GROUPS WITH THE WEAK MINIMAL CONDITION FOR NONABELIAN SUBGROUPS

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In the present paper the author establishes: a nonabelian periodic group with the weak minimal condition for nonabelian subgroup is Chernikov if $\langle g, g^h \rangle$ is finite for any prime p and its p-element g and its element h such that $[g^p, h] = 1$.

Recall that a group satisfies the minimal condition for some subgroups if it has no infinite descending chain of these subgroups. Below min, min-ab, min-$\overline{ab}$ are the minimal conditions for (all) subgroups, abelian and nonabelian subgroups respectively. Groups with min are also called Artinian.

Recall that a group satisfies the weak minimal condition for some subgroups if it has no infinite descending chains with infinite indices of these subgroups [1], [2]. Below wmin, wmin-ab, and wmin-$\overline{ab}$ are the weak minimal conditions for subgroups, abelian and nonabelian subgroups respectively. In view of [1], [2], periodic abelian groups with wmin are just Chernikov abelian.

The main result of the present paper is the following new theorem, including a number of theorems that positively solve in various classes of groups S. N. Chernikov's Problem: is an arbitrary group with min necessarily Chernikov? Among these theorems, we mention S. N. Chernikov's [3], [4] (see also Theorem 1.1 of [5]), O. Yu. Shmidt's – S. N. Chernikov's [3], [4], [6], Shunkov – Kegel – Wehrfritz [7], [8], N. S. Chernikov's [9] theorems asserting that a locally solvable, 2-, locally finite, binary finite group is Artinian if and only if it is Chernikov. Remind that the Problem was negatively solved by A. Yu. Ol'sanskiĭ [10], [11] (see also [12]).

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In what follows, $\mathbb{P}$ is the set of all primes, $J(G)$ is the intersection of all subgroups of finite index of a group G. Other notations in the present paper are standard.

Theorem. *Let G be a periodic group with wmin or nonabelian periodic group with wmin-$\overline{ab}$. Then G is Chernikov if and only if $\langle g, g^h \rangle$ is finite whenever g is a p-element for some prime p and $[g^p, h] = 1$ for any $g, h \in G$.*

The Theorem also generalizes Shunkov [13] and Shunkov – Zaitsev [13], [14] theorems asserting that nonabelian locally finite groups respectively with min-$\overline{ab}$ and wmin-$\overline{ab}$ are Chernikov.

The following proposition is an immediate consequence of the Theorem.

Corollary 1. *Let G be a 2-group with wmin or nonabelian 2-group with wmin-$\overline{ab}$. Then G is Chernikov.*

Remark 1. The group $G = \langle a \rangle \rightthreetimes \langle b \rangle$ with infinite cyclic subgroups $\langle a \rangle$ and $\langle b \rangle$ and $a^b = a^{-1}$ is nonabelian torsion-free and satisfies wmin. So, the condition of periodicity of G is essential in the Theorem.

Remark 2. A. Yu. Ol'shanskiĭ's examples [10], [11] (see also [12]) of infinite periodic groups in which any proper subgroup has a prime order show that the restriction imposed on $\langle g, g^h \rangle$ is essential in the Theorem.

Remark 3. In connection with the Theorem, note that there exist a lot of A. Yu. Ol'shanskiĭ's examples of periodic non-(Artinian or abelian) groups with proper abelian subgroups (see, for instance, Theorem 35.1 of [12]). Any such a group satisfies min-$\overline{ab}$ and does not satisfy wmin.

Further, remind that a group G is called Shunkov if for any its finite subgroup K every subgroup of the quotient group $N_G(K)/K$ generated by two of its conjugate elements of prime order is finite (V. D. Mazurov, 1998). The class of Shunkov groups is wide and contains, for instance, all locally finite, binary finite, 2-groups.

