DARBOUX TRANSFORMATIONS AND SCATTERING OPERATORS FOR DIRAC'S SYSTEMS

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A scattering operator for the Dirac's system is constructed using binary Darboux transformation. It is shown that this operator, in special case, coincides with the scattering operator obtained by L.Nizhnik [6,7].

The direct and inverse scattering problems for the Dirac system with operator L_1 given by

$$L_1 = \begin{pmatrix} \partial_x & u_1 \\ u_2 & \partial_y \end{pmatrix}, \quad u_1, u_2 \in L_2(\mathbb{R}^2),\ \partial_x = \frac{\partial}{\partial x},\ \partial_y = \frac{\partial}{\partial y}, \tag{1}$$

were studied by L. Nizhnik in [6] and [7], where he described the properties of the scattering operator S which is defined by the equation $b = Sa$, where

$$b = \begin{pmatrix} b_1(y) \\ b_2(x) \end{pmatrix},\ a = \begin{pmatrix} a_1(y) \\ a_2(x) \end{pmatrix},\ a_i,\ b_i \in L_2(\mathbb{R}),\ i = 1, 2,$$

are the asymptotic values of the solutions $\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}$ of the Dirac system $L_1 Y{=}0$:

$$\begin{aligned} a_1(y) &= Y_1(-\infty, y),\ \ b_1(y) = Y_1(+\infty, y), \\ a_2(x) &= Y_2(x, -\infty),\ \ b_2(x) = Y_2(x, +\infty). \end{aligned} \tag{2}$$

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In [7,8] the inverse scattering method (ISM) for the Dirac operator was used to solve the Schrödinger system in two spatial dimensions. As well as the ISM, which in its classical form is based on the solving of the Gelfand–Levitan–Marchenko integral equations [1, 2, 13, 14], one can also use more „algebrized" methods, for example, method of binary Darboux transformations ([3–5, 11, 12]), to obtain large classes of solutions to integrable systems. Indeed, in [9], [10] it was shown that exact solutions of the Schrödinger system in two-dimensional space obtained by the ISM were a subclass of solutions obtained using binary Darboux transformations.

Further, in [10] it was shown that binary Darboux transformation (BDT) [3–5, 11, 12] generate those transformations which Nizhnik obtained in his study of the direct and inverse scattering problem for the corresponding Dirac system [6, 7].

Let us expose some of the results of [9, 10] which we shall use in this article. In [10] it was shown that the two-dimensional Schrödinger system [8] admits the restriction $u_2 = \mu\bar{u}_1$, $u_1 := u$. Then the system becomes

$$\begin{aligned} iu_t &= u_{xx} + u_{yy} - 2(v_1 - v_2)u, \\ v_{1x} &= \mu(|u|^2)_y,\ v_{2y} = -\mu(|u|^2)_x, \end{aligned} \tag{3}$$

and it admits the Lax pair $[L, A] = 0$, where (cf. (1))

$$L = \begin{pmatrix} \partial_x & u \\ \mu\bar{u} & \partial_y \end{pmatrix}, \tag{4}$$

$$A = \begin{pmatrix} i\partial_t + (\partial_x - \partial_y)^2 - 2v_1 & -2\partial_y u + 2u\partial_x \\ 2\mu\partial_x\bar{u} - 2\mu\bar{u}\partial_x & -i\partial_t - (\partial_x - \partial_y)^2 - 2v_2 \end{pmatrix}.$$

It was also shown that the operators L, A admit the reductions

$$L^* = -\sigma L\sigma^{-1},\ A^* = \sigma A\sigma^{-1},$$

where

$$\sigma = \begin{pmatrix} 1 & 0 \\ 0 & -\mu^{-1} \end{pmatrix},\ \mu \in \mathbb{R}\setminus\{0\},$$

and, consequently, the fixed solutions

$$\varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} = \begin{pmatrix} \varphi_{11} & \cdots & \varphi_{1k} \\ \varphi_{21} & \cdots & \varphi_{2k} \end{pmatrix},\ \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} \psi_{11} & \cdots & \psi_{1k} \\ \psi_{21} & \cdots & \psi_{2k} \end{pmatrix}$$

