambda_1}{\beta_{i-2}^0 - \gamma_1}\right)\left(1 + \frac{0.5\lambda_2}{\beta_{i-2}^0 - \gamma_2}\right), \quad i = 3,\dots,7, \qquad (14)$$

β_j^0, $j = 1,\dots,5$, are roots of equation

$$1 - 0.25\sum_{i=1}^{5} \frac{\lambda_i'}{\beta_i' - \beta^0} = 0,$$

where

$$\lambda_1' = -\frac{\sqrt{E_1^0/E_2^0}}{æ} \lambda_1 \left(1 + \frac{0.5\lambda_2}{\beta_1' - \beta_2'}\right),$$

$$\lambda_2' = -\frac{\sqrt{E_1^0/E_2^0}}{æ} \lambda_2 \left(1 + \frac{0.5\lambda_1}{\beta_1' - \beta_2'}\right),$$

$$\lambda_3' = -\frac{E_{11}^0/G_{12}^0}{æ} \lambda_1 \left(1 - \frac{\lambda_G}{\beta_5' - \beta_3'}\right),$$

$$\lambda_4' = -2\frac{\nu_{21}^0}{æ} \lambda_\nu,$$

$$\lambda_5' = \frac{E_{11}^0/G_{12}^0}{æ} \lambda_G \left(1 - \frac{\lambda_1}{\beta_5' - \beta_3'}\right),$$

$$\beta_i' = \beta_i - 0.75\lambda_i, \quad i = 1,2, \quad \beta_3' = \beta_1 - \lambda_1, \quad \beta_4' = \beta_\nu, \quad \beta_5' = \beta_G,$$

$$æ = 2\left(\sqrt{\frac{E_1^0}{E_2^0}} - \nu_{21}^0\right) + \frac{E_1^0}{G_{12}^0}.$$

λ_j^0, $j = 1,\dots,5$, are solutions of the system of equation

$$1 - \sum_{j=1}^{5} \frac{\lambda_j^0}{\beta_i' - \beta_j^0} = 0, \quad i = 1,\dots,5. \qquad (15)$$

Consider as the kernel of integral operator (8) the Rabotnov function

$$R(t-\tau,\gamma_j)=E_\alpha(t-\tau,\gamma_j)=\sum_{n=0}^{\infty}\frac{\gamma_j^n(t-\tau)^{n(1-\alpha)-\alpha}}{\Gamma[(n+1)(1-\alpha)]},\qquad 0\le\alpha<1,\quad \gamma_j<0\,,$$

where $\Gamma(x)$ is Euler's gamma-function.

As numerical example a composite with quartz material and a polyethylene matrix is investigated. Rheological characteristics of the material are as follows

$$E_{11}^0=11.7\cdot10^3\,\text{MPa},\qquad \lambda_1=0.0608\sec^{\alpha-1},\quad \beta_1=-0.1283\sec^{\alpha-1},$$

$$E_{22}^0=19.7\cdot10^3\,\text{MPa},\qquad \lambda_2=0.0180\sec^{\alpha-1},\quad \beta_2=-0.0928\sec^{\alpha-1},$$

$$G_{12}^0=0.637\cdot10^3\,\text{MPa},\qquad \lambda_G=0.1398\sec^{\alpha-1},\quad \beta_G=-0.0407\sec^{\alpha-1},$$

$$\nu_{12}^0=0.14\,,\ \lambda_\nu=\beta_\nu=0\,,\ \alpha=0.717\,.$$

Safe loading. In the case when deformation of viscoelastic bodies is described by the bounded integral operators there is a safe level of external loadings when crack growth does not occur for an arbitrarily large time [1]. The safe loading P_s is given by

$$\frac{\delta_c}{\delta(\ell,P_s)}=\frac{T_\infty}{T_0}\,,\tag{16}$$

where $T_\infty=T^*\cdot1\big|_{t=\infty}$, $T_0=T^*\cdot1\big|_{t=0}$.

In examined case according to (13) the long-term value T_∞ is defined as follows

$$T_\infty=T_0\left\{1+\sum_{i=1}^{7}\frac{\mu_i}{|\gamma_i|}\right\},\tag{17}$$

where μ_i, γ_i, $i=1,\ldots,7$ are defined by (14).

In this case in view of (10) the safe loading is determined by

$$\frac{4\sigma_y^0(p_s,q_s)\ell_0}{\pi}\ln\sec\frac{\pi p_s}{2\sigma_y^0(p_s,q_s)}=\frac{T_0}{T_\infty}\delta_c\,,\tag{18}$$

where (p_s,q_s) is the safe loading field.