It is easy to see that for any prime p and p-element g of the Shunkov group G and any $h \in C_G(g^p)$, $\langle g, g^h \rangle$ is finite. So, the Theorem generalizes the Shunkov – Ostylovskiĭ – Suchkova theorem [15], [16], [17] (see Theorem 4.3.3) asserting that every Artinian group is Chernikov if it is Shunkov.

The next proposition is also an immediate consequence of the Theorem.

Corollary 2 ([15], see also Theorem 4.2.2 of [17]). *For a periodic Shunkov group G the following conditions are equivalent:*

(i) *wmin.*

(ii) *min.*

Preface the proof of the Theorem with some propositions.

Lemma 1. *Let G be a periodic nonabelian group such that $\langle g, g^h\rangle$ is finite whenever g is a p-element with some prime p for any $g, h \in G$ and also $h \in C_G(g^p)$. Let T be a subgroup of $Z(G)$ and N be a normal subgroup of G. Then the following statements hold.*

(i) *G contains a finite nonabelian subgroup F.*

(ii) *In the case when G satisfies wmin-$\overline{ab}$ and N is solvable, N is Chernikov.*

(iii) *In the case when G satisfies wmin-$\overline{ab}$, N/N' is Chernikov.*

(iv) *Any subgroup K/T of the quotient group G/T generated by two of its conjugate elements of prime order is finite.*

Proof. (i) Clearly, there exists a p-element $a \in G \setminus Z(G)$ for some prime p such that $a^p \in Z(G)$. Then all subgroups $\langle a, a^h\rangle$, $h \in G$ are finite.

Suppose that all $\langle a, a^h\rangle$ are abelian. In this case $A = \langle a^h \colon h \in G\rangle$ is an abelian normal subgroup of G. Let $b \in G \setminus C_G(a)$. Then $A\langle b\rangle$ is a solvable subgroup of G. Since $A\langle b\rangle$ is periodic, by S. N. Chernikov's theorem (see, for instance, Proposition 1.1 of [5]) it is locally finite. Consequently, $\langle a, b\rangle$ is nonabelian finite.

(ii) By O. Yu. Shmidt's theorem (see, for instance, Theorem 1.45 of [18]), FN is locally finite. Then, as it is shown in the proof of Corollary 1 of [14], FN satisfies min-$\overline{ab}$. So, in view of Corollary of Theorem 1 [19], N is Chernikov if it is nonabelian, and by Lemma 1 of [19], FN and simultaneously N are Chernikov if N is abelian.

(iii) If N' is abelian then by (ii) both N and N/N' are Chernikov.

Let N' be nonabelian. Since N satisfies wmin-$\overline{ab}$, it has no infinite descending chains with infinite indices of subgroups containing N'. Consequently, N/N' satisfies wmin. Since N/N' is periodic abelian, by [1], [2] it is Chernikov.

(iv) It is easy to see that $K = \langle g, g^h\rangle T$ for some $h \in G$ and prime p and p-element $g \in G$ with $g^p \in T$. Since obviously $\langle g, g^h\rangle$ is finite, K/T is finite.

The lemma is proven.

The following proposition is extracted from Theorem 2.13 of [20].

Lemma 2. *Let $G = A \leftthreetimes \langle b\rangle$ be a group, $|\langle b\rangle| = p \in \mathbb{P}$, $C_A(\langle b\rangle) = 1$ and $|\langle b, b^a\rangle| < \infty$ for all $a \in A$. Then A is periodic and has no elements of order p and*

$$G \setminus A = (\langle b\rangle \setminus \{1\})^A. \qquad (1)$$

Proof. In view of S. N. Chernikov's Lemma, $\langle b, b^a\rangle = L \leftthreetimes \langle b\rangle$ with $L = A \cap \langle b, b^a\rangle$ and $P = (P \cap L) \leftthreetimes \langle b\rangle$ for any Sylow p-subgroup $P \supseteq \langle b\rangle$ of $\langle b, b^a\rangle$. Since $C_{P\cap L}(\langle b\rangle) = 1$, in view of Sylow's Theorem, p does not divide $|L|$ and for some $c \in L$, $\langle b^a\rangle^c = \langle b\rangle$. Then obviously $ac \in C_A(\langle b\rangle)$. So, $ac = 1$. Thus, $a = c^{-1} \in L$. At the same time, a is a p'-element. Therefore, with regard to arbitrariness of a, the first conclusion of the present lemma is valid.

