УДК 512.64+512.56

V. M. Bondarenko[1], Yu. M. Orlovskaja[2], M. V. Styopochkina[3]

ON HASSE DIAGRAMS CONNECTED WITH THE 1-OVERSUPERCRITICAL POSET (1, 3, 5)

We study combinatorial properties of Hasse diagrams of posets minimax equivalent to the smallest oversupercritical poset with trivial group of automorphisms.

Introduction. M. M. Kleiner [6] proved that a poset S has finite representation type if and only if it does not contain the subsets of the form $(1,1,1,1), (2,2,2), (1,3,3), (1,2,5)$ and $(N,4)$, which are called the critical posets. By (P,Q) is denoted the poset which is the direct sum of P and Q; by $(i_1, i_2, ..., i_p)$ is denoted the poset wich is the direct sum of chains of length $i_1, i_2, ..., i_p$. In [2] it is proved that a poset is critical with respect to the positivity of the Tits quadratic form if and only if it is minimax equivalent to a critical poset (the notion of minimax equivalence was introduced in [1]); in [2] all such posets was completely described.

A similar situation occurs for the tame posets. L. A. Nazarova [7] proved that a poset S is tame if and only if it does not contain subsets of the form $(1,1,1,1,1), (1,1,1,2), (2,2,3), (1,3,4), (1,2,6)$ and $(N,5)$; these sets are called supercritical. In [3] it is proved that a poset is critical with respect to the non-negativity of the Tits form if and only if it is minimax equivalent to a supercritical poset; all such critical posets was described in [4].

The posets that are differ from the supercritical sets in the way as the supercritical sets differ from critical posets are called 1-supersupcritical. More precisely (see [5]), it is the following posets: 1) $(1,1,1,1,1,1)$, 2) $(1,1,1,1,2)$, 3) $(1,1,2,2)$, 4) $(1,1,1,3)$, 5) $(2,3,3)$, 6) $(2,2,4)$, 7) $(1,4,4)$, 8) $(1,3,5)$, 9) $(1,2,7)$, 10) $(6,N)$.

We study combinatorial properties of the posets which are minimax equalently to the smallest 1-oversupercritical poset with trivial group of automorphisms, i. e. the poset $(1,3,5)$.

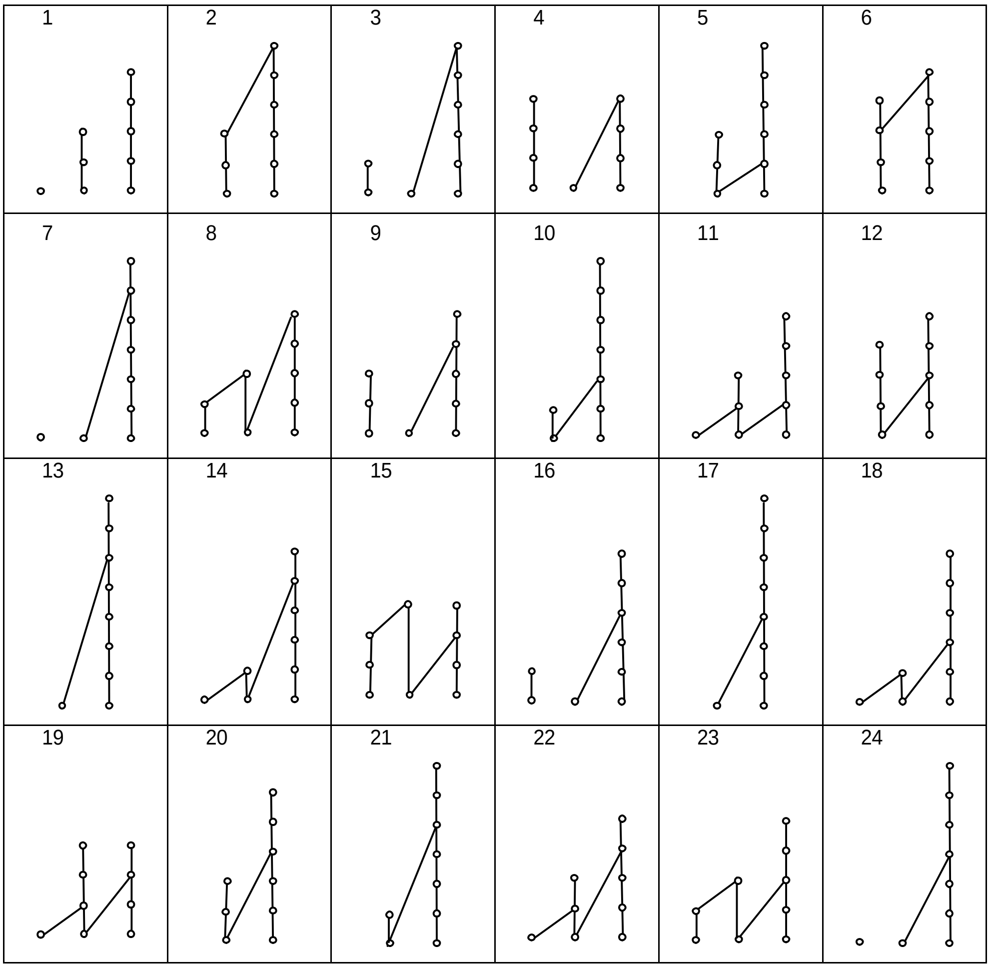
Formulation of the main theorems. Let S be a finite poset. The Hasse diagram of S is by definition the directed graph $H(S)$ with the vertices $x \in S$ and the arrows (x, y), $x, y \in S$, where y covers x (i. e. $x < y$ and there is no z satisfying $x < z < y$). We call the 0-length of an oriented path of $H(S)$ the number of its vertices, and denote by $l_{\min}(S)$ (respectively, $l_{\max}(S)$) the 0-length of a most short (respectively, long) oriented path of $H(S)$. We denote by [S]~ the set of all posets minimax equivalent to S (see [1]) and put