Since fracture of elastic plate with crack under uniaxial tension $(q=0)$ is described by [14]

$$\frac{4T_0\sigma_y^0(p_*^{(0)},0)\ell_0}{\pi}\ln\sec\frac{\pi p_*^{(0)}}{2\sigma_y^0(p_*^{(0)},0)}=\delta_c\,,\tag{19}$$

where $p_*^{(0)}$ is the ultimate load in uniaxial tension, then by comparing (18) and (19) and taking into account (16) the safe loading field (p_s,q_s) can be defined by

$$\sigma_y^0(p_s,q_s)\ln\cos\frac{\pi p_s}{2\sigma_y^0(p_s,q_s)}=\frac{T_0}{T_\infty}\sigma_y^0(p_*^{(0)},0)\ln\cos\frac{\pi p_*^{(0)}}{2\sigma_y^0(p_*^{(0)},0)}\,.\tag{20}$$

Fig. 2 shows the safe loading fields obtained on base of (19) for various loads $p_*^{(0)}/\sigma_2^0=0.1,\,0.5,\,0.9$. Dash-dotted line corresponds to Mises – Hill strength condition (2), solid lines correspond to the safe loads (p_s,q_s), dashed lines correspond to ultimate loads (p_*, q_*) defined by according to the δ_c-criterion as [14]

$$\sigma_y^0(p_*, q_*) \ln\cos\frac{\pi p_*}{2\sigma_y^0(p_*, q_*)} = \sigma_y^0(p_*^{(0)}, 0) \ln\cos\frac{\pi p_*^{(0)}}{2\sigma_y^0(p_*^{(0)}, 0)}. \tag{21}$$

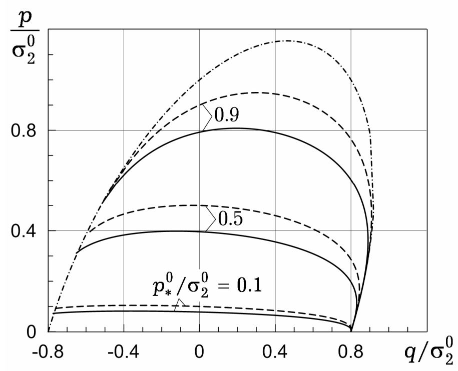


Fig. 2

It should be noted that development of crack in viscoelastic body with limited creep takes place only in the range of load change limited by curves defined by (19) and (21).

Incubational period. During the incubation period $(0 < t \le t_*)$ there is crack opening without growth. When the external loading does not change in time, the duration of the incubation period t_* is determined by [1]

$$\int_0^{t_*} R(\theta)\, d\theta = \frac{\delta_c}{\delta(\ell_0)} - 1, \tag{22}$$

where $\delta(\ell) = T_0\delta_0(\ell)$ is the elastic crack opening displacement in $x = \ell$.

Taking into account (16) the durability of the incubational period t_* can be defined by

$$\sum_{j=1}^{7} \mu_j \int_0^{t_*} R(\theta, \gamma_j)\, d\theta = \frac{\sigma_y^0(p_*^{(0)}, 0)}{\sigma_y^0(p, q)} \frac{\ln\cos\dfrac{\pi p_*^{(0)}}{2\sigma_y^0(p_*^{(0)}, 0)}}{\ln\cos\dfrac{\pi p}{2\sigma_y^0(p, q)}} - 1. \tag{23}$$

Consider new function

$$\Phi_2(\alpha, x) = \frac{1}{t^{1-\alpha}} \int_0^{t} E_\alpha(\theta, \gamma_i)\, d\theta = \sum_{n=0}^{\infty} \frac{x^n}{\Gamma[(n+1)(1-\alpha)+1]}, \quad x = \gamma_j t^{1-\alpha}. \tag{24}$$

Due to (22), (24) the durability of the incubational period t_* can be written as

$$t_*^{1-\alpha} \sum_{j=1}^{7} \mu_j \Phi_2(\alpha, \gamma_j t_*^{1-\alpha}) = \frac{\sigma_y^0(p_*^{(0)}, 0)}{\sigma_y^0(p, q)} \frac{\ln\cos\dfrac{\pi p_*^{(0)}}{2\sigma_y^0(p_*^{(0)}, 0)}}{\ln\cos\dfrac{\pi p}{2\sigma_y^0(p, q)}} - 1. \tag{25}$$

Fig. 3 shows the durability of the incubational period obtained on the basis of equation (25) for $p_*^{(0)}/\sigma_2^0 = 0.5$. Fig. 3*a* shows the durability of the incubational period t_* related to external loading p/σ_2^0 for different values of $q/\sigma_2^0 = -0.2, 0.0, 0.2$. Fig. 3*b* shows the durability of the incubational period t_*

related to external loading q/σ_2^0 for different values of $p/\sigma_2^0 = 0.44, 0.45, 0.46$.