of the systems

$$L\varphi = 0,\ L^\tau\psi = 0,\ A\varphi = 0,\ A^\tau\psi = 0 \tag{5}$$

admit the relation

$$\psi = \sigma\bar{\varphi}. \tag{6}$$

In [5, 11, 12] it was shown that (5) implies the existence (up to a constant) of the matrix potential

$$\Omega[\psi,\varphi] := \int_{M_0}^{M} (-\psi_2^{\top}\varphi_2)\,dx + \psi_1^{\top}\varphi_1\,dy,$$

where $M = (x,y,t)$, $M_0 = (x_0,y_0,t)$ (in the following we omit the parameter t). Therefrom and relation (6) we obtain

$$\Omega[\psi,\varphi] := \Omega[\sigma(\mu)\bar{\varphi},\varphi] = \int_{M_0}^{M} \varphi_1^{*}\varphi_1\,dy + \mu^{-1}\varphi_2^{*}\varphi_2\,dx. \tag{7}$$

Let $Y_0 = \begin{pmatrix} Y_1(y) \\ Y_2(x) \end{pmatrix}$ be an arbitrary solution and $\varphi = \begin{pmatrix} \varphi_1(y) \\ \varphi_2(x) \end{pmatrix}$ be a fixed $(2\times K)$-matrix solution of the system

$$\begin{cases} L_0 Y_0 = 0, \\ A_0 Y_0 = 0, \end{cases} \tag{8}$$

where $L_0 = \begin{pmatrix} \partial_x & 0 \\ 0 & \partial_y \end{pmatrix}$ is the unperturbed Dirac operator, $A_0 = [i\partial_t + (\partial_x - \partial_y)^2]\sigma_3$, $\sigma_3 := \mathrm{diag}(1,-1)$. The integral operator W defined on the space of solutions of (8) by the formula

$$Y = WY_0 = Y_0 - \varphi(C + \Omega[\sigma\bar{\varphi},\varphi])^{-1}\Omega[\sigma\bar{\varphi},Y_0], \tag{9}$$

(C is an arbitrary constant Hermitian matrix) maps the operator L_0 into the operator L given by (4)

$$L = WL_0W^{-1} = \begin{pmatrix} \partial_x & u \\ \mu\bar{u} & \partial_y \end{pmatrix},$$

where

$$u = \mu^{-1}\varphi_1(C + \Omega[\sigma\bar{\varphi},\varphi])^{-1}\varphi_2^{*}. \tag{10}$$

Note that formula (10) yields the solution of equation (3).

The main result of the present work is the explicit construction of the scattering operator S for the system

$$LY = 0 \tag{11}$$

using the binary Darboux transformations W given by (9).

From formula (7) we have the matrix potentials $\Omega_1[\sigma\bar{\varphi},\varphi]$ (putting $x_0 = -\infty$, $y_0 = -\infty$) and $\Omega_2[\sigma(\mu)\bar{\varphi},\varphi]$ (putting $x_0 = +\infty$, $y_0 = +\infty$):

$$\begin{aligned}\Omega_1[\sigma\bar{\varphi},\varphi] &= \int_{-\infty}^{y} \varphi_1^*(s)\varphi_1(s)\,ds + \mu^{-1}\int_{-\infty}^{x} \varphi_2^*(\tau)\varphi_2(\tau)\,d\tau,\\ \Omega_2[\sigma\bar{\varphi},\varphi] &= \int_{+\infty}^{y} \varphi_1^*(s)\varphi_1(s)\,ds + \mu^{-1}\int_{+\infty}^{x} \varphi_2^*(\tau)\varphi_2(\tau)\,d\tau.\end{aligned} \tag{12}$$

One may represent an arbitrary solution of (11) in many ways by choosing a concrete realization of (9). For example, taking into account (12) we have

$$Y = W_1 p = W_2 q, \tag{13}$$

where

$$p = \begin{pmatrix} p_1(y) \\ p_2(x) \end{pmatrix},\ q = \begin{pmatrix} q_1(y) \\ q_2(x) \end{pmatrix},\ p_1, p_2, q_1, q_2 \in L_2(\mathbb{R})$$

