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MATHEMATICAL MODEL OF SCATTERING OF A POLARIZED WAVE ON IMPEDANCE STRIPS LOCATED ON SCREENED DIELECTRIC LAYER

Description of the processes of interaction of electromagnetic waves with a non-perfectly conducting gratings leads to the consideration of boundary value problems for the Helmholtz equation with boundary conditions of the third kind. The original scattering problem of polarized wave at the reflecting structure was reduced to a system of boundary integral equations. Derivation of integral equations is based on the method of parametric representations of integral operators.

The description of the properties of real electrodynamic systems is impossible without taking into account the conductivity of materials [10]. The existence of the energy loss is often described by using the Shchukin – Leontovich boundary-value condition on the surface of non-perfectly conducting screens:

$$[\mathbf{n},\mathbf{E}] = -Z_s[\mathbf{n},[\mathbf{n},\mathbf{H}]], \tag{1}$$

where $(\mathbf{E},\mathbf{H})$ is the total electromagnetic field, Z_s is the surface impedance of the structure, $\mathbf{n}$ is the normal vector to the surface. One effective way construct the mathematical models of interaction of electromagnetic fields with non-perfectly conducting electrodynamic structures is to use the method of parametric representation of singular integral transforms [2, 9]. The obtained systems of boundary integral equations are solved numerically by the method of discrete singularities [2, 11].

With this approach, the electromagnetic waves scattering by various electromagnetic structures has been investigated in [3, 4, 7, 12]. It is interesting to model scattering and diffraction of electromagnetic waves on lattices, located in inhomogeneous media [2, 8]. In particular, the construction of the mathematical model of wave scattering on impedance strips located on screened dielectric layer is of interest.

Formulation of the Problem. Consider the following diffraction structure (see Fig. 1). There is an infinite screen in the plane $z = -D$.

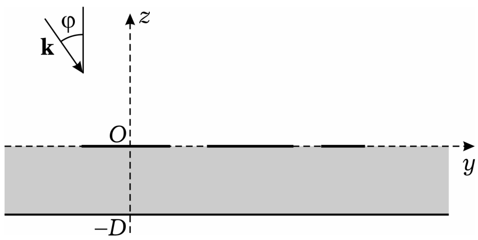

Fig. 1. A cross-section of the electrodynamic structure by the plane yOz.

The layer of dielectric lies on this screen (it occupies the domain $-D < z < 0$). The permittivity of the dielectric is equal to ε_1. The system consisting of M impedance thin strips is located in the plane $z = 0$. Let

$$\Lambda = \left\{(y,z) \in \mathbb{R} \mid y \in \bigcup_{q=1}^{M} [\alpha_q, \beta_q],\ z = 0\right\}$$

is the set of the points of the plane $z = 0$ in which the tapes are located. The permittivity ε_0 of the medium is equal to one in a half-space $z > 0$.

Let us denote $\Omega_0 = \{(y,z) \mid z > 0,\ y \in \mathbb{R}\}$, $\Omega_1 = \{(y,z) \mid y \in \mathbb{R},\ -D < z < 0\}$, $L = \bigcup_{q=1}^{M} (\alpha_q, \beta_q)$.

The time dependence of the fields is given by the factor $e^{-i\omega t}$.

The Case of H-Polarization. A plane monochromatic H-polarized electromagnetic wave of unit amplitude is falling from the infinity on the top of the diffraction structure. Single nonzero component of the magnetic field $H_x^{\text{ini}}(y,z)$ has the form

$$H_x^{\text{ini}}(y,z) = \exp(ik(y\sin\varphi - z\cos\varphi)).$$

It is necessary to find the total field $u(y,z) = H_x(y,z)$, appeared as a result of waves scattering on the considered structure. The function $u(y,z)$ satisfies the Helmholtz equation:

$$\Delta u + k^2\varepsilon_i u = 0, \qquad (y,z)\in\Omega_i, \qquad i = 0,1, \qquad k = \frac{\omega}{c},$$