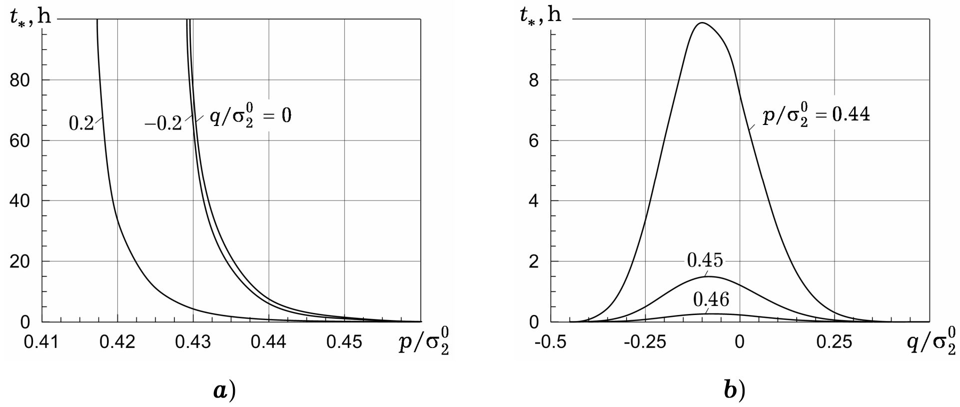


Fig. 3

Transitional period. During the transition period ($t_* < t \le t_I$), the crack begins its motion and travels a distance equal to the initial size of the process zone. This period starts at $t = t_*$ and continues until t_I when the crack length reaches the value $\ell_1 = \ell_0 + d$. Under time-constant load the equation of crack growth during the transition period has the form [1]

$$\delta[\ell(t)] + \delta[\ell(t), \ell_0] \int_0^{t_*} R(t-\tau)\, d\tau + \int_{t_*}^{t} R(t-\tau)\delta[\ell(t), \ell(\tau)]\, d\tau = \delta_c, \tag{26}$$

where $\delta[\ell(t), \ell_0]$ is the elastic crack opening displacement with length ℓ_0 in $x = \ell(t)$; $\delta[\ell(t), \ell(\tau)]$ is the elastic crack opening displacement with length $\ell(\tau)$ in $x = \ell(t)$.

The durability of transitional period $\Delta t_I = t_I - t_*$ is determined as

$$\delta(\ell_1) + \int_0^{\Delta t_I} R(\Delta t_I - \theta)\delta(\ell_1, \ell(\theta + t_*))\, d\theta = \delta_c. \tag{27}$$

As shown in [1] the next approximation is valid for some integral operators (for example, for Rabotnov operator)

$$R^* \cdot f(t) \cong k(\alpha)(R^* \cdot 1) f(t),$$

$$k(\alpha) = \frac{\sqrt{\pi}\,\Gamma(2-\alpha)}{2(2-\alpha)\Gamma(2.5-\alpha)}. \tag{28}$$

Then the durability of transitional period Δt_I is defined by

$$\frac{\delta_c}{\delta(\ell_1)} = 1 + k(\alpha) \int_0^{\Delta t_1} R(\theta)\, d\theta \tag{29}$$