are solutions of the unperturbed equation (8),

$$W_1 = I - \begin{pmatrix} \varphi_1(y)\Delta_1^{-1}\int_{-\infty}^{y}\varphi_1^*(s)\cdot ds; & \mu^{-1}\varphi_1(y)\Delta_1^{-1}\int_{-\infty}^{x}\varphi_2^*(\tau)\cdot d\tau \\ \varphi_2(x)\Delta_1^{-1}\int_{-\infty}^{y}\varphi_1^*(s)\cdot ds; & \mu^{-1}\varphi_2(x)\Delta_1^{-1}\int_{-\infty}^{x}\varphi_2^*(\tau)\cdot d\tau \end{pmatrix}, \tag{14}$$

$$\Delta_1 = C_1 + \int_{-\infty}^{y}\varphi_1^*(s)\varphi_1(s)\,ds + \mu^{-1}\int_{-\infty}^{x}\varphi_2^*(\tau)\varphi_2(\tau)\,d\tau, C_1 = const, C_1^* = C_1.$$

$$W_2 = I - \begin{pmatrix} \varphi_1(y)\Delta_2^{-1}\int_{+\infty}^{y}\varphi_1^*(s)\cdot ds; & \mu^{-1}\varphi_1(y)\Delta_2^{-1}\int_{+\infty}^{x}\varphi_2^*(\tau)\cdot d\tau \\ \varphi_2(x)\Delta_2^{-1}\int_{+\infty}^{y}\varphi_1^*(s)\cdot ds; & \mu^{-1}\varphi_2(x)\Delta_2^{-1}\int_{+\infty}^{x}\varphi_2^*(\tau)\cdot d\tau \end{pmatrix}, \tag{15}$$

$$\Delta_2 = C_2 + \int_{+\infty}^{y}\varphi_1^*(s)\varphi_1(s)\,ds + \mu^{-1}\int_{+\infty}^{x}\varphi_2^*(\tau)\varphi_2(\tau)\,d\tau,\ C_2 = const.$$

From equation (13) we find the relation

$$C_2 = C_1 + \int_{-\infty}^{+\infty}\varphi_1^*(s)\varphi_1(s)\,ds + \mu^{-1}\int_{-\infty}^{+\infty}\varphi_2^*(\tau)\varphi_2(\tau)\,d\tau. \tag{16}$$

Direct calculation shows that W_2^{-1} is given by

$$W_2^{-1} = I+$$

$$+\begin{pmatrix} \varphi_1(y)\int\limits_{+\infty}^{y}\Delta_2^{-1}(x,s)\varphi_1^*(s)\cdot ds; & \mu^{-1}\varphi_1(y)\int\limits_{+\infty}^{x}\Delta_2^{-1}(\tau,+\infty)\varphi_2^*(\tau)\cdot d\tau \\ \varphi_2(x)\int\limits_{+\infty}^{y}\Delta_2^{-1}(x,s)\varphi_1^*(s)\cdot ds; & \mu^{-1}\varphi_2(x)\int\limits_{+\infty}^{x}\Delta_2^{-1}(\tau,+\infty)\varphi_2^*(\tau)\cdot d\tau \end{pmatrix} \tag{17}$$

In order to define the scattering operator, equation (2) must be satisfied. To this end we put:

$$p = S_1^{-1}a, \quad q = S_2^{-1}b,$$

where

$$S_1 = \begin{pmatrix} 1-\varphi_1(y)\Delta_1^{-1}(-\infty,y)\int\limits_{-\infty}^{y}\varphi_1^*(s)\cdot ds & 0 \\ 0 & 1-\mu^{-1}\varphi_2(x)\Delta_1^{-1}(x,-\infty)\int\limits_{-\infty}^{x}\varphi_2^*(\tau)\cdot d\tau \end{pmatrix}, \tag{18}$$

$$S_1^{-1} = \begin{pmatrix} 1+\varphi_1(y)\int\limits_{-\infty}^{y}\Delta_1^{-1}(-\infty,s)\varphi_1^*(s)\cdot ds & 0 \\ 0 & 1+\mu^{-1}\varphi_2(x)\int_{-\infty}^{x}\Delta_1^{-1}(\tau,-\infty)\varphi_2^*(\tau)\cdot d\tau \end{pmatrix},$$