in the domains Ω_i, $i = 0,1$, and Meixner condition on edges. The scattered field (the difference between total and incident field) satisfies the Sommerfeld radiation conditions. Also, the total field satisfies the Shchukin – Leontovich boundary conditions on strips and screen:

$$\frac{\partial u}{\partial z}(y,+0) - h_0\, u(y,+0) = \frac{\partial u}{\partial z}(y,-0) + h_1\, u(y,-0) = 0, \qquad y\in L, \tag{2}$$

$$\frac{\partial u}{\partial z}(y,-D) - h_1\, u(y,-D) = 0, \qquad y\in\mathbb{R}, \tag{3}$$

and conditions of conjugation (conditions of connection of fields and their derivatives) on the boundary of the domains Ω_0 and Ω_1 outside the strips:

$$u(y,+0) = u(y,-0), \qquad y\in C\bar{L}, \tag{4}$$

$$\frac{1}{\varepsilon_0}\frac{\partial u}{\partial z}(y,+0) = \frac{1}{\varepsilon_1}\frac{\partial u}{\partial z}(y,-0), \qquad y\in C\bar{L}. \tag{5}$$

Let us consider the auxiliary diffraction structure. It differs from the structure of the main problem by the lack of strips. Introduce the function $u_0(y,z)$, which is the solution of the scattering problem of H-polarized plane electromagnetic wave $H_x^{\text{ini}}(y,z)$ on auxiliary diffraction structure.

We are looking for the total field $u(y,z)$ in the form:

$$u(y,z) = \begin{cases} u_0(y,z) + u^+(y,z), & (y,z)\in\Omega^+, \\ u_0(y,z) + u^-(y,z), & (y,z)\in\Omega^-. \end{cases}$$

Fields $u^+(y,z)$ and $u^-(y,z)$ will be sought in the form of integrals:

$$u^+(y,z) = \int_{-\infty}^{\infty} C^+(\lambda)\, e^{i\lambda y - \gamma_0(\lambda) z}\, d\lambda, \qquad (y,z)\in\Omega^+, \tag{6}$$

$$u^-(y,z) = \int_{-\infty}^{\infty} C^-(\lambda)\, Z(\lambda,z)\, e^{i\lambda y}\, d\lambda, \qquad (y,z)\in\Omega^-, \tag{7}$$

where

$$Z(\lambda,z) = \frac{h_1\,\mathrm{sh}(\gamma_1(\lambda)(z+D)) + \gamma_1(\lambda)\,\mathrm{ch}(\gamma_1(\lambda)(z+D))}{h_1\,\mathrm{ch}(\gamma_1(\lambda)D) + \gamma_1(\lambda)\,\mathrm{sh}(\gamma_1(\lambda)D)},$$

$$\gamma_i(\lambda) = \sqrt{\lambda^2 - k^2\varepsilon_i}, \quad \lambda\in\mathbb{R}, \quad \mathrm{Re}(\gamma_i(\lambda)) \ge 0, \quad \mathrm{Im}(\gamma_i(\lambda)) \le 0, \quad i = 0,1.$$

Choosing a branch of a complex function $\gamma_i(\lambda)$ provides the fulfillment of the Sommerfeld's radiation conditions.

Introduce the functions

$$F_1(y) = \frac{\partial u^+}{\partial y}(y,0) - \frac{\partial u^-}{\partial y}(y,0), \qquad y \in \mathbb{R},$$

$$F_2(y) = -\left(\frac{1}{\varepsilon_0}\frac{\partial u^+}{\partial z}(y,0) - \frac{1}{\varepsilon_1}\frac{\partial u^-}{\partial z}(y,0)\right), \qquad y \in \mathbb{R}.$$

Functions $F_1(y)$ and $F_2(y)$ have the following integral representation:

$$F_1(y) = \int_{-\infty}^{\infty} (C^+(\lambda) - C^-(\lambda)Z(\lambda,0))(i\lambda)e^{i\lambda y}\, d\lambda, \qquad y \in \mathbb{R}, \tag{8}$$

$$F_2(y) = \int_{-\infty}^{\infty} \left(\frac{\gamma_0(\lambda)}{\varepsilon_0}C^+(\lambda) + \frac{\gamma_1(\lambda)}{\varepsilon_1}C^-(\lambda)\right)e^{i\lambda y}\, d\lambda, \qquad y \in \mathbb{R}. \tag{9}$$