$$L_{\min}(S) = \min_{X \in [S]\sim} l_{\min}(X), \qquad L_{\max}(S) = \max_{X \in [S]\sim} l_{\max}(X).$$

Theorem 1. *For $S=(1,3,5)$, $L_{\min}(S) = 1$, $L_{\max}(S) = 8$.*

Theorem 2. *Let $X \in \left[(1, 3, 5)\right]\sim$. Then $1 \le l_{\min}(X) \le 6$, $4 \le l_{\max}(X) \le 8$, $n(X) \in \{2, 3, 4\}$.*

Proofs of the theorems. The posets minimax equivalent to $(1,3,5)$ were classified in [5]. They are given (up to isomorphism and anti-isomorphism) by the following table.



We have the next theorem.

Theorem 3. *The following holds for posets* 1–24:

N	$l_{\min}$	$l_{\max}$	n	N	$l_{\min}$	$l_{\max}$	n	N	$l_{\min}$	$l_{\max}$	n
1	1	5	3	9	3	5	3	17	6	8	2
2	4	6	2	10	2	7	3	18	2	6	4
3	2	6	3	11	3	5	4	19	3	4	4
4	2	4	3	12	4	5	3	20	3	6	3
5	3	6	3	13	4	8	2	21	2	7	3
6	4	5	3	14	2	6	4	22	3	5	4
7	1	7	3	15	2	4	4	23	2	5	4
8	2	5	4	16	2	6	3	24	1	7	3

Here the numbers $l_{\min}$, $l_{\max}$ and \$ denote, respectively, $l_{\min}(S_i)$, $l_{\max}(S_i)$ and $n(S_i)$ for $i = N$, where N runs from 1 to 24 and S_i is the i th poset in the table.

The proof is carried out by direct calculations.

Theorems 1 and 2 follow from Theorem 3.

1. *Bondarenko V. M.* On (min, max)-equivalence of posets and applications to the Tits forms // Bull of the University of Kiev (series: Physics & Mathematics). – 2005. – № 1. – P. 24–25.
2. *Bondarenko V. M., Stepochkina M. V.* (Min, max)-equivalence of partially ordered sets and the Tits quadratic form // Zb. Pr. Inst. Mat. NAN Ukr./ Problems of Analysis and Algebra /. – 2005. – 2, № 3. – P. 18–58 (in Russian).
3. *Bondarenko V. M., Stepochkina M. V.* (Min-, max)-equivalency of posets and non-negative Tits forms // Ukrainian Math. J. – 2008. – 60, № 9. – P. 1349–1359.
4. *Bondarenko V. M., Stepochkina M. V.* Description of posets critical with respect to the nonnegativity of the quadratic Tits form // Ukrainian Math. J. – 2009. – 61, № 5. – P. 734–746.
5. *Bondarenko V. V., Bondarenko V. M., Stepochkina M. V., Chervyakov I. V.* 1-over-supercritical partially ordered sets with trivial group of automorphisms and min-equivalence. I. // Nauk. Visn. Uzhgorod. Univ., Ser. Mat. – 2011. – 22, № 2. – P. 17–25 (in Russian).
6. *Kleiner M. M.* Partially ordered sets of finite type // Zap. Nauch. Semin. LOMI. – 1972. – 28. – P. 32–41 (in Russian).
7. *Nazarova L. A.* Partially ordered sets of infinite type // Izv. Akad. Nauk SSSR Ser. Mat. – 1975. – 39, № 5. – P. 963–991 (in Russian).

ПРО ДІАГРАМИ ХАССЕ, ПОВ'ЯЗАНІ З 1-НАДСУПЕРКРИТИЧНОЮ ЧАСТКОВО ВПОРЯДКОВАНОЮ МНОЖИНОЮ (1, 3, 5)

Вивчали комбінаторні властивості діаграм Хассе частково впорядкованих множин, мінімаксно еквівалентних найменшій надсуперкритичній частково впорядкованій множині з тривіальною групою автоморфізмів.

О ДИАГРАММАХ ХАССЕ, СВЯЗАННЫХ С 1-НАДСУПЕРКРИТИЧЕСКИМ ЧАСТИЧНО УПОРЯДОЧЕННЫМ МНОЖЕСТВОМ (1, 3, 5)

Изучали комбинаторные свойства диаграмм Хассе частично упорядоченных множеств, минимаксно эквивалентных наименьшему надсуперкритическому частично упорядоченному множеству с тривиальной группой автоморфизмов.

[1]Institute of Mathematics of NAN of Ukraine, Kyiv
[2]Koroliov Military Institute of Zhytomyr, Zhytomyr
[3]Zhytomyr National University of Agriculture and Ecology, Zhytomyr

Obtained
01.10.18