$$S_2 = \begin{pmatrix} 1-\varphi_1(y)\Delta_2^{-1}(+\infty,y)\int\limits_{+\infty}^{y}\varphi_1^*(s)\cdot ds & 0 \\ 0 & 1-\mu^{-1}\varphi_2(x)\Delta_2^{-1}(x,+\infty)\int\limits_{+\infty}^{x}\varphi_2^*(\tau)\cdot d\tau \end{pmatrix}, \tag{19}$$

$$S_2^{-1} = \begin{pmatrix} 1+\varphi_1(y)\int\limits_{+\infty}^{y}\Delta_2^{-1}(+\infty,s)\varphi_1^*(s)\cdot ds & 0 \\ 0 & 1+\mu^{-1}\varphi_2(x)\int\limits_{+\infty}^{x}\Delta_2^{-1}(\tau,+\infty)\varphi_2^*(\tau)\cdot d\tau \end{pmatrix},$$

where operators (18)–(19) are a special auto-Darboux-type transformation operators for unperturbed Dirac system (8).

The operator S given by $b = Sa$ is then a composition of the binary Darboux-type transformation operators of the form

$$S = S_2 W_2^{-1} W_1 S_1^{-1}. \tag{20}$$

Direct calculation, taking into account formulas (14)–(19), gives the operator S (20) as follows:

$$S = I + \begin{pmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{pmatrix}, \tag{21}$$

where

$$F_{11} = \mu^{-1}\varphi_1(y)\Delta_2^{-1}(+\infty, y) \int_{-\infty}^{+\infty} \varphi_2^*\varphi_2\, dx \cdot \int_{-\infty}^{y} \Delta_2^{-1}(-\infty, s)\varphi_1^*(s) \cdot ds;$$

$$F_{12} = -\mu^{-1}\varphi_1(y)\Delta_2^{-1}(+\infty, y)\Delta_2(+\infty, -\infty) \int_{-\infty}^{+\infty} \Delta_2^{-1}(\tau, -\infty)\varphi_2^*(\tau) \cdot d\tau;$$

$$F_{21} = -\varphi_2(x)\Delta_2^{-1}(x, +\infty)\Delta_2(-\infty, +\infty) \int_{-\infty}^{+\infty} \Delta_2^{-1}(-\infty, s)\varphi_1^*(s) \cdot ds;$$

$$F_{22} = \mu^{-1}\varphi_2(x)\Delta_2^{-1}(x, +\infty) \int_{-\infty}^{+\infty} \varphi_1^*\varphi_1\, dy \cdot \int_{-\infty}^{x} \Delta_2^{-1}(\tau, -\infty)\varphi_2^*(\tau) \cdot d\tau.$$

From formula (20) we obtain

$$S^{-1} = S_1 W_1^{-1} W_2 S_2^{-1}. \tag{22}$$

By direct calculation, taking into account (14) – (19) and

$$W_1^{-1} = I +$$

$$+ \begin{pmatrix} \varphi_1(y) \int_{-\infty}^{y} \Delta_1^{-1}(x, s)\varphi_1^*(s) \cdot ds; & \mu^{-1}\varphi_1(y) \int_{-\infty}^{x} \Delta_1^{-1}(\tau, -\infty)\varphi_2^*(\tau) \cdot d\tau \\ \varphi_2(x) \int_{-\infty}^{y} \Delta_1^{-1}(x, s)\varphi_1^*(s) \cdot ds; & \mu^{-1}\varphi_2(x) \int_{-\infty}^{x} \Delta_1^{-1}(\tau, -\infty)\varphi_2^*(\tau) \cdot d\tau \end{pmatrix},$$

we present operator S^{-1} (22) in the form

$$S^{-1} = I + \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix}, \tag{23}$$

where

$$G_{11} = -\mu^{-1}\varphi_1(y)\Delta_1^{-1}(-\infty, y) \int\limits_{-\infty}^{+\infty} \varphi_2^*\varphi_2\, dx \cdot \int\limits_{+\infty}^{y} \Delta_1^{-1}(+\infty, s)\varphi_1^*(s) \cdot ds;$$

$$G_{12} = \mu^{-1}\varphi_1(y)\Delta_1^{-1}(-\infty, y) \int\limits_{-\infty}^{+\infty} \Delta_1(-\infty, +\infty)\Delta_1^{-1}(\tau, +\infty)\varphi_2^*(\tau) \cdot d\tau;$$

$$G_{21} = \varphi_2(x)\Delta_1^{-1}(x, -\infty) \int\limits_{-\infty}^{+\infty} \Delta_1(+\infty, -\infty)\Delta_1^{-1}(+\infty, s)\varphi_1^*(s) \cdot ds;$$

$$G_{22} = -\mu^{-1}\varphi_2(x)\Delta_1^{-1}(x, -\infty) \int\limits_{-\infty}^{+\infty} \varphi_1^*\varphi_1\, dy \cdot \int\limits_{+\infty}^{x} \Delta_1^{-1}(\tau, +\infty)\varphi_2^*(\tau) \cdot d\tau.$$