It follows from the conjugation conditions (4) and (5), that

$$F_1(y) = 0, \qquad F_2(y) = 0, \qquad y \in C\overline{L},$$

$$\int_{\alpha_q}^{\beta_q} F_1(t)\,dt = 0, \qquad q = 1,\dots,M, \tag{10}$$

$$u^+(y,0) - u^-(y,0) = \int_{-\infty}^{y} F_1(t)\,dt, \qquad y \in \mathbb{R}. \tag{11}$$

Introduce the function

$$\rho(\lambda) = \frac{\gamma_1(\lambda)}{\varepsilon_1\,\gamma_0(\lambda)} + \frac{Z(\lambda,0)}{\varepsilon_0}, \qquad \rho(\lambda) = \frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_0} + O\left(\frac{1}{\lambda}\right), \qquad \lambda \to \infty.$$

The following integral representations we obtain from (8)–(11):

$$C^+(\lambda) = \frac{Z(\lambda,0)}{2\pi\gamma_0(\lambda)\rho(\lambda)}\int_L F_2(t)\,e^{-i\lambda t}\,dt + \frac{\gamma_1(\lambda)}{\varepsilon_1\gamma_0(\lambda)\rho(\lambda)}\frac{1}{2\pi i}\int_L F_1(t)\frac{e^{-i\lambda t}-1}{\lambda}\,dt, \quad \lambda \in \mathbb{R}, \tag{12}$$

$$C^-(\lambda) = \frac{1}{2\pi\gamma_0(\lambda)\rho(\lambda)}\int_L F_2(t)e^{-i\lambda t}\,dt - \frac{1}{\varepsilon_0\rho(\lambda)}\frac{1}{2\pi i}\int_L F_1(t)\frac{e^{-i\lambda t}-1}{\lambda}\,dt, \quad \lambda \in \mathbb{R}. \tag{13}$$

Taking into account the consequences of Shchukin – Leontovich boundary conditions (2) at the strips and conjugation conditions (4), (5), we have

$$\left(\frac{\partial u^+}{\partial z}(y,0) + \frac{\partial u^-}{\partial z}(y,0)\right) - h_0\int_{-\infty}^{y} F_1(t)\,dt - (h_0 - h_1)u^-(y,0) = -f_1(y), \quad y \in L, \tag{14}$$

$$F_2(y) + \frac{h_0}{\varepsilon_0}\int_{-\infty}^{y} F_1(t)\,dt + \left(\frac{h_0}{\varepsilon_0} + \frac{h_1}{\varepsilon_1}\right)u^-(y,0) = -f_2(y), \qquad y \in L, \tag{15}$$

where

$$f_1(y) = \left(\frac{\varepsilon_1}{\varepsilon_0} + 1\right)\frac{\partial u_0}{\partial z}(y,+0) - (h_0 - h_1)u_0(y,+0),$$

$$f_2(y) = \left(\frac{h_0}{\varepsilon_0} + \frac{h_1}{\varepsilon_1}\right)u_0(y,+0).$$

Based on the properties of the parametric representation of the Hilbert transform [6], we obtain

$$\int_{-\infty}^{\infty} (C^+(\lambda) - C^-(\lambda)\,Z(\lambda,0))|\lambda|\,e^{i\lambda y}\,d\lambda = -\frac{1}{\pi}\int_L \frac{F_1(t)\,dt}{t-y}. \tag{16}$$