Further, $(\langle b\rangle \setminus \{1\})^L \subseteq \langle b, b^a\rangle \setminus L$ and, obviously, $|(\langle b\rangle \setminus \{1\})^L| = (p-1)|L| = |\langle b, b^a\rangle \setminus L|$. So, $(\langle b\rangle \setminus \{1\})^L = \langle b, b^a\rangle \setminus L$. Therefore, for any $b^* \in \langle b\rangle \setminus \{1\}$ we have that

$$b^*a \in b^*L \subseteq \langle b, b^a\rangle \setminus L \subseteq (\langle b\rangle \setminus \{1\})^A.$$

Thus,

$$G \setminus A = \{b^*a\colon\ b^* \in \langle b\rangle \setminus \{1\},\ \ a \in A\} \subseteq (\langle b\rangle \setminus \{1\})^A.$$

Since obviously $(\langle b\rangle \setminus \{1\})^A \subseteq G \setminus A$, (1) is true.

Lemma is proven.

Recall that for a prime p, a p'-group is a periodic group which has no elements of order p.

Proposition 1. *Let $G = A \leftthreetimes \langle b\rangle$ be a group with locally graded A, $C_A(\langle b\rangle) = 1$, $|\langle b\rangle| = p \in \mathbb{P}$ and $|\langle b, b^a\rangle| < \infty$ for all $a \in A$. Then G is locally finite and A is a nilpotent p'-subgroup.*

(Moreover, A is, as it is well-known, abelian if $p = 2$, and in consequence of [21], [22] *and* [23], [24], *A is nilpotent of class ≤ 2, ≤ 6 and $\leq \frac{(p-1)^{2p-1}-1}{p-1}$ respectively when $p = 3, 5$ and $p > 5$).*

Proof. By Lemma 2, A is a p'-group. Therefore, G is periodic.

Let X be a finite set of elements of G. Obviously, $X \subseteq H = \langle S^{\langle b\rangle}\rangle \leftthreetimes \langle b\rangle$ for some finite $S \subseteq A$. Put $\langle S^{\langle b\rangle}\rangle = F$. Clearly, F and H are finitely generated.

Let M be any subgroup of finite index of F and $N = \bigcap_{h\in H} M^h$ and φ be the natural homomorphism of H onto H/N. Then $H^\varphi = F^\varphi \leftthreetimes \langle b^\varphi\rangle$, and by Poincare's theorem, H^φ is finite.

Clearly, $|(H \setminus F)^\varphi| = |F^\varphi|(|\langle b^\varphi\rangle| - 1)$. In view of Lemma 2, $H \setminus F = (\langle b\rangle \setminus \{1\})^F$. Therefore, obviously, $(H \setminus F)^\varphi = (\langle b^\varphi\rangle \setminus \{1\})^{F^\varphi}$. Also it is clear that

$$|(\langle b^\varphi\rangle \setminus \{1\})^{F^\varphi}| = |F^\varphi : C_{F^\varphi}(\langle b^\varphi\rangle)|(|\langle b^\varphi\rangle| - 1).$$

Thus,

$$|F^\varphi|(|\langle b^\varphi\rangle| - 1) = |F^\varphi : C_{F^\varphi}(\langle b^\varphi\rangle)|(|\langle b^\varphi\rangle| - 1).$$

So, $|F^{\varphi}| = |F^{\varphi} : C_{F^{\varphi}}(\langle b^{\varphi}\rangle)|$. Consequently, $C_{F^{\varphi}}(\langle b^{\varphi}\rangle) = 1$. Therefore, by Thompson's theorem [25], F^{φ} is nilpotent, and in view of Higman's theorem [22], the nilpotent class of F^{φ} does not exceed some $k(p) \in \mathbb{N}$ (which depends on p only).