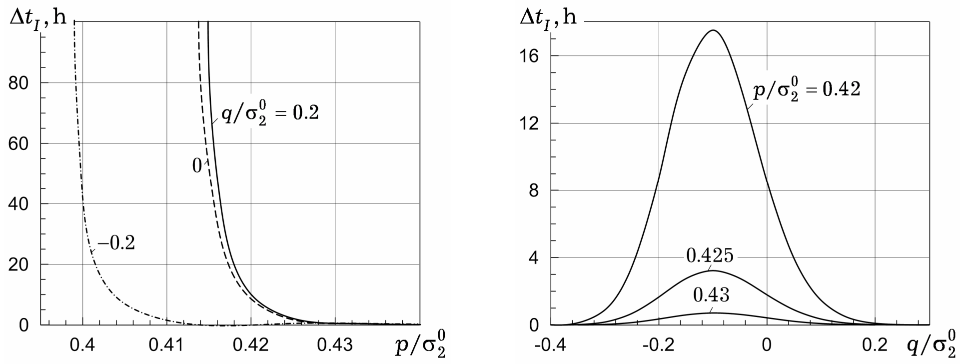
or, basing on (10) and (19),

$$1 + k(\alpha) \sum_{j=1}^{7} \mu_j \int_0^{\Delta t_1} R(\theta, \gamma_j)\, d\theta = \frac{\sigma_y^0(p_*^{(0)}, 0)}{\sigma_y^0(p, q)} \frac{\ln\cos\dfrac{\pi p_*^{(0)}}{2\sigma_y^0(p_*^{(0)}, 0)}}{\ln\cos\dfrac{\pi p}{2\sigma_y^0(p, q)}}. \tag{30}$$

Due to (24) transitional period durability Δt_I can be written as

$$k(\alpha)(\Delta t_I)^{1-\alpha}\sum_{j=1}^{7}\mu_j\Phi_2\left(\alpha,\gamma_j(\Delta t_I)^{1-\alpha}\right)=\frac{\sigma_y^0(p_*^{(0)},0)}{\sigma_y^0(p,q)}\frac{\ln\cos\dfrac{\pi p_*^{(0)}}{2\sigma_y^0(p_*^{(0)},0)}}{\ln\cos\dfrac{\pi p}{2\sigma_y^0(p,q)}}-1\,. \tag{31}$$

Fig. 4 shows the durability of transitional period Δt_I obtained on the basis of equation (31) for $p_*^{(0)}/\sigma_2^0 = 0.5$. Fig. 4*a* shows the durability of the transitional period Δt_I related to external loading p/σ_2^0 for different values of $q/\sigma_2^0 = -0.2, 0.0, 0.2$. Fig. 4*b* shows the durability of transitional period Δt_I related to external loading q/σ_2^0 for different values of $p/\sigma_2^0 = 0.42, 0.425, 0.43$.



a) ***b***)

Fig. 4

Conclusions. As can be seen from the results, a component of the external loading acting along the crack has a significant influence on the level of safe loading and the duration of different periods of subcritical crack growth in a viscoelastic body.

Thus, the proposed generalization of the Dugdale crack model on the case of orthotropic materials can efficiently solve not only problems of fracture mechanics of elastic bodies, but also the problem of long-term fracture of bodies with cracks.

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ПОЧАТКОВІ ЕТАПИ ДОКРИТИЧНОГО РОЗВИТКУ ТРІЩИНИ НОРМАЛЬНОГО ВІДРИВУ В ОРТОТРОПНІЙ В'ЯЗКОПРУЖНІЙ ПЛАСТИНІ В УМОВАХ ДВОВІСНОГО НАВАНТАЖЕННЯ

На основі модифікованої моделі Леонова – Панасюка – Дагдейла досліджено початкові етапи докритичного росту тріщини нормального відриву в ортотропній в'язкопружній пластині під дією постійного в часі двовісного зовнішнього навантаження. Для визначення реологічних характеристик матеріалу застосовано метод ланцюгових дробів. На основі принципу Вольтерра та розв'язку відповідної пружної задачі отримано співвідношення для визначення інкубаційного та перехідного періодів докритичного росту тріщини. У рамках δ_c *-критерію руйнування досліджено вплив двовісності зовнішнього навантаження на безпечне навантаження пластини з тріщиною і докритичний розвиток тріщини.*

НАЧАЛЬНЫЕ ЭТАПЫ ДОКРИТИЧЕСКОГО РОСТА ТРЕЩИНЫ НОРМАЛЬНОГО ОТРЫВА В ОРТОТРОПНОЙ ВЯЗКОУПРУГОЙ ПЛАСТИНЕ В УСЛОВИЯХ ДВУХОСНОГО НАГРУЖЕНИЯ

На основании модифицированной модели трещины Леонова – Панасюка – Дагдейла исследованы начальные этапы докритического роста трещины нормального отрыва в ортотропной вязкоупругой пластине под действием постоянного во времени двухосного внешнего нагружения. Для определения реологических параметров материала использован метод цепных дробей. На основании принципа Вольтера и решения соответствующей упругой задачи получены соотношения для определения длительности инкубационного и переходного периодов докритического роста трещины. В рамках δ_c *-критерия разрушения исследовано влияние двухосности внешней нагрузки на безопасное нагружение пластины с трещиной и докритический рост трещины.*

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