Let us introduce the functions

$$m_1(\lambda) = \gamma_0(\lambda) - \lambda, \quad m_2(\lambda) = \frac{\gamma_0(\lambda)Z(\lambda,0) - \gamma_1(\lambda)}{\rho(\lambda)}, \quad m_3(\lambda) = \frac{Z(\lambda,0) - 1}{\rho(\lambda)},$$

$$I_1(t) = \int_0^\infty \left(\frac{m_2(\lambda)}{\varepsilon_0} - m_1(\lambda) \right) \frac{\sin(\lambda t)}{\lambda} d\lambda, \qquad I_2(t) = \int_0^\infty m_2(\lambda) \frac{\cos(\lambda t)}{\gamma_0(\lambda)} d\lambda,$$

$$I_3(t) = \frac{1}{\varepsilon_0} \int_0^\infty m_3(\lambda) \cdot \frac{\sin(\lambda t)}{\lambda} d\lambda, \qquad I_4(t) = \int_0^\infty m_3(\lambda) \frac{\cos(\lambda t)}{\gamma_0(\lambda)} d\lambda.$$

It follows from (12), (13), and (16), that

$$\frac{\partial u^+}{\partial z}(y,0) + \frac{\partial u^-}{\partial z}(y,0) = \frac{1}{\pi} \int_L \frac{F_1(t)\,dt}{t-y} + \frac{1}{\pi} \int_L I_1(y-t) F_1(t)\,dt - \\ - \frac{1}{\pi} \int_L I_2(y-t) F_2(t)\,dt, \tag{17}$$

$$u^-(y,0) = \frac{A}{\pi} \int_L \frac{i\pi}{2} H_0^1(k|y-t|) F_2(t)\,dt - \frac{A}{\pi} \int_L \frac{\pi}{2\varepsilon_0} \operatorname{sgn}(y-t) F_1(t)\,dt - \\ - \frac{1}{\pi} \int_L I_3(y-t) F_1(t)\,dt + \frac{1}{\pi} \int_L I_4(y-t) F_2(t)\,dt, \tag{18}$$

where $H_0^1(y)$ is the Hankel function of the first kind of zero order.

Introduce notations and functions

$$A = \frac{\varepsilon_0 \, \varepsilon_1}{\varepsilon_0 + \varepsilon_1}, \qquad \frac{Z(\lambda,0)}{\rho(\lambda)} = A + O\left(\frac{1}{|\lambda|}\right), \qquad \lambda \to \infty,$$

$$Q_1(y,t) = I_1(y-t) + (h_0 - h_1) I_3(y-t), \tag{19}$$

$$Q_2(y,t) = -I_2(y-t) - (h_0 - h_1) I_4(y-t) - \\ - A(h_0 - h_1) \left(\frac{i\pi}{2} H_0^1(k|y-t|) + \ln|y-t| \right), \tag{20}$$

$$Q_3(y,t) = -\left(\frac{h_0}{\varepsilon_0} + \frac{h_1}{\varepsilon_1} \right) I_3(y-t), \tag{21}$$

$$Q_4(y,t) = \left(h_0 \varepsilon_0^{-1} + h_1 \varepsilon_1^{-1} \right) I_4(y-t) + \\ + A\left(h_0 \varepsilon_0^{-1} + h_1 \varepsilon_1^{-1} \right) \left(\frac{i\pi}{2} H_0^1(k|y-t|) + \ln|y-t| \right). \tag{22}$$

Note, that $Q_i(y,t) \in C^{0,\chi}(L \times L)$, $\chi > 0$, $i = 1, \dots, 4$.

Substituting the integral representations (17), (18) in (14), (15) we obtain the system of singular integral equations

$$\frac{1}{\pi} \int_L \frac{F_1(t)\,dt}{t-y} + \frac{A(h_0 - h_1)}{\pi} \int_L \ln|y-t| F_2(t)\,dt - h_0 \int_{\alpha_1}^{y} F_1(t)\,dt + \\ + \frac{1}{\pi} \int_L \frac{A(h_0 - h_1)\pi}{2\varepsilon_0} \operatorname{sgn}(y-t) F_1(t)\,dt + \frac{1}{\pi} \int_L Q_1(y,t) F_1(t)\,dt + \\ + \frac{1}{\pi} \int_L Q_2(y,t) F_2(t)\,dt = -f_1(y), \qquad y \in L, \tag{23}$$