Consequently, with regard to arbitrariness of M, $F/J(F)$ is nilpotent of class $\leq k(p)$. Since $F/J(F)$ is also periodic finitely generated, by S. N. Chernikov's theorem (see, for instance, Proposition 1.1 [5]), it is finite. Therefore $J(F)$ has no subgroups of finite nonidentity index. Further, in view of Schreier's theorem, $J(F)$ is finitely generated. So, $J(F) = 1$, because $J(F)$ is locally graded. Thus, F is finite and nilpotent of class $\leq k(p)$. Simultaneously, H is finite. So, $\langle X\rangle$ is finite, and in the case when $X \subseteq A$, $\langle X\rangle$ is nilpotent of class $\leq k(p)$.

Consequently, by arbitrariness of X, the present proposition is true.

The proposition is proven.

Proposition 2. *Let G be a residually finite group with min-ab such that for every $g, h \in G$ the subgroup $\langle g, g^h\rangle$ is finite whenever g is of prime order. Then G is finite.*

Proof. Obviously, G is periodic. We may assume that $G \neq 1$.

Obviously, G contains an abelian subgroup A with the following properties: for any $p \in \mathbb{P}$, A has no element of order p^2; for any abelian subgroup $B \supseteq A$ of G, either $B = A$ or B has an element of order p^2 for some $p \in \mathbb{P}$. Since A satisfies min, it is finite (for instance, by Lemma 1.1 of [5]). Since G is residually finite, there exists a normal subgroup N of G such that $A \cap N = 1$ and $|G : N| < \infty$.

Let $g \in C_N(A)$. Since $A \cap C_N(A) = 1$, one has that $A\langle g\rangle = A \times \langle g\rangle$. So, $|\langle g\rangle| \notin \mathbb{P}$. Therefore, with regard to arbitrariness of g, $C_N(A) = 1$. Since $|G : N| < \infty$, we obtain that $C_G(A)$ is finite.

Suppose that G is infinite. Then A contains a subgroup B such that $C_G(B)$ is finite, and $C_G(D)$ is infinite for some maximal subgroup D of B. Obviously, for some element $b \in B$ of prime order, $B = \langle b\rangle \times D$, and for some normal subgroup M of finite index of G, $C_G(B) \cap M = 1$.

Put $L = C_M(D)$. Then L is infinite. Since $C_M(B) = 1$, $C_L(\langle b\rangle) = 1$. Then, in view of Proposition 1, L is nilpotent. Since L satisfies min-ab, by S. N. Chernikov's theorem (see, for instance, Corollary 4.2 of [5]), L is Chernikov. Since L is residually finite, it is finite, which is a contradiction.

The proposition is proven.

Proof of Theorem. S u f f i c i e n c y is reduced to the case of wmin-$\overline{ab}$.

Indeed, a periodic group with wmin satisfies wmin-$\overline{ab}$ and wmin-ab and is either nonabelian or, in view of [1], [2], Chernikov abelian.

Assume that G is not Chernikov. Since G satisfies wmin-$\overline{ab}$, it contains a non-(Chernikov or abelian) subgroup H such that any subgroup of H of infinite index is Chernikov or abelian. Without loss of generality, we may assume that $G = H$. Then any subgroup of G of infinite index is Chernikov or abelian.

(a) Show that G satisfies min-ab and wmin.

Suppose that G does not satisfy min-ab. Then some maximal abelian subgroup A of G is non-Chernikov.

If $N_G(A) \neq A$ then $N_G(A)$ is a nonabelian group with wmin-$\overline{ab}$. But so, in view of Lemma 1 (ii), A is Chernikov, which is a contradiction.