Operators (21) and (23) can be considered as a generalization of the scattering operator S and the inverse scattering operator S^{-1} obtained by L. Nizhnik [2] in the case of the finite-dimensional scattering data.

Let the solution φ be given as the $(2 \times 2n)$-matrix

$$\varphi = \begin{pmatrix} \varphi_1(y) \\ \varphi_2(x) \end{pmatrix} = \begin{pmatrix} -f_1(y) & 0 \\ 0 & g_2(x) \end{pmatrix},$$

where $f_1(y) = (f_{11}(y), f_{12}(y), \ldots, f_{1n}(y))$, $g_2(x) = (g_{21}(x), g_{22}(x), \ldots, g_{2n}(x))$. Then the $(2 \times 2n)$-matrix potentials $\Omega_1[\sigma\bar{\varphi}, \varphi]$, $\Omega_2[\sigma\bar{\varphi}, \varphi]$, given in (12), become

$$\Omega_1[\sigma\bar{\varphi}, \varphi] = \begin{pmatrix} \int\limits_{-\infty}^{y} f_1^*(s)f_1(s)\, ds & 0 \\ 0 & \mu^{-1} \int\limits_{-\infty}^{x} g_2^*(\tau)g_2(\tau)\, d\tau \end{pmatrix},$$

$$\Omega_2[\sigma\bar{\varphi}, \varphi] = \begin{pmatrix} \int\limits_{+\infty}^{y} f_1^*(s)f_1(s)\, ds & 0 \\ 0 & \mu^{-1} \int\limits_{+\infty}^{x} g_2^*(\tau)g_2(\tau)\, d\tau \end{pmatrix}.$$

We have the $(2n \times 2n)$-matrix C_1 given as

$$C_1 = \begin{pmatrix} 0 & I_n \\ I_n & -\mu^{-1} \int\limits_{-\infty}^{-\infty} g_2^*(\tau) g_2(\tau)\, d\tau \end{pmatrix}, \quad I_n = \mathrm{diag}(1, \dots, 1).$$

Then the potential u is found from (10):

$$u = -\mu^{-1} f_1(y) \left[I_n - \mu^{-1} \int\limits_{+\infty}^{x} g_2^*(\tau) g_2(\tau)\, d\tau \cdot \int\limits_{-\infty}^{y} f_1^*(s) f_1(s)\, ds \right]^{-1} g_2^*(x). \tag{24}$$

We note that the solution (24) coincides with the solution obtained by L.Nizhnik and M.Pochynayko using the ISM [8].

The scattering operator S of (21) then has the form

$$S^r = I + \begin{pmatrix} F_{11}^r & F_{12}^r \\ F_{21}^r & F_{22}^r \end{pmatrix}, \tag{25}$$

where

$$F_{11}^r = \mu^{-1} f_1(y) \int\limits_{-\infty}^{+\infty} g_2^* g_2\, dx \int\limits_{-\infty}^{y} \left[I_n + \mu^{-1} \int\limits_{-\infty}^{s} f_1^* f_1\, d\xi \cdot \int\limits_{-\infty}^{+\infty} g_2^* g_2 dx \right]^{-1} f_1^*(s) \cdot ds;$$

$$F_{12}^r = \mu^{-1} f_1(y) \int\limits_{-\infty}^{+\infty} g_2^*(\tau) \cdot d\tau;$$

$$F_{21}^r = g_2(x) \left[I_n + \mu^{-1} \int\limits_{-\infty}^{+\infty} f_1^* f_1\, dy \cdot \int\limits_{x}^{+\infty} g_2^* g_2\, d\tau \right]^{-1} \times$$