$$F_2(y) - \frac{A}{\pi}\left(\frac{h_0}{\varepsilon_0} + \frac{h_1}{\varepsilon_1}\right)\int_L \ln|y-t|\, F_2(t)\, dt - \left(\frac{h_0}{\varepsilon_0} + \frac{h_1}{\varepsilon_1}\right)\frac{A}{2}\int_L \operatorname{sgn}(y-t) F_1(t)\, dt +$$

$$+ \frac{h_0}{\varepsilon_0}\int_{-\infty}^{y} F_1(t)\, dt + \frac{1}{\pi}\int_L Q_3(y,t) F_1(t)\, dt +$$

$$+ \frac{1}{\pi}\int_L Q_4(y,t) F_2(t)\, dt = -f_2(y), \qquad y \in L\,, \tag{24}$$

$$\int_{\alpha_q}^{\beta_q} F_1(t)\, dt = 0, \qquad q = 1,\ldots,M\,, \tag{25}$$

The Case of E-Polarization. A plane monochromatic E-polarized electromagnetic wave of unit amplitude is falling from the infinity on the top of the diffraction structure. Single nonzero component of the electric field $E_x^{\text{ini}}(y,z)$ has the form

$$E_x^{\text{ini}}(y,z) = \exp\left(ik(y\sin\phi - z\cos\phi)\right).$$

It is necessary to find the total field $U(y,z) = E_x(y,z)$, appeared as a result of waves scattering on the considered structure. Function $U(y,z)$ satisfies the Helmholtz equation in the domains Ω_i, $i = 0,1$, and Meixner condition on edges. The scattered field satisfies the Sommerfeld radiation conditions. Also, the total field satisfies the Shchukin – Leontovich boundary conditions on strips and screen:

$$\frac{\partial U}{\partial z}(y,+0) - h_2\, U(y,+0) = \frac{\partial U}{\partial z}(y,-0) + h_3\, U(y,-0) = 0, \qquad y \in L\,, \tag{26}$$

$$\frac{\partial U}{\partial z}(y,-D) - h_3\, U(y,-D) = 0, \qquad y \in \mathbb{R}\,, \tag{27}$$

and conditions of conjugation on the boundary of the domains Ω_0 and Ω_1 outside the strips:

$$U(y,+0) = U(y,-0), \qquad y \in C\bar{L}\,, \tag{28}$$

$$\frac{\partial U}{\partial z}(y,+0) = \frac{\partial U}{\partial z}(y,-0), \qquad y \in C\bar{L}\,. \tag{29}$$

Introduce the function $U_0(y,z)$, which is the solution of the scattering problem of E-polarized plane electromagnetic wave on auxiliary diffraction structure.

We are looking for the total field of $U(y,z)$ in the form:

$$U(y,z) = \begin{cases} U_0(y,z) + U^{+}(y,z), & (y,z) \in \Omega^{+}, \\ U_0(y,z) + U^{-}(y,z), & (y,z) \in \Omega^{-}. \end{cases}$$

Fields $U^{+}(y,z)$ and $U^{-}(y,z)$ will be sought in the form of integrals:

$$U^{+}(y,z) = \int_{-\infty}^{\infty} D^{+}(\lambda)\, e^{i\lambda y - \gamma_0(\lambda) z}\, d\lambda, \qquad (y,z) \in \Omega^{+}\,,$$

$$U^{-}(y,z) = \int_{-\infty}^{\infty} D^{-}(\lambda)\, Zh(\lambda,z)\, e^{i\lambda y}\, d\lambda, \qquad (y,z) \in \Omega^{-}\,,$$

where

$$Zh(\lambda,z) = \frac{h_3\, \operatorname{sh}(\gamma_1(\lambda)(z+D)) + \gamma_1(\lambda)\operatorname{ch}(\gamma_1(\lambda)(z+D))}{h_3\operatorname{ch}(\gamma_1(\lambda)D) + \gamma_1(\lambda)\operatorname{sh}(\gamma_1(\lambda)D)}\,.$$

Introduce the functions

$$G_1(y) = \frac{\partial U^+}{\partial y}(y,0) - \frac{\partial U^-}{\partial y}(y,0), \qquad y \in \mathbb{R},$$

$$G_2(y) = -\left(\frac{\partial U^+}{\partial z}(y,0) - \frac{\partial U^-}{\partial z}(y,0)\right), \qquad y \in \mathbb{R}.$$