Further, if $|G : A|$ is finite then in consequence of Poincare's theorem, $|G : \bigcap\limits_{x\in G} A^x|$ is finite. Then by Lemma 1 of [19], $\bigcap\limits_{x\in G} A^x$ is Chernikov. But so is A, a contradiction.

Thus,

$$N_G(A) = A \tag{2}$$

and $|G : A|$ is infinite.

Further, for any $g \in G \setminus A$, $\langle A, A^g\rangle$ is non-Chernikov and, with regard to (2), nonabelian. Therefore, $|G : \langle A, A^g\rangle| < \infty$.

Let $a \in A \cap A^g$. Then $\langle A, A^g\rangle \subseteq C_G(a)$. So, $|G : C_G(a)| < \infty$, i. e. a^G is finite. Therefore, by Dietzmann's lemma (see, for instance, [18]), $\langle a^G\rangle$ is finite.

Let $u \in G \setminus A$. Since $A^u\langle a^G\rangle$ is not Chernikov and $|G : A^u\langle a^G\rangle|$ is obviously infinite, $A^u\langle a^G\rangle$ is abelian. Therefore, since A^u is a maximal abelian subgroup of G, we obtain that $a \in A^u$. So, $A \cap A^g \subseteq A \cap A^u$.

Thus, for every $g, u \in G \setminus A$, $A \cap A^g = A \cap A^u$. Hence, with regard to (2), it easily follows that for $R = \langle A^u \colon u \in G\rangle$, $Z(R) \subset A$ and

$$A/Z(R) \cap (A/Z(R))^x = 1, \ \ \forall x \in G/Z(R) \setminus A/Z(R). \tag{3}$$

Since R is normal in G, $Z(R)$ is normal in it too. Therefore, by Lemma 1 (ii), Z(R) is Chernikov. Since A is not Chernikov, by S. N. Chernikov's theorem (see, for instance, Theorem 1.4 of [5]), $A/Z(R)$ is non-Chernikov.

Suppose that $A/Z(R)$ is not a 2-group. Then it contains an element v of an odd prime order. In view of Lemma 1 (iv), $\langle v, v^x\rangle$ is finite for any $x \in R/Z(R)$. Therefore, with regard to (3), by the Shunkov – Sozutov –

theorem [26], for some normal subgroup $N/Z(R)$ of $R/Z(R)$ we have that

$$R/Z(R) = (A/Z(R))(N/Z(R)) \text{ and } A/Z(R) \cap N/Z(R) = 1.$$

Consequently, $N \trianglelefteq R = AN$ and $A \cap N = Z(R)$. Therefore, $R/N \simeq A/Z(R)$. So, R/N is not Chernikov, which is a contradiction (see Lemma 1 (iii)). Thus, $A/Z(R)$ is a 2-group.

Since $A/Z(R)$ is a non-Chernikov abelian 2-group, it contains some distinct elements b and c of order 2. Let $h \in R/Z(R) \setminus A/Z(R)$. Then

$$\langle b, c^h \rangle = \langle bc^h \rangle \leftthreetimes \langle b \rangle = \langle bc^h \rangle \leftthreetimes \langle c^h \rangle. \tag{4}$$

Assume that bc^h is of odd order. Then, with regard to (4), by Sylow's theorem, $b = c^{hs}$ for some $s \in \langle bc^h \rangle$. So, since $A/Z(R)$ is abelian and $b, c \in A/Z(R)$, we obtain that $hs \notin A/Z(R)$. Consequently,

$$A/Z(R) \cap (A/Z(R))^{hs} = 1$$

(see (3)). But $1 \neq b \in A/Z(R) \cap (A/Z(R))^{hs}$, which is a contradiction. Thus, $\langle bc^h \rangle$ contains some element w of order 2.