$$\times \left[I_n + \mu^{-1} \int\limits_{-\infty}^{+\infty} f_1^* f_1\, dy \cdot \int\limits_{-\infty}^{+\infty} g_2^* g_2\, dx \right] \times$$

$$\times \int\limits_{-\infty}^{+\infty} \left[I_n + \mu^{-1} \int\limits_{-\infty}^{s} f_1^* f_1\, d\xi \cdot \int\limits_{-\infty}^{+\infty} g_2^* g_2\, dx \right]^{-1} f_1^*(s) \cdot ds;$$

$$F_{22}^r = \mu^{-1} g_2(x) \left[I_n + \mu^{-1} \int\limits_{-\infty}^{+\infty} f_1^* f_1\, dy \cdot \int\limits_{x}^{+\infty} g_2^* g_2\, d\tau \right]^{-1} \times$$

$$\times \int\limits_{-\infty}^{+\infty} f_1^* f_1 \, dy \cdot \int\limits_{-\infty}^{x} g_2^*(\tau) \cdot d\tau.$$

The inverse scattering operator $(S^\tau)^{-1}$ (23) is given by

$$(S^\tau)^{-1} = I + \begin{pmatrix} G_{11}^\tau & G_{12}^\tau \\ G_{21}^\tau & G_{22}^\tau \end{pmatrix}, \tag{26}$$

where

$$G_{11}^\tau = -\mu^{-1} f_1(y) \left[I_n + \mu^{-1} \int\limits_{-\infty}^{+\infty} g_2^* g_2 \, dx \cdot \int\limits_{-\infty}^{y} f_1^* f_1 \, ds \right]^{-1} \times$$

$$\times \int\limits_{-\infty}^{+\infty} g_2^* g_2 \, dx \cdot \int\limits_{+\infty}^{y} f_1^*(s) \cdot dx;$$

$$G_{12}^\tau = -\mu^{-1} f_1(y) \left[I_n + \mu^{-1} \int\limits_{-\infty}^{+\infty} g_2^* g_2 \, dx \cdot \int\limits_{-\infty}^{y} f_1^* f_1 \, ds \right]^{-1} \times$$

$$\times \left[I_n + \mu^{-1} \int\limits_{-\infty}^{+\infty} g_2^* g_2 \, dx \cdot \int\limits_{-\infty}^{+\infty} f_1^* f_1 \, dy \right] \times$$

$$\times \int\limits_{-\infty}^{+\infty} \left[I_n + \mu^{-1} \int\limits_{\tau}^{+\infty} g_2^* g_2 \, d\eta \cdot \int\limits_{-\infty}^{+\infty} f_1^* f_1 \, dy \right]^{-1} g_2^*(\tau) \cdot d\tau;$$

$$G_{21}^\tau = -g_2(x) \int\limits_{-\infty}^{+\infty} f_1^*(s) \cdot ds;$$

$$G_{22}^\tau = -\mu^{-1} g_2(x) \int\limits_{-\infty}^{+\infty} f_1^* f_1 \, dy \times$$

$$\times \int\limits_{+\infty}^{x} \left[I_n + \mu^{-1} \int\limits_{\tau}^{+\infty} g_2^* g_2 \, d\eta \cdot \int\limits_{-\infty}^{+\infty} f_1^* f_1 \, dy \right]^{-1} g_2^*(\tau) \cdot d\tau.$$

Note that from (25) and (26) we have the relation:

$$G^{r}_{21} = -\mu F^{r*}_{12},\ G^{r}_{12} = -\mu^{-1} F^{r*}_{21},\ G^{r}_{11} = F^{r*}_{11},\ G^{r}_{22} = F^{r*}_{22},$$

obtained by Nizhnik [7].

Conclusion. We show in the paper a number of results on the connections of Darboux transformations and Scattering theory. It appears that they are related very closely. They have common problems and common methods to study them. We believe that considerations of these two domains of mathematical physics from a single point of view may be helpful for both of them.

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ПЕРЕТВОРЕННЯ ДАРБУ І ОПЕРАТОРИ РОЗСІЮВАННЯ ДЛЯ СИСТЕМИ ДІРАКА

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Методом бінарних перетворень Дарбу побудовано оператор розсіяння для системи Дірака. Показано, що цей оператор у частковому випадку співпадає з оператором розсіяння, отриманим Л.Нижником.