It follows from the conjugation conditions (28) and (29), that

$$G_1(y) = 0, \quad G_2(y) = 0, \quad y \in C\bar{L}, \quad \int_{\alpha_q}^{\beta_q} G_1(t)\,dt = 0, \qquad q = 1,\ldots,M, \tag{30}$$

$$U^+(y,0) - U^-(y,0) = \int_{\alpha_1}^{y} G_1(t)\,dt, \qquad y \in \mathbb{R}. \tag{31}$$

Fourier amplitudes $D^+(\lambda)$ and $D^-(\lambda)$ have the following integral representations:

$$D^+(\lambda) = \frac{Z(\lambda,0)}{2\pi\,\gamma_0(\lambda)\,\psi(\lambda)} \int_L G_2(t)\,e^{-i\lambda t}\,dt + \\ + \frac{\gamma_1(\lambda)}{\gamma_0(\lambda)\,\psi(\lambda)} \frac{1}{2\pi i} \int_L G_1(t) \frac{e^{-i\lambda t} - 1}{\lambda}\,dt, \qquad \lambda \in \mathbb{R}, \tag{32}$$

$$D^-(\lambda) = \frac{1}{2\pi\,\gamma_0(\lambda)\,\psi(\lambda)} \int_L G_2(t)\,e^{-i\lambda t}\,dt - \\ - \frac{1}{\psi(\lambda)} \frac{1}{2\pi i} \int_L G_1(t) \frac{e^{-i\lambda t} - 1}{\lambda}\,dt, \qquad \lambda \in \mathbb{R}, \tag{33}$$

where

$$\psi(\lambda) = \frac{\gamma_1(\lambda)}{\gamma_0(\lambda)} + Zh(\lambda,0), \qquad \psi(\lambda) = 2 + O\left(\frac{1}{|\lambda|}\right), \qquad \lambda \to \infty.$$

It follows from the boundary conditions (26), (27) and conjugation conditions (4), (5), that

$$\left(\frac{\partial U^+}{\partial z}(y,0) + \frac{\partial U^-}{\partial z}(y,0)\right) - h_2 \int_{-\infty}^{y} G_1(t)\,dt - (h_2 - h_3)U^-(y,0) = -f_3(y), \\ y \in L, \tag{34}$$

$$G_2(y) + h_2 \int_{\alpha_1}^{y} G_1(t)\,dt + (h_2 + h_3)U^-(y,0) = -f_4(y), \qquad y \in L, \tag{35}$$

where

$$f_3(y) = 2\frac{\partial U_0}{\partial z}(y,+0) - (h_2 - h_3)U_0(y,+0), \quad f_4(y) = (h_2 + h_3)U_0(y,+0),$$

Introduce the functions

$$m_4(\lambda) = \frac{\gamma_0(\lambda)\cdot Zh(\lambda,0) - \gamma_1(\lambda)}{\psi(\lambda)}, \qquad m_5(\lambda) = \frac{Zh(\lambda,0) - 1}{\psi(\lambda)}.$$

Performing the integral transforms such which was carried out in the case of H-polarization, we obtain the system of integral equations:

$$\frac{1}{\pi}\int_L \frac{G_1(t)\,dt}{t-y} + \frac{h_2 - h_3}{2\pi}\int_L \ln|y-t|\,G_2(t)\,dt - h_2\int_{\alpha_1}^{y} G_1(t)\,dt + \\ + \frac{h_2 - h_3}{4}\int_L \operatorname{sgn}(y-t)G_1(t)\,dt + \frac{1}{\pi}\int_L R_1(y,t)\,G_1(t)\,dt + \\ + \frac{1}{\pi}\int_L R_2(y,t)\,G_2(t)\,dt = -f_3(y), \qquad y \in L, \tag{36}$$