Clearly, $w \in C_{G/Z(R)}(b) \cap C_{G/Z(R)}(c^h)$. But relations (3) and $(A/Z(R))' = 1$ imply that $C_{G/Z(R)}(b) = A/Z(R)$ and $C_{G/Z(R)}(c^h) = (A/Z(R))^h$. So, $1 \neq w \in A/Z(R) \cap (A/Z(R))^h$, which is a contradiction.

Thus, G satisfies min-ab. Since also G satisfies wmin-$\overline{ab}$, it satisfies wmin.

(b) Show that

$$|G : J(G)| < \infty. \tag{5}$$

Suppose that $|G : J(G)|$ is infinite. Then $J(G)$ is Chernikov or abelian. By Lemma 1 (ii), in the second case $J(G)$ is Chernikov too. Thus, $J(G)$ is anyway Chernikov.

Besides, the Baer – Polovičkiĭ – S. N. Chernikov theorem [27] – [29] states that every periodic group of automorphisms of a Chernikov group is Chernikov. In view of this theorem, $G/C_G(J(G))$ is Chernikov. By Lemma 1 (ii), $Z(J(G))$ is also Chernikov. Consequently, by Theorem 1.4 of [5], $C_G(J(G))/Z(J(G))$ is non-Chernikov. Since $G/J(G)$ is residually finite and

$$C_G(J(G))/Z(J(G)) = C_G(J(G))/(C_G(J(G)) \cap J(G)) \simeq$$

$$\simeq J(G)C_G(J(G))/J(G),$$

$C_G(J(G))/Z(J(G))$ is residually finite. Since G is periodic and satisfies wmin (see (a)), $C_G(J(G))/Z(J(G))$ is also periodic and satisfies wmin.

Therefore, in view of [1], [2], this quotient group satisfies min-ab. Further, by virtue of Lemma 1 (iv), any two conjugate elements of prime order of $C_G(J(G))/Z(J(G))$ generate finite subgroups. Consequently, by Proposition 2, $C_G(J(G))/Z(J(G))$ is finite, which is a contradiction.

(c) Show that $J(G)$ is Artinian and Shunkov.

Clearly, with regard to (5), $J(G)$ has no proper subgroups of finite index. So, any proper subgroup of $J(G)$ is Chernikov or abelian with min (see (a)). Therefore $J(G)$ is Artinian.

Let F be a finite subgroup of $J(G)$. If $F \subseteq Z(J(G))$, then by Lemma 1 (iv), any two conjugate elements of prime order of $N_{J(G)}(F) = J(G)/F$ generate a finite subgroup.

Suppose that $F \nsubseteq Z(J(G))$. Then $|J(G) : C_{J(G)}(F)|$ is infinite. Since F is finite, $|N_{J(G)}(F) : C_{J(G)}(F)|$ is finite. Consequently, $|J(G) : N_{J(G)}(F)|$ is infinite. Therefore $N_{J(G)}(F)$ is Chernikov or abelian. At the same time, $N_{J(G)}(F)/F$ is also Chernikov or abelian. Thus, any two conjugate elements of prime order of $N_{J(G)}(F)/F$ generate a finite subgroup.

Thus, $J(G)$ is Shunkov.

(d) Final contradiction.

In view of Shunkov-Ostylovskii-Suchkova Theorem mentioned above, with regard to (c), $J(G)$ is Chernikov. So because of $|G : J(G)| < \infty$ (see (b)), G is Chernikov, with is a contradiction.

Necessity is obvious.

Theorem is proven.

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ГРУПИ ЗІ СЛАБКОЮ УМОВОЮ МІНІМАЛЬНОСТІ ДЛЯ НЕАБЕЛЕВИХ ПІДГРУП

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У даній роботі автор встановлює: неабелева періодична група зі слабкою умовою мінімальності для неабелевих підгруп є черніковською, якщо для простого числа p та її p-елемента g і її елемента h, таких, що $[g^p, h] = 1$, $\langle g, g^h \rangle$ скінченна.