$$G_2(y) - \frac{(h_2+h_3)}{2\pi}\int_L \ln|y-t|\,G_2(t)\,dt - \frac{h_2+h_3}{4}\int_L \operatorname{sgn}(y-t)G_1(t)\,dt +$$
$$+ h_2\int_{\alpha_1}^{y} G_1(t)\,dt + \frac{1}{\pi}\int_L R_3(y,t)G_1(t)\,dt +$$
$$+ \frac{1}{\pi}\int_L R_4(y,t)G_2(t)\,dt = -f_4(y), \qquad y\in L\,, \tag{37}$$

$$\int_{\alpha_q}^{\beta_q} G_1(t)\,dt = 0, \qquad q = 1,\dots,M\,, \tag{38}$$

where

$$R_1(y,t) = \int_0^\infty (m_4(\lambda) - m_1(\lambda) + (h_2-h_3)m_5(\lambda))\frac{\sin(\lambda(y-t))}{\lambda}\,d\lambda\,,$$

$$R_2(y,t) = -\int_0^\infty (m_4(\lambda) + (h_2-h_3)m_5(\lambda))\frac{\cos(\lambda(y-t))}{\gamma_0(\lambda)}\,d\lambda -$$
$$-\frac{1}{2}(h_2-h_3)\left(\frac{i\pi}{2}H_0^1(k|y-t|) + \ln|y-t|\right),$$

$$R_3(y,t) = -(h_2+h_3)\int_0^\infty m_5(\lambda)\frac{\sin(\lambda(y-t))}{\lambda}\,d\lambda\,,$$

$$R_4(y,t) = (h_2+h_3)\int_0^\infty m_5(\lambda)\frac{\cos(\lambda(y-t))}{\gamma_0(\lambda)}\,d\lambda +$$
$$+\frac{1}{2}(h_2+h_3)\left(\frac{i\pi}{2}H_0^1(k|y-t|) + \ln|y-t|\right).$$

Note, that $R_i(y,t)\in C^{0,\chi}(L\times L)$, $\chi>0$, $i=1,\dots,4$.

Conclusions. Mathematical models of the initial problems have been constructed on the basis of the system of Fredholm integral equations of the second kind and the system of singular integral equations. The main parameters of scattered electromagnetic waves had been expressed via the solutions of these systems. The system of integral equations (23)–(25) and (36)–(38) differs from the system of integral equations obtained in the article [8]. This difference is caused by the presence of terms with logarithmic singularities and terms with discontinuities of the first kind in the kernels of integral equations. The numerical solution of the system of integral equations (23)–(25) and (36)–(38) can be obtained with the help of quadrature formulas of interpolation type [1, 2, 5].

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МАТЕМАТИЧНА МОДЕЛЬ РОЗСІЯННЯ ПОЛЯРИЗОВАНОЇ ХВИЛІ НА ІМПЕДАНСНИХ СТРІЧКАХ, РОЗТАШОВАНИХ НА ЕКРАНОВАНОМУ ДІЕЛЕКТРИЧНОМУ ШАРІ

Опис процесів взаємодії електромагнітних хвиль з неідеально провідними гратками приводить до розгляду крайових задач для рівнянь Гельмгольца з граничними умовами третього роду. Вихідну задачу розсіювання H-поляризованої хвилі на відбиваючій структурі зведено до системи граничних інтегральних рівнянь. Виведення інтегральних рівнянь ґрунтується на застосуванні методу параметричних зображень інтегральних операторів.

МАТЕМАТИЧЕСКАЯ МОДЕЛЬ РАССЕЯНИЯ ПОЛЯРИЗОВАННОЙ ВОЛНЫ НА ИМПЕДАНСНЫХ ЛЕНТАХ, РАСПОЛОЖЕННЫХ НА ЭКРАНИРОВАННОМ ДИЭЛЕКТРИЧЕСКОМ СЛОЕ

Описание процессов взаимодействия электромагнитных волн с не идеально проводящими решётками приводит к рассмотрению краевых задач для уравнений Гельмгольца с граничными условиями третьего рода. Исходная задача рассеяния H-поляризованной волны на отражающей структуре сведена к системе граничных интегральных уравнений. Вывод интегральных уравнений основан на применении метода параметрических представлений интегральных операторов.

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